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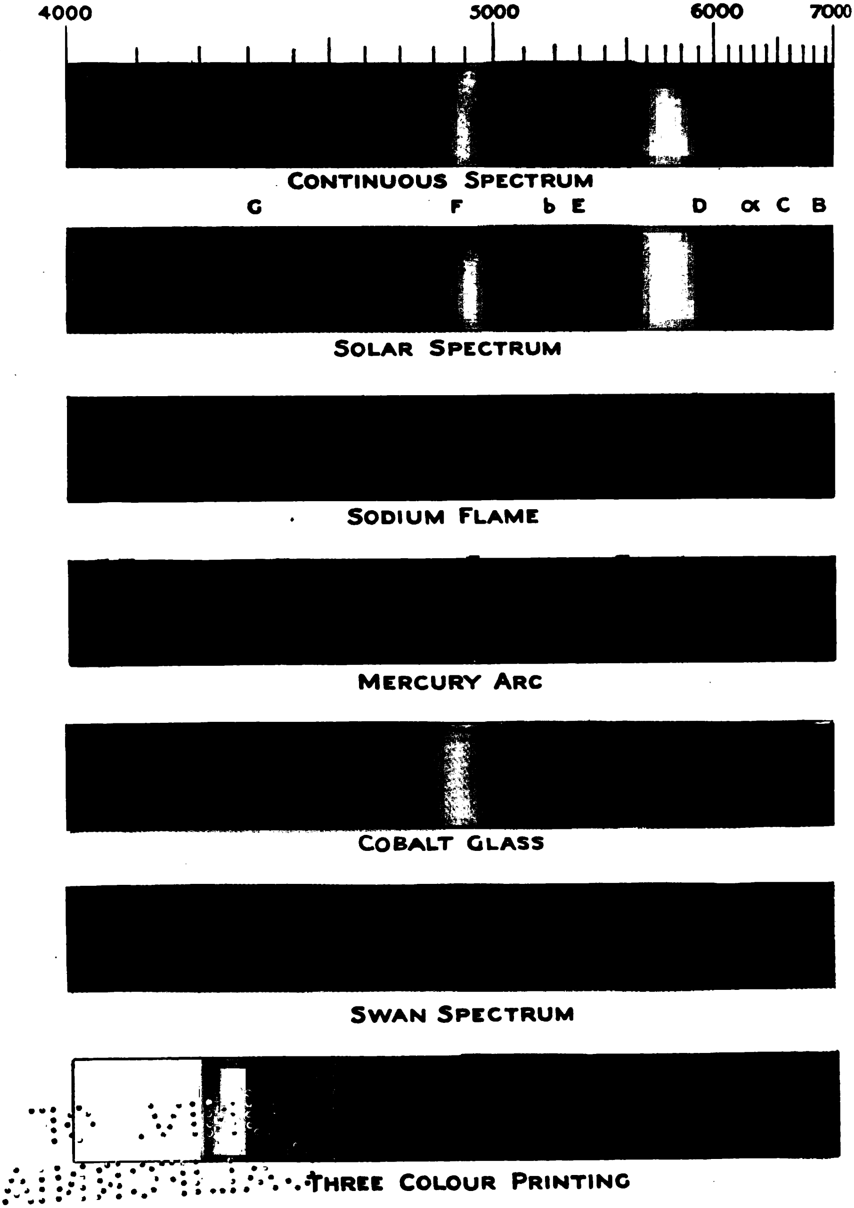

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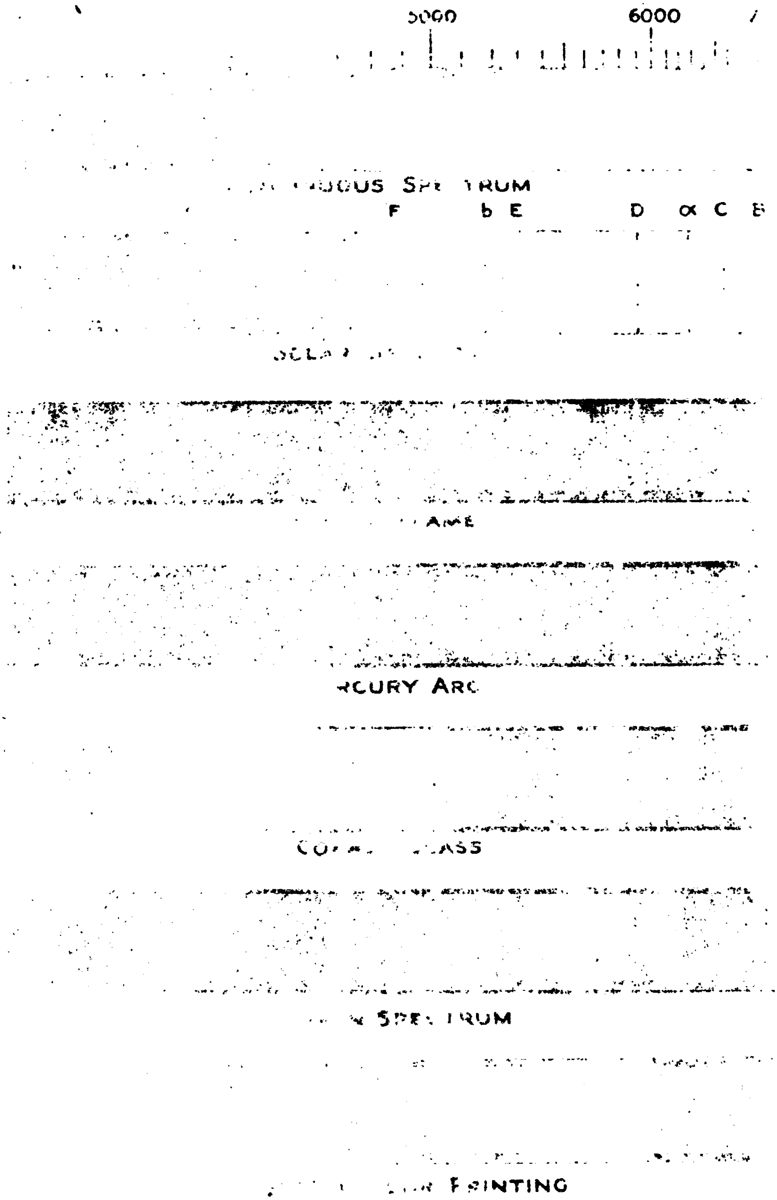
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BY

R. A. HOUSTOUN, M.A., PH.D., D.SC.

LECTURER ON PHYSICAL OPTICS IN THE UNIVERSITY OF GLASGOW

WITH A COLOURED SPECTRUM PLATE AND 202 FIGURES

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~~Physics Dept~~

PREFACE

THIS book is intended as a text-book for schools and for beginners generally. The work covered is approximately that required for the First Professional Examination in Medicine. A slight knowledge of trigonometry is assumed on the part of the student.

While there are many systematic books on the subject much larger than the present one, and also many elementary books on practical physics, there is no systematic treatment of the subject of exactly the same range. Consequently I hope the book may fill a want, especially since the report of the recent Royal Commission on the Position of Natural Science in the Educational System of Great Britain makes it probable that physics will in future be taught more from a systematic standpoint and less by the laboratory notebook method.

Mr. Alex. H. Gray, M.A., B.Sc., has read all the proof-sheets, and has worked all the examples. Some of the figures have been taken from Messrs. Longmans, Green and Co.'s other publications, and I am indebted to Messrs. Baird & Tatlock (London), Ltd., John J. Griffin & Sons, Ltd., and W. Butcher & Sons, Ltd., for the loan of trade blocks, but the greater number of the figures have been drawn specially for the book.

The book is the result of experience gained in assisting Prof. A. Gray, F.R.S., at Glasgow University, and I am indebted to Prof. Gray for many of the methods employed. Prof. James Muir, D.Sc., and Dr. G. E. Allan read part of the manuscript and the proofs, and I am indebted to them for many valuable suggestions.

R. A. HOUSTOUN.

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ELEMENTS OF PHYSICS

CHAPTER I

DYNAMICS

1. UNITS AND MEASUREMENT

PHYSICS is an exact science, and it deals with the relations between the measurements of different quantities. Now before we measure any quantity we must have a unit, in which to measure it. We must therefore begin our study of the subject by describing the units employed in it. They are of two kinds, fundamental units and derived units. The fundamental units are those of length, mass, and time; as examples of derived units we may take the unit of volume, which can be expressed in terms of the unit of length, or we may take the unit of velocity, which can be expressed in terms of the unit of length and the unit of time.

There are two systems of fundamental units established by law in this country, the foot-pound-second system, sometimes referred to as the British system, and the centimetre-gram-second system or metric system, which is usually referred to for short as the c.g.s. system.

The standard unit of length on the ft.-lb.-sec. system is the yard, which is defined by Act of Parliament as follows: "The straight line or distance between the centres of the transverse lines in the two gold plugs in the bronze bar deposited in the Office of the Exchequer shall be the genuine standard at 62° F., and if lost it shall be replaced by means of its copies." The foot is, of course, $\frac{1}{3}$ of the yard. The standard unit of mass on the ft.-lb.-sec. system is the pound; this is defined as the mass of a certain piece of platinum, marked "P.S., 1844, 1 lb.," which is kept at the same place as the standard yard.

The yard and pound are arbitrary units, that is, they are not intended to be the length or mass of any object existing in nature; they are the descendants of units employed in ancient times. The c.g.s. system, on the contrary, was a new system devised by a scientific committee appointed in France after the great revolution. The standard unit of length on this system, the metre, was based on the earth's dimensions, and was intended to be one ten-millionth part of the distance from the North Pole to the Equator measured along the meridian passing through Paris. A rod of platinum was made by Borda which should fulfil this condition at 0° C., and as it would be inconvenient to alter the standard of length slightly every time a more accurate determination of the length of the earth's meridian was made, the metre is defined, not as the ten-millionth part of the quadrant of the meridian, but as the distance between the ends of Borda's rod at 0° C. The kilogram was originally intended by Borda to be the mass of one cubic decimetre of water at 4° C., the decimetre being $\frac{1}{10}$ of the metre. It does not, however, fulfil this condition exactly, and is defined as the mass of a certain lump of platinum, preserved at Paris and called the "Kilogram des Archives."

The advantage of the metric system over the ft.-lb.-sec. system is that all the multiples and submultiples of its standard units are powers of 10; thus the centimetre is $\frac{1}{100}$ of the metre, the millimetre $\frac{1}{1000}$ of the metre, and the kilometre, the unit on the metric system corresponding to the mile, is 1000 metres, while the kilogram is 1000 grams. Length and mass can be easily changed from the one system to the other if we remember two relations, namely, that

1 foot = 30·48 centimetres
1 lb. = 453·6 grams

The unit of volume on the c.g.s. system is the litre, which is one cubic decimetre or 1000 cubic centimetres. Owing to the ease with which calculations can be made by the metric system it has been adopted by most civilized countries, and is used almost exclusively for scientific work.

The unit of time on both systems is the mean solar second, which is defined as the one 86,400th part of the mean solar day. Owing to different causes the time between successive transits of the sun across the meridian, that is, the interval between the times at which the sun is highest in the heavens on two successive days, is not constant throughout the year. The mean solar day is the average value of this interval.

The Vernier.—We shall now describe certain measuring instruments, namely, the vernier, the micrometer screw gauge, and the spherometer.

If an object is being measured by a scale divided into centimetres and millimetres, and its end comes between two divisions, it is possible to estimate the length to about a quarter of a millimetre. The eye does this by mentally subdividing the millimetre into quarters and estimating by how many of these imaginary quarters the end of the object exceeds the nearest whole division. If we wish, however, to have the result correct to $\frac{1}{20}$ of a millimetre, it is difficult to obtain this degree of accuracy by judgment and the eye alone, and it is usual to employ an auxiliary scale, which slides along the edge of the scale and is called a vernier, after the name of its inventor, a seventeenth-century French mathematician.

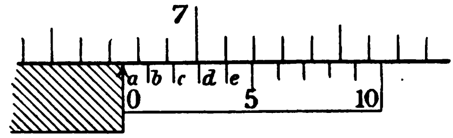

FIG. 1.

The principle of the vernier will be made clear by consideration of Fig. 1, which represents a portion of a centimetre scale considerably magnified. AB is the object to be measured, and we see that its end B lies between 6·7 and 6·8 cm. Let v = length of a vernier division and s = length of the smallest scale division. Then it will be found by inspection that $9s = 10v$ or $v = \frac{9}{10}s$. That is, $v = s - \frac{1}{10}s$, and the difference in length of a vernier division and scale division, or "least count" as it is called, is equal to $\frac{1}{10}$ scale division or $\frac{1}{10}$ mm.

Now look along the vernier to see which division on it is most nearly opposite to a division on the scale. It is the fourth, the one marked *e* in the diagram. If *e* coincides with a scale division, *d* is consequently $\frac{1}{10}$ mm. to the right of a scale division, *c* $\frac{2}{10}$ mm. to the right of a scale division, *b* $\frac{3}{10}$ mm. to the right of the corresponding scale division, and *a* $\frac{4}{10}$ mm. to the right of the corresponding scale division. Thus B exceeds the 7 mm. mark by $\frac{4}{10}$ mm., and the correct reading of the scale is 6·74 cms. The rule, therefore, in using this vernier is simply to look for the point where coincidence occurs; if it is at the fourth division on the vernier, the fraction is $\frac{4}{10}$ mm. If it is at the seventh division on the vernier, the fraction is $\frac{7}{10}$ mm.

The vernier represented in Fig. 1 is a particularly simple one. More generally if n vernier divisions equal $n - 1$ scale divisions

$$nv = (n - 1)s \quad \text{and} \quad v = s - \frac{1}{n}s$$

Thus the least count is s/n, and if coincidence occurs at the mth division the fraction is $(ms)/n$. The method of reading a strange vernier may always be made out by inspecting it, and the first few readings should always be checked by estimating the fraction by the eye.

Sometimes, but very rarely, $nv = (n + 1)s$ and the vernier division is larger than the scale division. This is the form which was given to the vernier by its inventor. It has the advantage of a "more open scale," *i.e.* the divisions are further apart than in the other form and consequently easier to read, but this is outweighed by the disadvantage that the numbers on the vernier run the opposite way to the numbers on the scale itself.

The Micrometer Screw Gauge.—The micrometer screw gauge, which is shown in Fig. 2, is an instrument used for measuring small thicknesses, for example, the thickness of a metal plate. The object to be measured is inserted in the gap at the end of the screw. The latter is moved in and out by turning its head. As the screw moves out, it uncovers a scale. When the end of the screw is pressed against the piece opposite

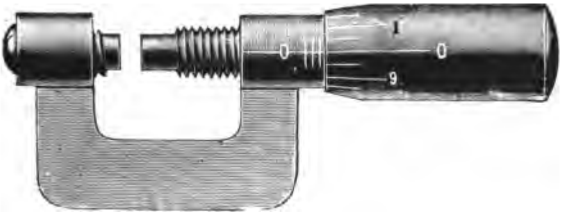

FIG. 2.

Baird & Tatlock (London).

to it, the reading on the scale is zero. In the form of screw gauge most commonly used in schools the pitch of the screw is $\frac{1}{2}$ mm., and the head of the screw is divided into 50 equal parts, enabling fractions of a rotation to be read. Thus for every complete turn of the screw the end moves out one half-millimetre. As each division on the "drum-head" corresponds to a movement of the end of the screw through a distance of $\frac{1}{50} \times \frac{1}{2}$ mm., the instrument obviously reads to $\frac{1}{100}$ mm.

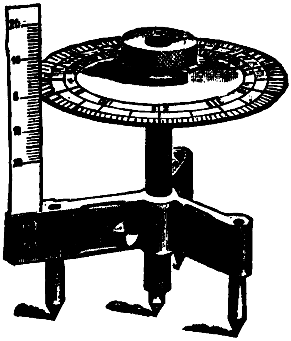

FIG. 3.

Baird & Tatlock (London).

The Spherometer.—The spherometer is founded on the same principle as the screw gauge. The usual construction of the instrument is shown in Fig. 3. It rests on three fixed legs, the feet of which are at the same distance l from one another. These three legs support a framework in which a micrometer screw turns. The micrometer screw is equidistant from the three fixed legs. The pitch of the screw is usually $\frac{1}{2}$ mm. The number of half-millimetres through which the screw is raised is given by the vertical scale, and the fraction of a half-millimetre is given by the circular scale.

The spherometer is used for determining the radius of curvature of curved surfaces such as the surfaces of lenses. Let us suppose that the surface to be measured is a concave one. The instrument is first placed upon a plane surface, usually the surface of a glass plate, and the micrometer point screwed down until it just touches the plane. The four feet of the instrument are then all in the same plane. The reading of the circle and of the scale is then taken. The instrument is next placed upon the curved surface and the micrometer point screwed down until it makes contact again. The reading of the circle and of the scale is then taken again. Let d be the vertical distance traversed by the point of the screw. Then the radius of curvature is given by

$$r = \frac{l^2}{6d} + \frac{d}{2}$$

It is sometimes difficult to decide when the micrometer screw is just making contact. If it goes too far, of course, the instrument rocks, but this is a clumsy test. If the surface is a polished one, as contact is approaching, an image of the point in the surface can be seen approaching the point itself, and at contact point and image touch. If the screw is turned very steadily and carefully indeed, there is a slight jar at contact, and the whole instrument may swing round on the screw as a pivot. This is probably the best test.

In order to prove the formula consider Fig. 4. Let A, B, and C be the points of contact of the fixed feet, and D the point of contact of the screw on the curved surface. Let G be its position when it is in the same plane as the fixed feet, and let O be the centre of curvature of the spherical surface. Then GD $= d$, AB $=$ BC $=$ CA $= l$, and OD $= r$.

Join CG. Then

$$r^2 = \mathrm{OC}^2 = \mathrm{OG}^2 + \mathrm{GC}^2 = (r - d)^2 + \mathrm{GC}^2$$

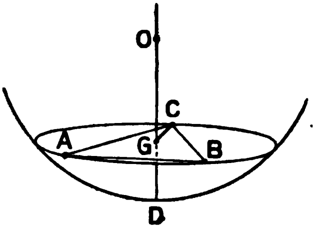

FIG. 4.

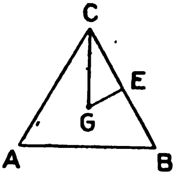

FIG. 5.

Draw GE perpendicular to CB. Then it is clear from Fig. 5 that

$$\mathrm{CG} = \frac{\mathrm{CE}}{\cos 30^\circ} = \frac{l}{\sqrt{3}}$$

Hence $$r^2 = (r - d)^2 + \frac{l^2}{3} = r^2 - 2rd + d^2 + \frac{l^2}{3}$$

i.e. $$2rd = \frac{l^2}{3} + d^2 \quad \text{or} \quad r = \frac{l^2}{6d} + \frac{d}{2}$$

the required formula. The second term in the formula may usually be neglected in comparison with the first one.

EXAMPLES I

1. Change a density of 50 lbs./ft.^{3} into lbs./inch^3 and gms./c.c.

$$\frac{50 \text{ lbs.}}{\text{ft.}^3} = \frac{50 \text{ lbs.}}{(12 \text{ inch})^3} = \frac{50 \text{ lbs.}}{1728 \text{ inch}^3} = \cdot 02894 \text{ lb./inch}^3$$

$$\frac{50 \text{ lbs.}}{\text{ft.}^3} = \frac{50 \times 453{\cdot}6 \text{ gms.}}{(30{\cdot}48 \text{ cms.})^3} = \frac{50 \times 453{\cdot}6 \text{ gms.}}{30{\cdot}48^3 \text{ cms.}^3} = \cdot 8011 \text{ gm./c.c.}$$

2. Change a velocity of 60 mls./hr. into ft./sec., kms./hr. and metres/sec.

3. A piece of copper wire 62 cms. long weighs 4·05 gms. What is its weight in kgs./km. and lbs./ml.?

4. A piece of lead sheeting measuring 5·0 cms. by 1·6 cms. weighs 14·02 gms. What is its weight in kgs./metre^2 and lbs./yd^2.?

5. A spherometer is used to measure the curvature of a surface of radius r. The vertical distance screwed through between contact on the curved surface and contact on a plane surface is d, and a is the horizontal distance between the movable point and any one of the fixed points. Show that

$$r = \frac{a^2}{2d} + \frac{d}{2}$$

6. A spherometer is used to measure the curvature of a cylindrical surface, *e.g.* the outside surface of a brass tube. When contact is made on the cylindrical surface, the line joining two of the fixed points is kept parallel to the axis of the cylinder. Will the ordinary formula give the correct result?

2. THE PARALLELOGRAM OF FORCES

Fig. 6 represents a drawing board which is supported vertically by a wooden frame. Round wooden pegs are attached to the drawing board at D and E. The upper ends of two spiral springs B and C are attached to these pegs by means of loops of thread, and to the lower ends of these springs are tied two pieces of thread which have each a loop at their lower ends at A. Both these loops are passed over a hook which carries a pan for holding weights. When weights are placed in the pan both the springs are pulled out. Now at the point A three forces meet, one due to the weights in the pan and the weight of the pan itself; this force acts vertically downwards through A. The other two forces act upwards, one along the thread AB and the other along the thread AC. A piece of drawing paper is pinned on the board behind the threads. The purpose of the apparatus is to determine the relations between these three forces, when the threads AB and AC are inclined at different angles to one another and to the vertical. The springs, threads, and scale pan hang clear of the drawing board.[1]

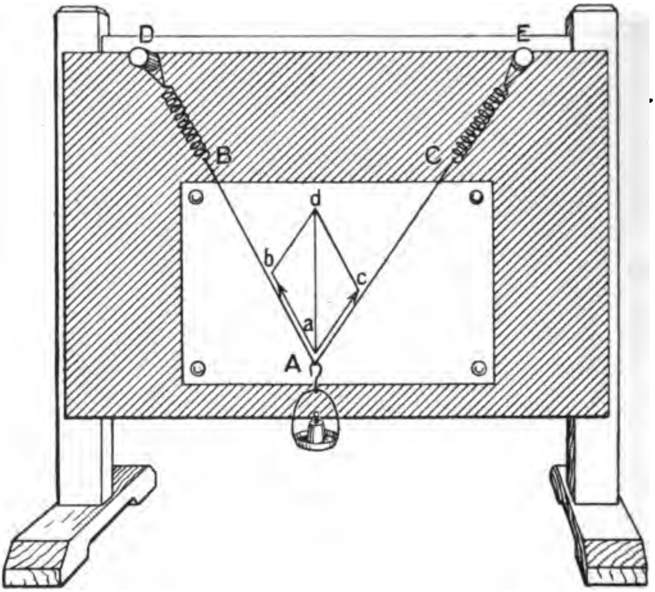

Fig. 6.

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[1] It should be noted that the springs B and C in Fig. 6 cannot be replaced by spring balances; owing to friction the latter do not give satisfactory readings when inclined to the vertical. Also it is not satisfactory to dispense with springs and pass the threads AB and AC over pulleys, attaching scale pans to their other ends, because owing to friction in the pulley the force in a thread is never the same on both sides of it.

The use of the apparatus will be understood best from the description of a particular case. The weight of the pan was 9·2 gms.; in it was placed a 200-gm. weight, and thus the total downward force was 209·2 gms. The springs were originally each about 7·85 cms. long. When the weight was placed in the pan, they were elongated and their lengths were measured very carefully with a centimetre rule. It was found then that the length of the left-hand one had become 12·7 cms., and the length of the right-hand one 11·75 cms. Pencil marks were made on the paper behind the two threads so as to get their line, and a thread carrying a weight was held up in front of the paper, and pencil marks made on it giving the line of the vertical. The scale pan was then unhooked from the thread AC, so that it hung on the thread AB vertically below D, and weights were placed in it until the same extension was produced as was obtained during the experiment The necessary weight, including the pan itself, was 129·2 gms. Similarly the weight necessary to produce the same extension in the spring CE, as was obtained in the experiment, was found to be 109·2 gms.

Now a force is completely defined when its magnitude, its direction, and the point at which it acts are given. Since the three characteristics of a straight line are its length, its direction, and the point at which it starts, it is clearly possible to represent forces by means of straight lines. The paper was unpinned from the drawing board, and the traces *ab*, *ac* of the threads AB and AC drawn. For the sake of clearness *ab* and *ac* are shown a little out of position in the figure. The force along AB was 129·2 gms.; *ab* was made exactly 129·2 mm. long. The force along AC was 109·2 gms.; *ac* was made 109·2 mm. long. Thus, by taking one millimetre to represent one gram, *ab* and *ac* represented the forces applied by the springs in magnitude, direction, and point of application.

The parallelogram was then completed and the diagonal *ad* joined. It was found to coincide very nearly with the trace of the vertical through *a*, the difference in the directions of the two lines being about half a degree. Its length was measured and found to be 208·8 mm. Now the downward force exerted by the weights in the pan was 209·2 gms., and 209·2 coincides with 208·8 within the error of experiment. Thus the diagonal of the parallelogram represents the force exerted by the weights and pan at A in magnitude and direction. If the experiment is repeated with other weights in the pan and with different inclinations of the threads to the vertical, the result is always found to be the same; if the sides of the parallelogram represent the forces exerted by the springs, the diagonal represents the force exerted by the weights.

The downward force at A is called the *equilibrant* of the two upward forces because it neutralizes them. The line *ad* does not represent the equilibrant in direction; it goes the wrong way. It represents a single force which produces the same effect as the two upward forces, and is called their *resultant*. We thus arrive at the proposition known as the parallelogram of forces—

> If two forces act on a point, and if lines be drawn from that point representing the forces in magnitude and direction, and a

parallelogram be constructed on these lines as sides, their resultant will be represented in magnitude and direction by that diagonal which passes through the point.

The forces represented by the sides are called the components of the force represented by the diagonal.

The Triangle of Forces.—Let AB and AC represent two forces. Then AD represents their resultant, and DA their equilibrant. Now AC and BD are equal and parallel, since they are the opposite sides of a parallelogram. Hence the force represented by AC may be also represented by BD, and the three forces represented by AB, BD, and DA are in equilibrium. Thus

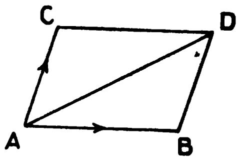


FIG. 7.

> If three forces represented in magnitude and direction by the sides of a triangle taken in order act on a particle, then the forces are in equilibrium.

This proposition is known as the triangle of forces. It merely expresses the same fact as the parallelogram of forces, but from a slightly different point of view.

The Polygon of Forces.—Let AB, BC, CD, and DE represent in magnitude and direction four forces which act on a point. Then, if we join AC, it is obvious by the triangle of forces that CA is the equilibrant and AC the resultant of AB and BC. Similarly, if we join AD, we find that AD is the resultant of AC and CD, and, if we join AE, AE is the resultant of AD and DE, that is, it is the resultant of the four forces AB, BC, CD, and DE, and EA is their equilibrant. Proceeding in this way we arrive at the proposition known as the polygon of forces, which is stated as follows:—

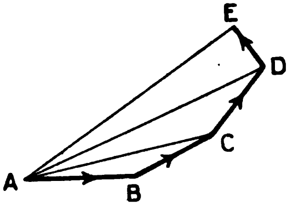


FIG. 8.

> If any number of forces acting on a particle are represented in magnitude and direction by the sides of a closed polygon taken in order, then the forces are in equilibrium. If the lines representing the forces do not form a closed polygon, the resultant will be represented by the line joining the ends of the first and last-drawn sides of the incomplete polygon.

EXAMPLES 2

1. A weight of 30 lbs. is suspended by two strings inclined to the vertical at 30° and 60°. Find the stretching force in each string.

Let the vertical line AD represent the equilibrant of the downward force, and let the lines AB and AC represent in magnitude and direction the upward forces exerted by the two strings inclined at 30° and 60° to the vertical respectively. Then BD = AC and

$$\frac{AB}{\sin 60^\circ} = \frac{BD}{\sin 30^\circ} = AD$$

The stretching force at 30° to the vertical is consequently ·30 sin 60°, *i.e.* 25·98 lbs., and the stretching force at 60° to the vertical 30 sin 30°, *i.e.* 15 lbs.

2. Two forces of 6 and 8 gms. act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

3. A 100-gm. weight is suspended by a thread. A second thread is then tied to the weight, and pulled in a horizontal direction until the first thread makes an angle of 45° with the vertical. Find the stretching forces in both threads.

4. The resultant of two forces at right angles is 26 lbs., and one of the forces is 10 lbs. Find the other.

5. Forces 1, 3, 4, 2 and 7 gms. act on a particle in one plane in directions making angles 30°, 60°, 90°, 135°, and 180° respectively, with a given straight line. Find their resultant in magnitude and direction.

Resolve each of the forces into two components, one parallel and the other perpendicular to the given straight line. The sum of the parallel components is

$$\begin{aligned}&\cos 30^\circ + 3\cos 60^\circ + 4\cos 90^\circ + 2\cos 135^\circ + 7\cos 180^\circ\\ &= {\cdot}8660 + 3 \times {\cdot}5000 - 2 \times {\cdot}7071 - 7\\ &= -6{\cdot}0482\end{aligned}$$

The sum of the perpendicular components is

$$\begin{aligned}&\sin 30^\circ + 3\sin 60^\circ + 4\sin 90^\circ + 2\sin 135^\circ + 7\sin 180^\circ\\ &= {\cdot}5000 + 3 \times {\cdot}8660 + 4 + 2 \times {\cdot}7071\\ &= 8{\cdot}5122\end{aligned}$$

The magnitude of the resultant is therefore

$$= \sqrt{6{\cdot}048^2 + 8{\cdot}512^2} = 10{\cdot}44$$

and its direction makes the angle, whose tan is $-\dfrac{8{\cdot}512}{6{\cdot}048}$, *i.e.* 125° 24′, with the given line.

The result may, of course, be obtained graphically.

6. Find in each of the following cases the magnitude and direction of the resultant:—

i. Forces 2, 4, 8, 12 acting respectively N., E., S., and W.

ii. Forces 3, 5, 9 acting on a particle parallel respectively to the sides of an equilateral triangle taken in order.

iii. Forces 1, 2, 3, 4 acting on a particle parallel respectively to the sides of a square taken in order.

7. Are three forces in equilibrium when they are represented in magnitude, direction, and position by the sides of a triangle taken in order?

3. THE LAW OF MOMENTS

Let AB represent a force of F units in magnitude, direction and position. From any point P draw PD perpendicular to AB, and let PD $= p$. Then the product Fp is called the moment of the force about the point.

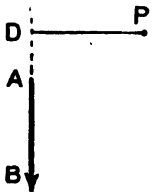


FIG. 9.

Law of Moments.—A metre stick was balanced on the sharp edge of a wedge-shaped piece of wood, and was found to come to rest when the edge was below 50·25 cms. This was due to the stick not having quite the same thickness at both ends; otherwise it would have balanced at the middle. When the stick was balanced in this way it was in equilibrium under the action of two forces—

(1) its weight acting vertically downwards,

(2) the upward force exerted by the wedge.

When two forces are in equilibrium they must be equal, and act in opposite directions in the same straight line. Hence the weight of the stick must act vertically downwards through 50·25 cms., and the upward force exerted by the wedge must be equal to the weight of the stick.

A loop of thread was tied to a 200-gm. weight and the latter hung over the stick at 40·25 cms. Loops of thread were tied to another 200-gm. weight, a 100-gm. weight, and a 50-gm. weight, and these were in succession one at a time hung on the stick on the other side of the wedge. It was found, when the wedge was at 50·25, that in order to produce equilibrium the second 200-gm. weight had to be placed at 60·20 cms., the 100-gm. weight at 70·15 cms., and the 50-gm. weight at 90·40 cms.

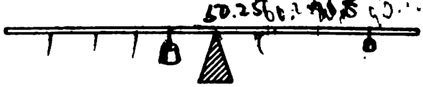


FIG. 10.

The moment of the first 200-gm. weight about the edge of the wedge was

$$200 \times (50{\cdot}25 - 40{\cdot}25) = 2000 \text{ units}$$

The moments of the other three weights about the same points were in succession

$$\begin{aligned} 200 \times (60{\cdot}20 - 50{\cdot}25) &= 1990 \text{ units} \\ 100 \times (70{\cdot}15 - 50{\cdot}25) &= 1990 \text{ units} \\ \text{and} \quad 50 \times (90{\cdot}40 - 50{\cdot}25) &= 2007 \text{ units} \end{aligned}$$

Now the numbers 2000, 1990, and 2007 agree with one another within the limits of experimental error. Hence, when the stick is balanced, the moment of the weight suspended from the one side is equal to the moment of the weight suspended from the other.

The weight on the right of the wedge tends to make that side fall, *i.e.* it tends to make the stick rotate in the direction of the hands of a clock, or in the clockwise direction as it is called. The weight on the left of the wedge tends to make the stick turn the other way, in the direction opposed to the hands of a clock, or in the counter-clockwise direction as it is called. The stick does not turn at all when the turning power of the force on each side is the same. Thus the moment of a force about a point measures the turning power of the force about the point.

Instead of hanging a weight on the other side the stick may be balanced by passing a loop of thread over the hook of a spring balance, and supporting it in the manner shown. In an actual experiment made it was found that when the loop of thread was at 28·7 cms. the spring balance read 94 gms. The spring balance in this case obviously exerted a clockwise moment of

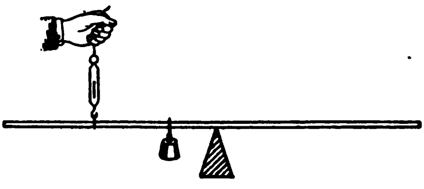


FIG. 11.

$$94 \times (50{\cdot}25 - 28{\cdot}7) = 2026 \text{ units}$$

about the edge. This result agrees with the former ones, the greater error in this respect being due to the difficulty of reading the spring balance accurately.

If we have several weights suspended on each side of the edge, and one or more spring balances acting on it, we find the stick balances when the sum of the moments tending to turn it in the clockwise direction is equal to the sum of the moments tending to turn it in the counter-clockwise direction.

We thus, as a result of experiment, arrive at the general law known as the law of moments, which may be stated as follows:—

> When a body which is free to turn about an axis is in equilibrium, the sum of the moments of the forces tending to turn it in the clockwise direction is equal to the sum of the moments tending to turn it in the counter-clockwise direction.

The Lever.—A lever is a rigid bar capable of rotation about a point called the fulcrum, and employed for changing one force into another, which is more suitable for the purpose in view. The first force is called the power or effort, the second the weight or resistance, and the point about which rotation takes place is called the fulcrum.

Fig. 12 depicts a lever which is being used to roll a large stone over. The point C where the end of the lever rests on the ground is the fulcrum. The force P represented as applied by the workman at right angles to the bar at A is the power, and the resistance of the stone represented as acting at right angles to the bar at B is the weight. The distances CA and CB, which we shall denote by *a* and *b*, are called the arms of the lever.

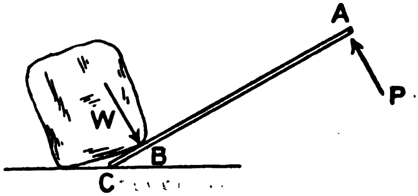


FIG. 12.

Let us suppose that the stone is poised halfway, that it is lifted a certain distance but that the workman is just holding his own, neither gaining nor losing. Then, if the weight of the lever itself is disregarded, the moments of the two forces about C are equal, that is

$$Pa = Wb$$

$$\text{or} \quad \frac{W}{P} = \frac{a}{b}$$

Now *a* is many times greater than *b*, consequently W is many times greater than P, and the workman is enabled to apply a much greater force at B than if he were to use the direct force of his hands. The ratio W/P is called the mechanical advantage of the lever.

In Fig. 12 P and W are represented as acting perpendicularly to the lever. This is not necessary. The equations hold whatever the directions of P and W, provided we regard *a* and *b* not as the lengths of CA and CB, but as the perpendicular distances from C to the lines of action of P and W.

In the example cited above the weight is between the fulcrum and the power. This arrangement is usually referred to as a lever of the second class. A lever of the first class is one in which the fulcrum is between the power and weight, as, for example, in a pair of scissors, which may be regarded as two levers crossing one another. An oar when used for splashing is another example of a lever of the same kind; the rowlock is then the fulcrum, the jerk exerted by the oarsman the force, and the resistance of the water to the blade the weight. But when used for rowing it becomes a lever of the second class. A lever of the third class is one in which the power is between the weight and the fulcrum, as in a pair of tongs or in the treadle of a lathe. In this case the mechanical advantage is less than 1, but, as will be shown later on, there is a gain in speed.

Equilibrium of Parallel Forces.—The law of moments may be regarded from a somewhat different standpoint. Let Fig. 13 represent the same metre stick as was used above. It is now balanced at the point C, a weight P is hung on at A, and B is the point at 50·25 cms. where it balanced before. It is now in equilibrium under the action of three forces—

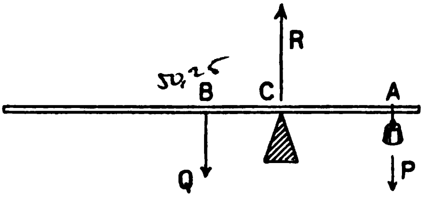


FIG. 13.

(1) P acting downwards through A,

(2) the weight of the whole stick itself, which we shall denote by Q, acting downwards through B,

(3) the upward force through C exerted by the wedge which we shall denote by R.

It is clear that the weight of the stick acts through B, because the stick would balance there if the weight P were removed. Also, since the stick does not move, the whole downward force must equal the upward force, *i.e.*

$$P + Q = R$$

This can also be shown experimentally by suspending the stick at C in a loop of thread hung from a spring balance, instead of resting it on the wedge. If, for example, the stick weighs 95 gms., a likely value, and P is 50 gms., it will then be found that the balance is pulled out to 95 + 50, that is, to 145 gms.

By the law of moments

$$P \times AC = Q \times BC$$

$$\text{or} \quad \frac{P}{BC} = \frac{Q}{AC} = \frac{P + Q}{BC + AC}$$

by a well-known algebraical result.

But $P + Q = R$ and $BC + AC = BA$.

Therefore we have the following symmetrical result, which is very easily remembered—

$$\frac{P}{BC} = \frac{Q}{CA} = \frac{R}{AB}$$

The meaning of this result will be more evident from Fig. 14, which represents the forces of Fig. 13 themselves, with the metre stick and wedge removed. We had a system of three parallel forces. We have arrived at the result, that for these three forces to be in equilibrium the middle one must have the opposite direction to the two outside ones, and each of the three must be proportional to the perpendicular distance between the other two.

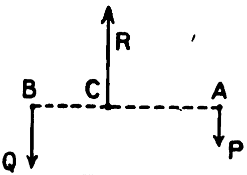


FIG. 14.

If the direction of the force R is reversed, it gives the resultant, not the equilibrant, of the forces P and Q.

EXAMPLES 3

1. A weight of 120 lbs. is slung from a pole which is carried on the shoulders of two men, A and B. The distance between the points where the pole presses on their shoulders is 6 ft., and the point where the weight is slung is 4 ft. from the point where the pole presses on A's shoulder. Find the weight borne by each, the weight of the pole being neglected.

Let P and Q be the weights carried by A and B respectively. Then by the rule at the foot of p. 12,

$$\frac{P}{2} = \frac{Q}{4} = \frac{120}{6}.$$

Thus A carries 40 lbs. and B 80 lbs.

2. Two men, A and B, carry a weight slung from a pole resting on their shoulders. If the maximum weight A can carry is 120 lbs. and the maximum weight B can carry is 90 lbs., what is the maximum load they can carry between them, and where must it be slung on the pole? The weight of the pole is to be neglected.

3. The arms of a balance are of slightly unequal length. When a body is put in the right-hand pan and the weights in the left-hand pan, its weight is found to be w_1; when the body is placed in the left-hand pan and the weights in the right-hand pan, its weight is found to be w_2. Show that the true weight is $\sqrt{w_1 w_2}$.

4. A metre stick weighs 92·0 gms., and balances on a knife-edge at the 50·2 cm. mark. A 50-gm. weight is slung on it by a piece of thread at the 10·0 cm. mark. Where will the stick now balance?

5. The same metre stick rests on a fulcrum or knife-edge at 10 cms., and carries a 50-gm. weight at 60 cms. It is supported at 90 cms. by a loop of thread, which is passed over the hook of a spring balance. What is the force exerted by the balance?

6. A metre stick weighing 88 gms. carries a 50-gm. weight at 10 cms., a 60-gm. weight at 20 cms., and a 70-gm. weight at 80 cms. At what point will it balance?

7. Weights of 7, 5, 3 and 1 lb. are hung at 1, 2, 3 and 4 ft. from one end of a rod of length 5 ft. and of negligible weight, the rod being hinged at this end. What force must act upwards at the other end to sustain them? What is the upward force exerted by the hinge?

8. A lever is to be cut from a bar weighing 2 lbs./ft. What must its length be, so that it may balance about a point distant 2 ft. from one end, when weighted at this end with 40 lbs.?

4. CENTRES OF GRAVITY

Bodies fall to the earth because the earth attracts them. This force of attraction on them is called the force of gravity. The direction of a plumb line at any point gives the direction of the force of gravity acting on the weight at the end of the line. Hence the direction of the force of gravity gives the vertical.

Let us consider a rigid body, that is, one the parts of which cannot be displaced relatively to one another. We can conceive of it as divided up into a great number of particles of weights w_1, w_2, w_3, etc. Now the weights of these particles are forces, the lines of action of which all pass through the centre of the earth, but the centre of the earth is very far away. Hence we can regard them as parallel forces.

Let A, B, and C be three of the particles of weights w_1, w_2, and w_3 respectively. Then by the theorem of p. 12 the parallel forces w_1, w_2 have a resultant equal to $w_1 + w_2$ acting through a point D which divides AB inversely in the ratio of w_1 to w_2. No matter how the body is turned, the resultant of w_1 and w_2 always passes through D. This point is called the centre of gravity of the two particles A and B. The resultant will be the same if we suppose the two particles collected at their centre of gravity D. Now combine this resultant with the weight of the third particle at C. We then obtain a force $w_1 + w_2 + w_3$ acting at some point E in DC, which divides DC inversely in the ratio of $w_1 + w_2$ to w_3. The weights of the three particles may now be supposed collected at E. By proceeding in this way we can show that the weights of the different particles, into which the body can be divided, all combine into one force equal in magnitude to their sum, which passes through the same single fixed point in the body. This point is called the centre of gravity of the whole body. It is that point through which the resultant action of gravity always passes, no matter how the body is turned relatively to the vertical.

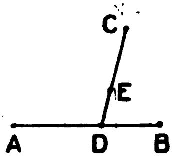


FIG. 15.

Particular Cases.—(1) The centre of gravity of a rod which has the same breadth, thickness, and density throughout is obviously at the middle point. For if it is divided into particles, to every particle on one side of the middle point there corresponds an equally heavy particle on the other side at the same distance from the middle.

(2) If we have a rectangular lamina or thin rectangular plate ABCD of uniform thickness and density, it can be divided into rods parallel to the ends. Let MN be such a rod. Then the centre of gravity of MN is at its middle point P, which is on the straight line FH drawn from F, the middle point of AD, to H, the middle point of BC. Similarly the centres of gravity of all the other rods are on the same straight line. We can suppose each rod concentrated at its middle point, and thus suppose the rectangle replaced by a line

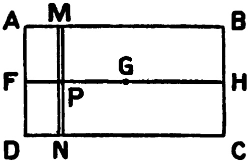


FIG. 16.

FH uniformly loaded with matter from end to end. And the centre of gravity of such a line is obviously at its middle point G.

(3) Let ABC be a triangular lamina, and let D be the middle point of BC. Join AD and divide the lamina into strips parallel to BC.

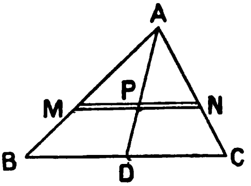


FIG. 17.

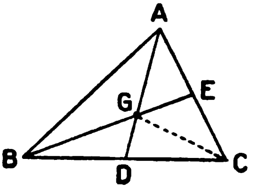


FIG. 18.

Let MN be any one of these strips. Then each side of MN is bisected by AD; hence P, the centre of gravity of MN, lies on the median AD. Similarly the centres of gravity of all the other strips lie on AD. It follows that the centre of gravity of the triangle lies on AD.

Similarly, if E is the middle point of AC, the centre of gravity of the triangle lies on BE. Hence it must be G, the point where AD and BE meet.

Join GC. Then since they are on equal bases $\triangle ABD = \triangle ADC$. Similarly $\triangle GBD = \triangle GDC$. Hence by subtraction $\triangle ABG = \triangle AGC$. Similarly it may be shown that $\triangle ABG = \triangle BGC$. Hence, since the three triangles are equal, each is equal to one third of $\triangle ABC$, that is, $\triangle ABG = \frac{2}{3}\triangle ABD$ or $AG = \frac{2}{3}AD$. We thus arrive at the result, that the centre of gravity of a triangular lamina is situated on any one of the three medians at a distance of $\frac{2}{3}$ of its length from the corner of the triangle.

If the triangle is replaced by three equal particles situated at its corners, the centre of gravity of the system is still at the same point. For the particles at B and C can be supposed collected at D, the point midway between them. Then, since the weight of the particle at D is twice that of the particle at A, their centre of gravity must divide DA in the ratio of 1 to 2.

(4) The centre of gravity of a circular lamina is obviously at its centre. Also the centres of gravity of a solid homogeneous sphere and solid cube are obviously at their centres.

The centre of gravity of a tetrahedron is on the line joining a vertex to the centre of gravity of the opposite face at a point distant $\frac{3}{4}$ of the length of this line from the vertex.

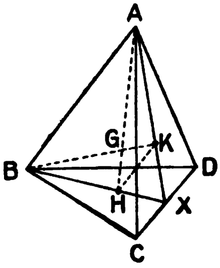


FIG. 19.

For let the tetrahedron ABCD be divided into laminæ parallel to the base. Then obviously the centre of gravity of each lamina lies in the point where it is intersected by the line joining the vertex to the centre of gravity of the base. Consequently the centre of gravity of the whole tetrahedron lies on this line. Let CX = XD, and let H be the centre of gravity of $\triangle BCD$.

Then the centre of gravity of the tetrahedron lies on AH. Similarly if K is the centre of gravity of $\triangle ACD$, the centre of gravity of the tetrahedron lies on BK. Consequently it is at G, the point of intersection of AH and BK.

Join HK. Now $BH = 2HX$ and $AK = 2KX$; hence HK is parallel to AB. Then by similar triangles—

$$\begin{aligned} AG : GH &= AB : HK \\ &= AX : KX \\ &= 3 : 1 \end{aligned}$$

It follows that the centre of gravity of a pyramid or cone is on the line joining the vertex to the centre of gravity of the base at a point distant $\frac{3}{4}$ of the length of this line from the vertex.

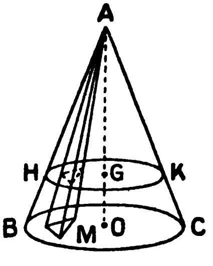


FIG. 20.

For if we consider the case of the right circular cone shown in Fig. 20, it is clear, by dividing it up into circular laminæ parallel to the base, that its centre of gravity lies on the line AO joining the vertex to the centre of gravity of the base. Also by dividing it up into tetrahedra such as MA it is clear that the centre of gravity of each tetrahedron lies on the plane HK drawn parallel to the base and cutting AB and AC in the ratio 3 : 1. Hence the centre of gravity of the whole cone is at G, the intersection of AO with this plane.

Since a triangle or other plane figure is infinitely thin, it has no weight and consequently in strictness no centre of gravity. It is, however, customary to speak of the centres of gravity of areas and lines, the area being identified in thought with a uniform plate and the line with a uniform wire.

Experimental Determination of Centre of Gravity.—If a body is suspended by a string, it comes to rest with its centre of gravity vertically under the point of support. The centre of gravity is thus restricted to one line in the body. By suspending the body now from another corner we can restrict the centre of gravity to another line. It lies consequently at the intersection of the two lines.

This method can be applied very well to a uniform plate. Holes are bored at two of its corners, it is suspended by passing one of the holes over a nail and a plumb line is hung on the same nail. When the latter comes to rest, its line is marked in chalk on one face of the plate. The plate is then suspended from the other hole, and the operation repeated. The intersection of the two chalk lines gives the centre of gravity.

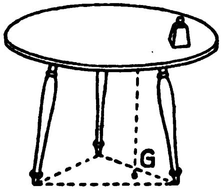


FIG. 21.

Stability.—For a body to stand on a given base it is necessary for the vertical through its centre of gravity to fall within the base. Thus if we have a table with a heavy weight resting on it (Fig. 21), the table remains erect, provided the vertical through the centre of gravity of table and weight meets the floor at a point G within the triangle formed by joining the feet of the table. If G lies outside this triangle, the table topples over.

A body is said to be in stable equilibrium if it returns to its original position when it is slightly disturbed from it. A cube resting on its side or a cone resting on its base is an example of a body in stable equilibrium.

A body is said to be in unstable equilibrium if on being slightly disturbed from its original position it tends to move further from it. A cone balanced on its point or an egg on its end are examples of unstable equilibrium.

A body is said to be in neutral equilibrium when after a slight displacement it neither returns to its original position nor moves further from it. A sphere resting on a horizontal plane is an example of neutral equilibrium.

In stable equilibrium the centre of gravity of the body is in its lowest position and any displacement tends to raise it. In unstable equilibrium the centre of gravity is in its highest position and any slight displacement tends to lower it. In neutral equilibrium the centre of gravity is neither raised nor lowered by a slight displacement.

If the body is free to roll on a supporting surface or is suspended from a point, the centre of gravity always tends to descend, just as it would do if all the weight of the body were collected there.

EXAMPLES 4

1. A circular hole, 4 inches in diameter, is cut out of a circular disc of wood, 10 inches in diameter, close to the edge. Find the centre of gravity of the remaining portion.

By symmetry the centre of gravity of the remainder is on the straight line joining the centre of the disc and the centre of the part cut out. Let it be a distance x from the centre of the disc, on the side opposite to the part cut out.

The area of the part cut out is 4π and of the remainder 21π. The whole disc may be regarded as composed of the remainder and the part cut out; hence, since the centre of gravity of the whole disc cuts the line joining the centres of gravity of its parts inversely as their areas,

$$\frac{x}{3} = \frac{4\pi}{21\pi} \quad \text{or} \quad x = \frac{4}{7}\text{ inch}$$

2. A sphere 10 inches in diameter contains a spherical hollow 4 inches in diameter, the centre of which is 3 inches distant from the centre of the sphere. Find the centre of gravity of the remaining portion.

3. Weights of 1, 2, and 4 gms. are placed at the corners A, B, and C of a triangle. Find the position of their centre of gravity.

4. A square of side a is divided by its diagonals into four triangles and one of these is cut out. Find the position of the centre of gravity of the remainder.

5. A square of side $2a$ is divided into four squares of side a by straight lines through its centre parallel to its sides. One of these squares is removed. Find the centre of gravity of the remainder.

6. A wooden cylinder of radius 4 cms. and density ·60 gm./c.c. has a cylindrical hole of radius 1 cm. bored in it, parallel to its axis. The axes of the two cylinders are 2 cms. apart. The cylindrical hole is filled with lead of density 11·4 gm./c.c. Find the centre of gravity of the whole figure.

5. FALLING BODIES

The velocity of a body is its rate of change of position. Velocity, like force, has both direction and magnitude; for example, we may say that a man is walking due north at three miles per hour. Hence velocities may be represented by straight lines in the same way as forces, and if a body has simultaneously two velocities in different directions, as, for example, in the case of a man walking across the deck of a ship, which is itself in motion, then their resultant can be found by the parallelogram construction in the same way as the resultant of two forces.

A velocity is said to be uniform when the body passes over equal spaces, however small, in equal times. A velocity is said to be variable when the body does not pass over equal spaces in equal times. If a body is moving with variable velocity, its velocity at any instant gives the number of units of space it would pass over in one unit of time, provided that its velocity were to remain constant for one unit of time at the value it has at the instant. Thus in general in the case of variable velocity the velocity at any instant does not give the actual distance traversed in the next unit of time. The average velocity during any time is the total distance traversed during the time divided by the time.

The acceleration of a body is its rate of change of velocity. Like velocity, acceleration may be either uniform or variable. Let us suppose that a train is moving with a velocity of 30 ft. per sec., that 1 minute later it is moving in the same straight line with a velocity of 60 ft. per sec. and that the acceleration is uniform. Then the acceleration is

$$\frac{60 \text{ ft./sec.} - 30 \text{ ft./sec.}}{60 \text{ sec.}} = \frac{30 \text{ ft./sec.}}{60 \text{ sec.}} = \tfrac{1}{2} \text{ ft./sec.}^2$$

If the acceleration had not been uniform, this would have been its average value during the minute in question.

Attention should be directed to the fact that acceleration is measured in ft./sec.^{2}. or feet per second per second, not in ft./sec. or feet per second. This follows from its definition. Velocity, which is rate of change of distance, is measured in units of distance or units of length per second; acceleration, which is rate of change of velocity, is measured in units of velocity per second, *i.e.* in (ft. per sec.) per sec.

Acceleration, like velocity and force, has both direction and magnitude, and so, like them, can be compounded by the parallelogram construction. We shall, however, confine ourselves here to uniformly accelerated motion in a straight line, and shall proceed to derive the formulæ for this case.

Body moving in a Straight Line with Uniform Acceleration.—Let u be the velocity of the body at a certain instant of time, and let a be the acceleration. It is required to find v, the velocity after a time t has elapsed, and s, the space traversed in this time.

The change of velocity in the time is $v - u$. By definition a is the rate of change of velocity. Hence

$$a = \frac{v - u}{t} \qquad - v = - u - at$$

$t - v - t$

which gives

$$v = u + at \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The space traversed is equal to the time multiplied by the average velocity. Since the velocity increases uniformly with the time, it follows that the average velocity is equal to half the sum of the velocities at the beginning and end of the time. Hence

$$\begin{aligned} \text{average velocity} &= \tfrac{1}{2}(u + v) \\ &= \tfrac{1}{2}(u + u + at) = u + \tfrac{1}{2}at \end{aligned}$$

and the space traversed during the time is given by

$$\begin{aligned} s &= \tfrac{1}{2}(u + v)t \\ \text{i.e.} \quad s &= (u + \tfrac{1}{2}at)t \\ \text{or} \quad s &= ut + \tfrac{1}{2}at^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2) \end{aligned}$$

If formula (1) be squared, we have—

$$\begin{aligned} v^2 &= u^2 + 2uat + a^2t^2 \\ &= u^2 + 2a(ut + \tfrac{1}{2}at^2) \\ &= u^2 + 2as \text{ from formula (2)} \end{aligned}$$

Hence

$$v^2 - u^2 = 2as \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This formula does not contain the time. It is of equal importance with the other two.

Special Cases.—If the body starts from rest, $u = 0$ and the formulæ become $v = at$, $s = \frac{1}{2}at^2$, and $v^2 = 2as$. If the motion is retarded instead of accelerated, then v is less than u and the formulæ become

$$\begin{aligned} v &= u - at \\ s &= ut - \tfrac{1}{2}at^2 \\ \text{and} \quad v^2 - u^2 &= -2as \end{aligned}$$

Falling Bodies.—The importance of uniformly accelerated motion in a straight line lies in the fact that it includes the case of falling bodies. If a body is allowed to fall freely under the earth's attraction, it moves downwards with a uniform acceleration of 32·2 ft./sec.^{2} or 981·4 cms./sec.^{2}; this acceleration is always denoted by the letter g.

The laws of falling bodies are due to Galileo (1564–1642). About 1590 he performed the famous experiment of the leaning tower of Pisa, dropping a 10-lb. shot and a 1-lb. shot from the top together and showing that they struck the ground simultaneously. The length of the fall was 180 ft. The time taken can be obtained from the formula $s = \frac{1}{2}gt^2$. Thus

$$t = \sqrt{\frac{2 \times 180}{32}} = \sqrt{\frac{45}{4}} = 3{\cdot}35 \text{ secs.}$$

if we substitute for g the round number 32 instead of 32·2. Nearly 2000 years before, Aristotle had taught that the time of falling was inversely as the weight of the body; *i.e.* that the 1-lb. weight would take ten times as long to reach the ground as the 10-lb. weight would, but no one had thought of trying the experiment. Of course some plausibility was lent to Aristotle's view by the fact that a feather flutters to the ground more slowly than a coin falls, but this is due to the resistance of the air. In a celebrated experiment, called the guinea and feather experiment, performed first long after Galileo's time, a guinea and feather were placed in a straight glass tube about four feet long from which the air was exhausted; when the tube was inverted the guinea and feather fell to the other end together.

By means of the experiment with the two shot and similar experiments Galileo was able to show that all bodies fell the same distance in the same time, the slight differences observed being due to the resistance of the air; thus and by experiments on inclined planes he was able to verify the formulæ (1), (2), and (3) derived in the last section.

EXAMPLES 5

1. A train moving at 30 mls./hr. is brought to rest in 100 secs., the retardation being uniform. Find how far it travels in this time. What is the retardation in ft./sec.^{2}?

30 miles per hour = 44 ft./sec.

$$\text{Distance} = \text{average velocity} \times \text{time} = 22 \times 100 = 2200 \text{ feet}$$

$$\text{Retardation} = \frac{44}{100} = \frac{11}{25} \text{ ft./sec.}^2$$

2. A body is thrown vertically upwards with a velocity of 128 ft./sec. Find the height to which it rises and the time it takes to reach the highest point.

$$t = \frac{128}{32} = 4 \text{ secs.}$$

$$\text{Height} = \text{average velocity} \times \text{time} = 64 \times 4 = 256 \text{ feet}$$

3. A body begins to move in a straight line with a velocity of 15 ft./sec. and an acceleration of 10 ft./sec.^{2} After what time will its velocity be tripled, and what space will it describe in this time?

$$t = \frac{45 - 15}{10} = 3 \text{ secs.}$$

$$\text{Space} = \text{average velocity} \times \text{time} = 30 \times 3 = 90 \text{ feet}$$

4. With what velocity must a body be thrown vertically upwards in order that it may rise to a height of 30 ft.? What time does it take to reach this height?

5. A body is moving with uniform acceleration in a straight line. Its velocity changes from 4 ft./sec. to 14 ft./sec. in 6 secs. Find the acceleration and the space passed over during the interval.

6. How far must a body fall to acquire a velocity of 10 ft./sec.?

7. A stone is falling with a velocity of 20 ft./sec. How much further must it fall for the velocity to increase to 30 ft./sec.?

8. A man on a motor cycle dashes into a wall at a speed of 40 mls./hr. How far would he have to fall to hit the ground with the same speed?

9. A train moving with uniform acceleration takes 4 mins. to travel the first mile. What velocity has it acquired at the end of that time? What is its acceleration in ft./sec.^{2}?

10. A body starts from rest, and moves with an acceleration of 2 ft./sec.^{2} What distances does it traverse in the first, second, third, fourth, and fifth seconds of its motion?

11. Change an acceleration of 32 ft./sec.^{2} to (i) mls./hr.^{2}, (ii) cms./sec.^{2}, (iii) kms./hr.^{2}

12. Change an acceleration of 100 cms./sec.^{2} to (i) kms./hr.^{2}, (ii) ft./sec.^{2} (iii) mls./hr.^{2}

13. A body, uniformly accelerated, starts from rest and passes over a ft. in b sec. How long will it take to pass over the next a ft.?

14. A stone falls over a cliff and travels 80 ft. in the last second before it strikes the ground at the foot of the cliff. How high is the cliff?

15. A man cycles at 10 mls./hr. and can walk pushing his cycle at 3 mls./hr. On a certain journey he cycles half the time and walks half the time. On the way back he cycles half the distance and walks half the distance. What is his average velocity, (i) on the way out, (ii) on the way back, (iii) on the whole journey?

16. Two trains are running in the same direction at the speed of 48 mls./hr., the first train leading by 4 mls. If the second train is brought to rest in $\frac{1}{3}$ of a mile, kept at rest for 1 min., and then brought to its former speed in 1 mile, find the extra lead gained by the first train.

6. THE LAWS OF MOTION

So far we have dealt only with forces in equilibrium. It is now necessary to consider the action of forces in causing bodies to move. The principles of this subject were first stated in Newton's three laws of motion, which run as follows:—

I. Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by impressed forces to change that state.

II. The change in momentum which takes place in unit time is proportional to the impressed force, and takes place in the direction in which the force acts.

III. To every action there is an equal and opposite reaction.

(i) The first law contains two statements. The first of these, namely, that a body at rest continues in a state of rest unless it is compelled by a force acting on it to change that state, appears self-evident. A book resting on the table remains at rest unless it is lifted or pushed. The second statement, namely, that a body already in motion continues in a state of uniform motion in a straight line, unless it is compelled by impressed forces to change that state, does not appear at first sight so evident, because, for example, a curling stone sliding over the ice, when once started, does not go on for ever, but gradually comes to a stop. A little consideration, however, shows, that it is the impressed force due to the surface of the ice rubbing on it, that brings it to rest. Also the smoother the ice, the farther it is possible to make the stone go. Hence we come to the conclusion, that if the ice were ideally smooth, that if there were no rubbing at all between it and the stone, then the latter would go on with undiminished velocity, until it came to the edge of the pond.

As illustrations of the first law we may take the case of a halfpenny resting on a card placed on a wine glass, or of a passenger standing in a tramcar when the brakes are applied suddenly, or of a passenger leaving a car in motion. If the card is flicked away smartly with the finger, the halfpenny does not go with it, but remains in a state of rest until the card is gone from beneath it, and then drops into the glass with a ringing noise. When the brakes are applied suddenly, the floor of the car and the passenger's feet are brought to rest, but the upper part of his body continues in its state of uniform motion, and thus he is thrown forward in the direction of the motion. In the same way, when the passenger leaves the car in motion, all his body has the same forward motion as the car has, but on touching the ground his feet are brought to rest, and, unless he runs forward, he will fall.

(ii) In order to understand the second law we must define momentum. The word has a more precise meaning in physics than in daily life. The momentum of a body in any direction is the product of its mass and its velocity in that direction. Thus a bullet may have a high momentum in virtue of its velocity, in spite of its small mass, and a steam road-roller may have a high momentum in virtue of its mass, in spite of its low velocity.

The second law states that a given force produces the same change in momentum in unit time, whether the body is at rest or in motion when the force begins to act, also that, if several forces are acting simultaneously on the body, each produces the same change of momentum that it would produce if it acted alone.

If a force acts on a body of mass m for a time t, and the velocity of the body in the direction in which the force acts changes from v_1 to v_2 in that time, then the change of momentum per unit time is

$$\frac{m(v_2 - v_1)}{t}$$

and, if a is the acceleration, this becomes simply

$$ma$$

Thè second law of motion states that F, the magnitude of the force, is proportional to ma. Hitherto we have measured force in grams weight or lbs. weight, the most natural units to take. These units are called the gravitational units of force. We shall now define two other units called the absolute units of force, which are based on the second law of motion. They are chosen so as to make F not merely proportional to, but equal to ma, and are called the dyne and the poundal.

> The dyne is that force which acting upon a mass of one gram produces an acceleration of one centimetre per second per second in the direction in which it acts.
>
> The poundal is that force which acting upon a mass of one pound produces an acceleration of one foot per second per second in the direction in which it acts.

Now the weight of one gram acting on one gram produces a downward acceleration equal to g or 981·4 cms./sec.^{2}, *i.e.* a value of $ma = 981{\cdot}4$. The gram weight is thererefore equal to 981·4 dynes. The weight of one lb. acting on one lb. produces a downward acceleration of 32·2 ft./sec.^{2} Hence the lb. weight is equal to 32·2 poundals.

The disadvantage of the gravitational units of force is that they are not constant. The acceleration of falling bodies is less at the equator than at the poles. Consequently if we took a spring balance reading to 250 gms. and hung an object weighing, say 200 gms., on to it, the spring would not always be pulled out to the same extent. If the reading on the scale were 200 gms. at Glasgow, it would be $\frac{7}{10}$ gm. less than this at the equator and $\frac{3}{10}$ gm. greater than this at the poles. Such differences would not be perceptible on a spring balance—the Salter's letter balance for 0 to 250 gms. is only divided for every 5 gms. and reads to about 1 gm.—and the gravitational units are always used for practical purposes, as their use avoids multiplication by g and simplifies calculation; thus engineers are accustomed to measure forces in tons weight and kilograms weight. But for scientific work it is extremely important that the unit should be constant, and hence in scientific work the dyne is always used as the unit of force. The poundal and its multiples, some of which, *e.g.* the tondal, have special names, are used only in text-books and in examination questions.

It should be noted, that if the object were weighed in an ordinary balance against a 200-gm. brass weight, the result would be the same both at the equator and the poles. For bodies which have equal weights at one place have equal weights at any other place. But the force with which the earth attracted both the object and brass weight would be less at the equator than at the poles. The weight of a pound of matter is therefore not a definite standard of force, but the pound of matter itself is a perfectly definite standard of mass.

(iii) The third law of motion states that forces always occur in pairs. The earth attracts the moon, but the moon also attracts the earth. A magnet attracts a piece of iron, but the iron also attracts the magnet. If you press your hand down on the table, the table presses up on your hand. If a horse tows a canal boat by means of a rope, the rope exerts a pull on the horse as well as on the boat. Each of these pairs of forces is said to constitute a stress. A force is simply one aspect of a stress; a force in one direction cannot exist by itself.

The third law asserts further that both aspects of the stress are equal, that, for example, the hand presses down on the table with the same force as the table presses up on the hand. This seems self-evident, because in the case of the table and the hand there is no relative motion, but beginners have a difficulty in seeing, if the canal boat pulls the horse back with the same force as the horse pulls it forward, how together they make any progress at all. The explanation of the difficulty is, that the stress between the horse and boat affects only their relative motion, and relatively to one another they are at rest. Their joint forward motion is due to the action between the horse's hoofs and the towing path.

If a 100-gm. weight is hung by a thread to a spiral spring, it pulls the lower end of the spring down with a force of 100 gms., and the spring pulls it up with the same force. Beginners are apt to think that the total force in the thread is either 200 gms. weight or zero. This is wrong; the two forces cannot be added, since they are in opposite directions. Also they do not annul one another, for obviously the thread is pulled tight and there is no slack in it, as there would be if there was no weight attached to it. The two aspects of the stress may be compared to a monetary transaction; a buyer gives £100, and a seller receives it; the total sum involved is not £200, nor is it as if no money had passed at all.

In his three laws of motion Newton put into definite form ideas which were, so to speak, in the air at his time; the best proof of the truth of the laws is that the whole of astronomy is based on them, and they have invariably led to correct results.

In working examples on the equation $F = ma$, care should be taken to express F always in absolute units.

EXAMPLES 6

1. A man weighing 12 stone stands on a hoist, which has a constant acceleration of 4 ft./sec.^{2} Find the force on the hoist due to his weight, (i) when it is ascending, (ii) when it is descending?

Let m be the mass of the man and a his acceleration. Let R be the upward force exerted on him by the hoist; then R is equal and opposite to the force on the hoist due to his weight.

When the hoist is ascending, F, the resultant force on the man $= R - mg$. Hence

$$R - mg = ma$$

$$R = m(g + a) = 12\frac{(32 + 4)}{32} \text{ stones weight} = 13\tfrac{1}{2} \text{ stones weight}$$

When the hoist is descending, F, the resultant force on the man $= mg - R$. Hence

$$mg - R = ma$$

$$R = m(g - a) = 12\frac{(32 - 4)}{32} \text{ stones weight} = 10\tfrac{1}{2} \text{ stones weight}$$

2. Change a force of 1 ton weight into dynes.

$$\begin{aligned} 1 \text{ ton weight} &= 2240 \text{ lbs. weight} \\ &= 2240 \times 453{\cdot}6 \text{ gms. weight} \\ &= 2240 \times 453{\cdot}6 \times 981{\cdot}4 \text{ dynes} \\ &= 9{\cdot}972 \times 10^8 \text{ dynes} \end{aligned}$$

3. A force of 500 dynes acting on a certain mass for one second gives it a velocity of 20 cms./sec. Find the mass in grams.

4. A force of 80 pdls. acts on a mass of 2 lbs. for 2 secs. Find the velocity generated.

5. A force of 6 lbs. wt. acts on a mass of 20 lbs. for 1 sec. Find the velocity generated.

6. A body is moving in a straight line with a velocity of 10 cms./sec. In 3 secs. the velocity increases to 19 cms./sec., and the motion is uniformly accelerated. Find the force acting on the body, (i) in dynes, (ii) in gms. wt., given that the mass of the body is 4 grams.

7. A force of 8 lbs. wt. acting on a certain mass for 3 secs. gives it a velocity of 6 ft./sec. Find the mass in lbs.

8. What horizontal force in tons wt. is required to stop a train of 200 tons mass running at 30 mls./hr., (i) in one minute, (ii) in 400 yds.?

9. A force F_1 acting on a mass of 5 gms. increases the velocity of the mass 11 cms./sec. every second. Another force F_2 acting on a mass of 12 gms. increases its velocity 8 cms./sec. every second. Find the ratio of F_1 to F_2.

10. The last waggon on a train weighs together with its load 10 tons and is being accelerated at the rate of 3 ft./sec.^{2} What is the force in the coupling in tons wt.? Frictional resistances are to be neglected.

11. A cage of mass 1 ton is lowered down a pit with an acceleration of 16 ft./sec.^{2} What is the stretching force in the rope? A mass of 8 lbs. lies on the bottom of the cage. With what force does it press on the latter?

12. The weight of a body read on a spring balance at the equator is 150 gms. What should it be on the same spring balance (i) at Glasgow, (ii) at the North Pole, supposing the balance sensitive enough to record the change in weight, given that the values of g for the equator, Glasgow, and the pole are respectively 978·0, 981·56, and 983·2 cms./sec.^{2}?

13. The mass of a gun is 2 tons and that of the shot 14 lbs. The shot leaves the gun with a velocity of 3000 ft./sec; what is the initial velocity of the recoil?

Let v be the initial velocity of the gun. Then, as a consequence of Newton's 3rd law, after the explosion the backward momentum of the gun is equal to the forward momentum of the shot. Thus

$$2 \times 2240\, v = 14 \times 3000$$

or

$$v \doteq \frac{14 \times 3000}{2 \times 2240} = 9{\cdot}375 \text{ ft./sec.}$$

14. The mass of a gun together with its carriage is 7 tons. It fires a 100-lb. shot with a velocity of 1000 ft./sec. What is the velocity of recoil of the gun?

15. Two bodies initially at rest attract each other with a force of $\frac{1}{100}$ dyne. If the masses are 10 and 50 gms. respectively, find the velocities acquired in 1 sec.

7. CIRCULAR MOTION AND THE PENDULUM

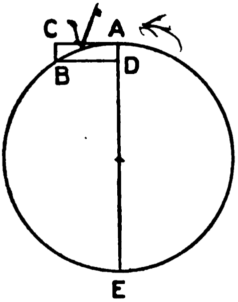


FIG. 22.

Uniform Motion in a Circle.—Let us suppose that a particle is moving in a circular path of radius r with uniform speed v, and that it goes from A to B in time t. When it is at A it is moving in the direction of AC, the tangent at A. Then it is clear it must be acted on by some force towards the centre of the circle. For otherwise, according to Newton's first law, it would continue in a state of uniform motion along the tangent.

Draw BC perpendicular to AC, and let us suppose that AB is very small. Then AB may be regarded as straight and sensibly equal to AC, and the particle would have been found at C had no force acted on it. CB is consequently the distance due to the force. Let a be the acceleration in the direction CB produced by the force. Then, since at the beginning of the time t the particle had no motion in the direction CB, and at the end of the time had traversed the distance CB, we have

$$CB = \tfrac{1}{2}at^2$$

But

$$AC = vt$$

The first of these equations gives

$$a = \frac{2\mathrm{CB}}{t^2}$$

and substituting for t from the second we obtain

$$a = \frac{2\mathrm{CB}v^2}{\mathrm{AC}^2}$$

Let AE be a diameter of the circle, and draw BD perpendicular to AE. Then $\mathrm{AC}^2 = \mathrm{DB}^2 = \mathrm{AD}\,.\,\mathrm{DE} = \mathrm{AD}\,.\,\mathrm{AE}$ since AD is small. Hence

$$a = \frac{2\mathrm{CB}v^2}{\mathrm{AD}\,.\,\mathrm{AE}} = \frac{v^2}{r}$$

i.e. a particle moving with velocity v in a circular path of radius r has an acceleration v^2/r towards the centre of curvature. If m is the mass of the particle, the force in the direction CB, that is, in the direction of AE, is consequently given by mv^2/r. The formula, of course, gives the force in absolute units.

It is a matter of common experience, that a force is necessary to keep a body in a circular path. If a stone is swung round at the end of a string we feel it tugging at the string, and if the string breaks, or is let go, the stone flies off at a tangent.

Simple Harmonic Motion.—Let a point Q travel with uniform speed v round a circle of radius r, and from its instantaneous position let a perpendicular QP be drawn to AB, any fixed diameter of the circle. Then, as the point Q travels round the circle, the auxiliary circle as it is called, the perpendicular QP moves parallel to itself, and the point P travels back and forwards along the diameter AB. The point P is then said to describe a simple harmonic motion, or S.H.M., as it is written for short. Simple harmonic motion is thus the projection upon a straight line of uniform motion in a circle. It must be noted that both the point Q and the auxiliary circle are mathematical fictions used for defining the S.H.M. When S.H.Ms. occur in nature there is only the one real motion, that of P in the straight line.

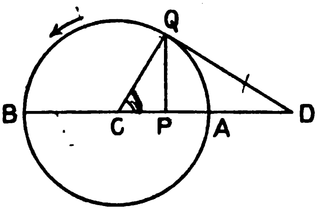


Fig. 23.

Simple harmonic motion occurs frequently in nature. It is described, for example, by the bob of the pendulum of a clock and by the prongs of a vibrating tuning fork. The angle QCP is called the phase of the S.H.M. The whole distance that P traverses, *i.e.* AB, is called the range of the S.H.M., and half the range, *i.e.* the radius of the auxiliary circle, is called the amplitude of the S.H.M. The time taken by P to go from one end of the range to the other end and back, or the time taken by Q to go once round the auxiliary circle, is called the period of the S.H.M. Let it be denoted by T. Then

$$\mathrm{T} = \frac{2\pi r}{v}$$

Let C be the centre of the circle, join QC and let $\angle QCP = \theta$. Then the displacement of P from its mean position is given by $CP = CQ \cos\theta = r\cos\theta$.

Draw QD, the tangent at Q, and produce it to meet BA at D. Then the velocity of Q is equal to v, and is in the direction of DQ. The velocity of P is the component of the velocity of Q along DB, that is, it is $v \cos QDP = v \sin\theta$.

The acceleration of Q is v^2/r, and is in the direction QC. The acceleration of P is the component of the acceleration of Q in the direction AB, that is, it is

$$\frac{v^2}{r}\cos QCP = \frac{v^2}{r}\cos\theta$$

We have therefore

$$\frac{\text{Acceleration of P}}{\text{Displacement of P}} = \frac{\frac{v^2}{r}\cos\theta}{r\cos\theta} = \frac{v^2}{r^2} = \frac{4\pi^2}{T^2}$$

Thus when a particle is describing a S.H.M. the ratio of its acceleration to its displacement is constant and equal to $4\pi^2$ divided by the square of the period.

Conversely, when a particle is moving in a straight line under an acceleration towards a fixed point in the line proportional to its distance from that point, it is describing a S.H.M. This statement, which is often taken as the definition of S.H.M., may easily be seen to be in harmony with the previous definition. For let a point P′ moving according to this statement start from the point A at the end of its range and move towards C, the centre of its range; at the same time let another point P start from A and move towards C, so that it is the projection of uniform motion in a circle with radius CA and centre C, and let P and P′ reach C at the same time. Then the positions of P and P′ must coincide for all intermediate times, for otherwise we would have two particles starting simultaneously from rest at the same point, accelerated under the same law, and covering the same distance in the same time, yet not coincident at intermediate times, which would be absurd.

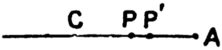


FIG. 24.

The Simple Pendulum.—If a body is suspended so that it is free to vibrate about a horizontal axis, it will oscillate back and forward until the resistance of the air and the friction of the axis gradually bring it to rest in its position of stable equilibrium, with the centre of gravity in the same vertical plane as the axis and below it. Such a body, whatever its form, is called a pendulum, or more correctly a compound pendulum, and Fig. 25 represents one of a type used in clocks. All pendulums have the property, that the period is always the same provided the arc of swing is small, not more than 2° or 3°.

The simple pendulum consists of a "heavy particle attached to one end of an inextensible, perfectly flexible, weightless thread," the other end of which is fixed. This description is realized with sufficient accuracy by a small lead sphere suspended by a thin thread. The distance between the point of support and the centre of the lead sphere is called the length of the pendulum, and will be denoted by l. Let m be the mass of the bob of the pendulum.

When the pendulum is at rest in its position of stable equilibrium, the weight of the bob acting vertically downwards is balanced by the stretching force in the string acting vertically upwards. But, suppose the pendulum is in the position shown in Fig. 26, making $\angle\theta$ with the vertical. Then the weight mg, acting vertically downwards and represented by BE, can be resolved into two components, one $mg\cos\theta$ represented by BD, and the other $mg\sin\theta$ represented by BC. The first of these components is neutralized by the stretching force in the string. The second, acting on the mass m of the bob, gives it an acceleration $g\sin\theta$ in the direction BA.

Now, if θ is small, we may write $\sin\theta = \theta$, and we may regard the arc AB as straight and $= l\theta$. The bob of the pendulum is therefore moving in a straight line towards A, and its acceleration divided by its displacement is equal to $g\theta/(l\theta)$, or g/l, *i.e.* the ratio is constant. The motion is consequently a S.H.M., and the period according to the equation on p. 27 is given by

$$\frac{\text{Acceleration}}{\text{Displacement}} = \frac{g\theta}{l\theta} = \frac{4\pi^2}{T^2}$$

or

$$T = 2\pi\sqrt{\frac{l}{g}}$$

It is thus independent both of the arc of swing, provided the latter is small, and of the mass of the bob. This formula forms our most accurate means of determining g.

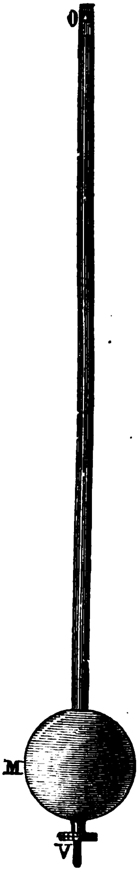


FIG. 25.

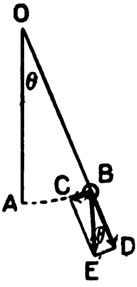


FIG. 26.

Half the period is sometimes referred to as the time of vibration of the pendulum. The seconds pendulum is that one which has the time of vibration equal to one second. We have for its length

$$l = \frac{gT^2}{4\pi^2}$$

If we substitute for g the value for Glasgow, namely, 981·56 cms./sec.², this gives

$$l = \frac{981{\cdot}56 \times 2^2}{4\pi^2} = 99{\cdot}49 \text{ cms.}$$

that is, it is just a little short of the metre.

The fact that the period of the pendulum remains constant as the amplitude diminishes in extent, or the law of the isochronism of the pendulum, as it is called, was discovered by Galileo in 1582.

EXAMPLES 7

1. A stone weighing 1 lb. is swung round in a horizontal circle at the end of a string 1 yard long, making a complete revolution every $\frac{2}{3}$ second. What is the stretching force in the string?

The stretching force in the string

$$= \frac{mv^2}{r} = \left(\frac{2\pi 3}{\frac{2}{3}}\right)^2 \frac{1}{3}$$

$$= 266{\cdot}5 \text{ pdls. or } 8{\cdot}27 \text{ lbs.}$$

2. What is the force in grams weight required to keep a mass of 10 gms. revolving in a circle of 5 cms. radius 40 times per second?

3. A cyclist turns a corner at 10 mls./hr. in a curve of 15 ft. radius. What is his acceleration towards the centre of curvature?

4. A "joy wheel" 8 ft. in radius makes 4 revolutions per minute. A 1-lb. wt. is placed on it. Find its acceleration, when it is (i) 1, (ii) 4, (iii) 8 ft. from the centre of the wheel.

5. Taking the earth's equatorial radius as 3962 mls. and the sidereal day, *i.e.* the time taken by the earth to make a complete revolution about its axis, as 86,164 secs., find the acceleration of a point on the equator in ft./sec.^{2}

6. Find the acceleration at Glasgow due to the earth's rotation. The latitude of Glasgow is 55° 52′.

7. What is the value of the ratio of acceleration to displacement for one of the prongs of a tuning fork, which makes 256 complete vibrations per second? If its range is $\frac{1}{10}$ mm., find its maximum velocity.

$$\frac{\text{Acceleration}}{\text{Displacement}} = \frac{4\pi^2}{T^2} = 4\pi^2 256^2 = 2{\cdot}587 \times 10^6$$

The maximum velocity = velocity of particle in auxiliary circle

$$= \frac{2\pi r}{T} = \frac{2\pi 256}{200}$$

$$= 8{\cdot}042 \text{ cms./sec.}$$

8. A tuning fork makes 512 complete vibrations per second. The range of the end of one of the prongs is $\frac{1}{20}$ mm. Find its maximum velocity and maximum acceleration.

9. Find the periods of simple pendulums 20, 40, 60, 80, and 100 cms. long. Take g as 981·4 cms./sec.^{2}

10. Find the periods of simple pendulums 2, 3, and 4 ft. long. Take g as 32·2 ft./sec.^{2}

11. A pendulum, which beats seconds at a place where g is 32·20 ft./sec.^{2}, is taken to a place where g is 32·28 ft./sec.^{2} How many seconds does it gain or lose in a day?

Let T be the original period of the pendulum and T_1 the new period. A pendulum which "beats seconds" is one which has a period of 2 secs., but this fact is not required:—

$$T = 2\pi\sqrt{\frac{l}{32{\cdot}20}}, \quad T_1 = 2\pi\sqrt{\frac{l}{32{\cdot}28}}$$

Hence $$T_1 = T\sqrt{\frac{32{\cdot}20}{32{\cdot}28}} = T\left(1 - \frac{8}{3228}\right)^{\frac{1}{2}}$$

$$\doteq T\left(1 - \frac{4}{3228}\right). \quad \text{(See Appendix.)}$$

Since $T_1 < T$, the pendulum will go too fast. Let n = no. of secs. gained per day. The number of secs. in a day is 86400.

$$86400T = (86400 + n)T_1$$

$$86400 = (86400 + n)\left(1 - \frac{4}{3228}\right)$$

or

$$n = \frac{86400 \times 4}{3228} = 107$$

since the other term is too small to be taken into account.

12. A pendulum which beats seconds at a place where g is 981·4 cms./sec.^{2} is taken to a place where g is 981·0 cms./sec.^{2} How many seconds does it gain or lose in a day?

13. The thread of a simple pendulum is shortened by 1 inch and the period diminishes by $\frac{1}{80}$ of its value. What was the original value of the length?

14. If a simple pendulum, which is intended to beat seconds, loses 20 minutes per week, by what percentage of its own length should it be shortened?

8. FRICTION

Let us suppose that a body of mass m is at rest on a smooth plane inclined at an angle θ to the horizontal. Let its weight mg acting downwards be represented by BE. Draw EC at right angles to AB and complete the rectangle ECBD; $\angle$CEB $= \theta$. Then the weight mg may be resolved into two components, one $mg \cos \theta$ represented by BD acting at right angles to the plane, and the other $mg \sin \theta$ represented by BC acting down the plane.

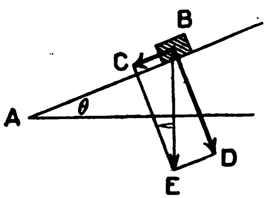


FIG. 27.

The first of these is neutralized by the upward reaction or normal force exerted by the plane; the second causes the body to move down the plane with the uniform acceleration $g \sin \theta$.

If, however, we take a wooden board, place a small block of wood on it and tilt it, we find that for small values of θ the block does not move at all. This is because the wooden board is not perfectly smooth. The perfectly smooth inclined plane is an ideal not realized in practice. If the mass m is placed on any actual plane, a force F called the force of friction acts up the plane in the direction CB and keeps it at rest, just as much friction being called into action as prevents motion. Consequently $F = mg \sin \theta$, and as θ increases, F increases. When, however, θ reaches a certain limiting value, which we shall call α, the component of gravity down the plane overcomes the resistance of friction, and the body begins to slide. The resistance of friction acts tangentially to the surfaces of contact. The limiting angle depends on the substances in contact; for an oak block upon an oak plank it is about 23° when the fibres are parallel, and 16° when they are cross.

In the same way, if a block of wood of mass m is placed on a horizontal plane, and a string attached to it and passed over a pulley, so as to carry a scale pan, and if weights are placed in the pan, it will be found that at first the body does not move. Only when the weights in the pan together with the weight of the pan itself reach a certain critical value, which we shall call P, is the resistance of friction overcome and the body begins to move.

Friction obeys the following laws:—

(i) It always opposes motion.

(ii) It depends on the nature of the surfaces in contact, but is independent of the extent of the areas in contact, as long as the reaction or total pressure between the two surfaces is the same.

(iii) The maximum friction between two surfaces is proportional to the total pressure between them.

These laws are the result of experiment. If a wooden block and a piece of metal of exactly the same weight and with the same extent of rubbing surface are placed on the inclined plane in succession, and the angle with the horizontal gradually increased, it is found that sliding begins at different angles, showing that the maximum resistance exerted by friction in each case is not the same. Also if the surfaces in contact are oily or greasy, friction is diminished and sliding starts at a smaller angle. It is because oil diminishes friction that it is used in machinery and between axles and their bearings.

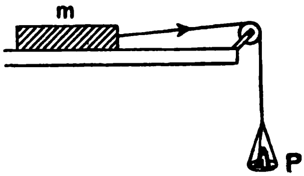


FIG. 28.

If the area in contact in the arrangement represented in Fig. 28 is halved, say by sawing the block in two, sliding will start when P has only half the value previously necessary. If, however, the block is loaded by placing weights on it until the force pressing it down on the horizontal plane is the same as before, it is found that sliding starts when P has its original limiting value.

If the weights which the block carries are altered, it is found that sliding always starts when $P/(mg)$ has a constant value. This value, the coefficient of static friction between the two surfaces in question, is the maximum resistance offered to relative motion divided by the force pressing the surfaces together. It is usually denoted by μ. It can be measured experimentally by the two methods depicted in Figs. 27 and 28. When the block in Fig. 27 is on the point of sliding, the resistance of friction to motion is $mg \sin \alpha$. The normal force is $mg \cos \alpha$. Hence

$$\mu = \frac{mg \sin \alpha}{mg \cos \alpha} = \tan \alpha$$

or the coefficient of friction is simply the tangent of the angle at which sliding begins. In the arrangement of Fig. 28, when mg is the total weight of the block and the weights it carries, and P the combined weight of scale pan and weights on it just sufficient to start motion,

$$\mu = \frac{P}{mg}$$

Kinetic Friction.—When the two surfaces are moving relatively to one another, the resistance of friction is much less than when motion is on the point of starting, even though the normal force is the same in both cases. The ratio of friction to normal force *after motion is started* is called the coefficient of kinetic friction. It can be determined by either of the arrangements shown in Figs. 27 and 28, if the inclined plane in Fig. 27 is continually tapped as its inclination is increased, or if the horizontal plane in Fig. 28 is continually tapped as the weights are added to the pan. The coefficient of kinetic friction is the limiting value of tan θ or of $P/(mg)$ sufficient to keep up motion after it is started. The tapping starts the motion.

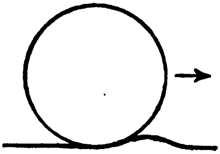


FIG. 29.

Rolling Friction.—Rolling friction is the resistance experienced when one surface rolls on another. It is a matter of common experience that it is very much less than sliding friction; witness the use of wheeled carriages on the roads instead of sledges and the use of ball bearings in bicycles. Rolling friction is due to the wheel being slightly flattened at the point of contact and to the plane being rolled up into a little ridge in front of the moving body.

EXAMPLES 8

1. A body of mass 10 lbs. is placed on a smooth plane, which is inclined at 5 vertical to 13 along the slope. How far does it slide in 2 secs., and what velocity does it acquire in this time? With what force does it press down on the plane?

2. A block of wood is placed on an inclined plane, and the inclination of the latter gradually increased. When it makes an angle of 32° with the horizontal, the block begins to slide. What is the coefficient of static friction between it and the plane?

3. A body weighing 10 lbs. is placed on a plane inclined at 30° to the horizontal. The coefficient of kinetic friction between the body and the plane is ·6. Find the force (i) acting up the plane, (ii) acting down the plane, necessary to give the body an acceleration of 10 ft./sec.^{2}

Component of weight down plane

$$= 10 \times 32 \sin 30^\circ = 160 \text{ pdls.}$$

Resistance of friction

$$= {\cdot}6 \times 10 \times 32 \cos 30^\circ = 166 \text{ pdls.}$$

Required rate of change of momentum = 100. Let F be the required force in pdls. Then

(i) $100 = F - 160 - 166, \quad \therefore F = 426 \text{ pdls.} = 13{\cdot}3 \text{ lbs.}$

(ii) $100 = F + 160 - 166, \quad \therefore F = 106 \text{ pdls.} = 3{\cdot}3 \text{ lbs.}$

4. A body is placed on a plane inclined at 30° to the horizontal. The coefficient of kinetic friction between the body and the plane is ·4. If the body is moving down the plane with a velocity of 100 cms./sec., what does this velocity increase to in the next 2 secs., and how far does the body travel in this time?

5. A body of mass 6 lbs. is placed on a plane inclined at 5 vertical to 13 along the slope. The coefficient of kinetic friction between it and the plane is ·3. What force applied up the incline is necessary to give it an acceleration of (i.) 4 ft./sec.^{2}, (ii) 6 ft./sec.^{2} up the plane?

6. A mass of 4 lbs. hangs over the edge of a smooth table. It is connected by a string, which passes over a pulley at the edge of the table, to a mass of 6 lbs. laid on the table. Find the space described and the velocity acquired in a quarter of a second.

$$a = \frac{F}{m} = \frac{4 \times 32}{6+4} = \frac{64}{5}\text{ft./sec.}^2$$

$$v = \frac{64}{5} \times \frac{1}{4} = \frac{16}{5}\text{ft./sec.}$$

$$s = \frac{1}{2} \times \frac{64}{5} \times \frac{1}{4^2} = \frac{2}{5}\text{ft.}$$

7. A block of wood, ~~which weighs~~ 400 gms., rests on a table. A string is attached to it, and passes horizontally to a pulley at the edge of the table, carrying a scale pan at its other end. The pan weighs 50 gms. Weights are gradually added to the pan, and, when the total weight in the pan is 182 gms., the block begins to slide. What is the coefficient of static friction between it and the table?

8. If in question 7 a weight of (i) 200, (ii) 300 gms. is placed in the pan, what is the acceleration of the block on the table? Take the coefficient of kinetic friction between the block and the table as ·4.

9. A train running down an incline of 1 vertical to 150 along the slope has a velocity of 10 ft./sec. at a certain time. What velocity will it have half a minute later, and how far will it have run in the time?

10. A body slides down an incline of 5 vertical to 13 along the slope, the coefficient of friction being ¼. After the body has moved a distance *d* from rest, the coefficient of friction changes to ½. Prove that the body will come to rest after traversing a further distance 2*d*.

9. WORK AND ENERGY

In physics the word "work" has a special meaning attached to it. If a force F acts on a body and its point of application moves a distance *s* in a direction making an angle θ with the direction of F, then the force is said to do an amount of work F*s* cos θ on the body.

Thus if a body of mass *m* is allowed to fall a height *h* from a table to the floor, the force acting on it is *mg* and the distance moved through by the point of application of the force is *h*, consequently the work done by gravity on it is *mgh*. If the body is lifted from the floor to the table, the work *mgh* is done against gravity.

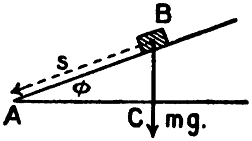


FIG. 30.

If a body of mass *m* is allowed to slide a distance *s* down a smooth plane inclined at an angle ϕ to the horizontal (Fig. 30), the distance *s* being measured along the slope, the cosine of the angle between the direction of F and *s*, *i.e.* cos ABC, is equal to sin ϕ; hence the work done by gravity is *mgs* sin ϕ.

If the bob of a simple pendulum of mass *m* and length *l* is allowed to swing from a position B making an angle ϕ with the vertical to its lowest position A, and BC be drawn perpendicular from B to OA, the force is mg and the component of the displacement in the direction of the force is CA. Now CA = OA − OC = $l(1 - \cos\phi)$. Hence the work done by gravity is $mgl(1 - \cos\phi)$.

The product $Fs \cos\theta$ may be regarded either as the product of the force and the component of the displacement in the direction of the force, or as the product of the displacement and the component of the force in the direction of the displacement.

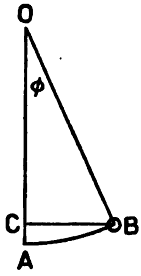


FIG. 31.

No work is done if the point of application of the force does not move, or if it moves only at right angles to the direction of the force. An example of the first case is a book resting on a table; it is acted on at its centre of gravity by its weight mg vertically downwards, but there is no displacement of the point of application of the force. An example of the second case is that of the stretching force in the string of a simple pendulum. The bob moves always at right angles to the direction of the string; $\cos\theta = \cos 90° = 0$, and the stretching force in the string does no work.

Units.—Just as there are four different kinds of units for measuring force, so there are four different kinds of units for measuring work.

The erg is the amount of work done by a force of one dyne acting through one centimetre.

The foot-poundal is the amount of work done by a force of one poundal acting through one foot.

The gram-centimetre is the amount of work done by a force of one gram weight acting through one centimetre.

The ft.-lb. is the amount of work done by a force of one lb. weight acting through one foot.

The first two of these units are absolute units, the erg being the unit used for scientific work. The last two are gravitational units; they or their multiples are used by engineers. The erg has the disadvantage that it is a very small unit; to lift a kilogram weight from the floor to a table one metre above the floor requires $1000 \times 981{\cdot}4 \times 100 = 9{\cdot}814 \times 10^7$ ergs. Hence for many purposes a multiple of it, called the joule and equal to 10^7 ergs, is used.

Machines.—Any appliance used for performing work can be called a machine in physics. The term includes appliances as elaborate as the steam locomotive, but here we shall consider three of the very simplest, namely, the lever, the single movable pulley, and the screw press. These and similar arragements were described at length in the older books on physics under the name of "mechanical powers."

The lever has, of course, already been described on page 11, and it was shown there that if P was the "power" and W the resistance, and CA and CB the perpendicular distances from the fulcrum to their respective lines of action, then

$$P \times CA = W \times CB$$

Let us suppose, for example, that the lever under discussion is one of the second class, and that it is rotated through a small angle about the fulcrum (Fig. 32). Then A traces out the arc AA′ and B traces out the arc BB′. The lengths of the arcs are in the ratio of their respective radii, *i.e.*

$$\frac{AA'}{BB'} = \frac{CA}{CB}$$

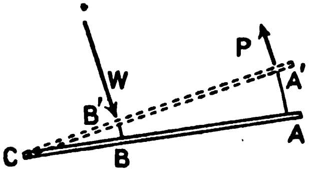


FIG. 32.

If the angle ACA′ is small, the arcs may be regarded as elements of straight lines. The work done by the power is then P × AA′, and the work done against the resistance W × BB′. But

$$\frac{AA'}{BB'} = \frac{CA}{CB} = \frac{W}{P}$$

or

$$P \times AA' = W \times BB'$$

That is, the work done by the power is equal to the work done against the resistance, or, in other words, you only get as much work out of the lever as you put into it. Although by its means a comparatively small power may be enabled to overcome a great resistance, yet the displacement of the point of application of the power must then be much greater than the displacement of the point of application of the resistance. Or, as it is popularly expressed, what is gained in force is lost in speed.

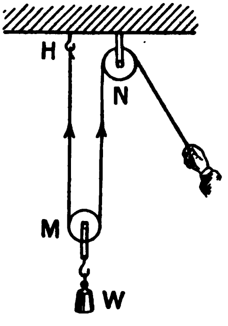


FIG. 33.

Fig. 33 represents the single movable pulley. A hand is shown pulling a cord which passes over the fixed pulley N, then round the movable pulley M to the hook H, to which its end is fastened. The pulley M carries a weight W. The pulley is the name given to the revolving wheel. The pulley and frame in which it is fixed together form the block.

Let P be the force with which the hand is pulling the cord. Then the stretching force is the same throughout the whole length of the cord to the hook H. Now consider the equilibrium of the block M, neglecting its weight. It is acted on by the weight W vertically downwards and by the stretching force P in the cord applied vertically upwards on both of its sides. The total upward force must balance the downward force. Hence

$$W = 2P$$

Let us suppose now that the weight is pulled up a distance h. Then the hand must move a distance $2h$. The work done by the hand is $P2h$, and the work done against the weight is Wh. By the equation above these are equal. Hence, although by the arrangement it is possible to lift a weight twice as heavy as could be done by direct lifting, yet no more work is obtained from the arrangement than is put into it. Indeed, if we take into consideration the weight of the block

M and the friction in both pulleys, we find that W is less than 2P, and the useful work obtained from the arrangement is less than the work put into it.

The study of other machines leads us to the same result, which is known as the principle of work and can be stated as follows: In all mechanical devices of any sort whatever, the work accomplished by the machine is never greater than the work expended upon it.

We shall now consider the application of the principle to the screw press shown in Fig. 34. When the screw is turned through one complete revolution, its point is lowered through a vertical distance d, equal to the distance between two adjoining threads. This distance is called the pitch of the screw; let r be the radius of the cross arm, and let a force P be applied at each end at right angles to the cross arm. In a complete revolution the point of application of each force moves round a circle of radius r; hence the work expended is

$$4\pi r\mathrm{P}$$

Let F be the force exerted by the point of the screw. Then the work accomplished in the same time is

$$\mathrm{F}d$$

The principle of work states that if friction is neglected, $\mathrm{F}d = 4\pi r\mathrm{P}$. In the screw press, however, much of the work is lost overcoming friction, and $\mathrm{F}d$ is considerably less than $4\pi r\mathrm{P}$.

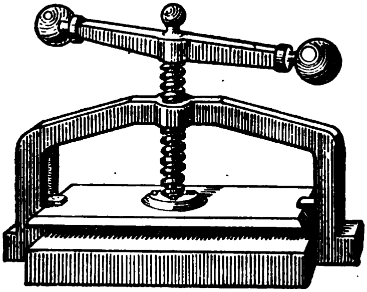


FIG. 34.

From Watson's Physics.

Activity.—The rate at which an agent does work is called its power or activity. Activity is measured in any unit of work per unit of time, in ergs per second or foot-lbs. per second. It has, however, three special units of its own. James Watt, the inventor of the steam engine, introduced a unit called the horse-power (H.P.), which is equal to 33,000 ft.-lbs. of work per minute or 550 ft.-lbs. of work per second. His estimate of the horse's activity was intentionally high, as he did not wish to over-advertise his engines; the average horse can work only at about $\frac{3}{4}$ of this rate, and the average man at about $\frac{1}{7}$ of this rate.

The other special units are the watt, named after James Watt, which is defined as 1 joule per second, and the kilowatt, which is defined as 1000 watts. The kilowatt is very nearly $\frac{4}{3}$ H.P.

Energy.—The energy of a body is the work that can be obtained from it in virtue either of its motion or position. If it is obtained in virtue of its motion, it is termed kinetic energy; if it is obtained in virtue of its position, it is termed potential energy. Energy, being work, is measured in the same units as work is.

Kinetic Energy.—Let us suppose that a bullet of mass m is moving with velocity v in a straight line, that it hits a block of wood perpendicularly, and is brought to rest inside it. Let us suppose, also, that the wood offers a constant resistance F to the motion of the bullet, that this resistance produces a negative acceleration a, and that the bullet is brought to rest in a distance s. Then the work it does against the wood is Fs.

Now $$F = ma$$

and, since the negative acceleration a annuls the velocity v in distance s,

$$v^2 = 2as$$

The work obtained from the bullet in consequence of its motion is therefore given by

$$\begin{aligned} Fs &= mas \\ &= \tfrac{1}{2}mv^2 \end{aligned}$$

The same reasoning can be applied to any body of mass m moving with velocity v which is brought to rest by a constant force F. It can also be extended to include the case when the force F is not constant. Hence the kinetic energy of a body of mass m moving with velocity v is $\frac{1}{2}mv^2$.

Potential Energy.—Fig. 35 represents a pile-driver. A massive lump of iron called the ram is raised by means of a rope and pulley several feet above the pile which is to be driven in. It is then let go, and falls with increasing velocity until it hits the pile, and drives it a certain distance into the ground.

Let m be the mass of the ram and v the velocity with which it hits the pile; let h be the height it falls before hitting the pile. Immediately before hitting the pile it has the kinetic energy $\frac{1}{2}mv^2$, which is suddenly spent in doing work on the pile.

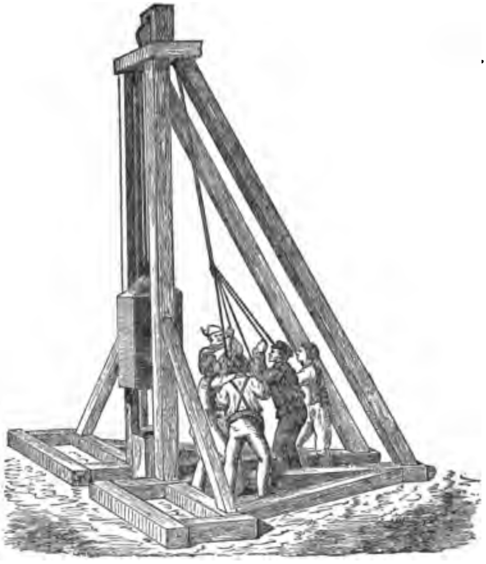


FIG. 35.

Before starting to fall the ram had only potential energy, energy due to its position, and this was all converted into kinetic energy during the fall. Now $v^2 = 2gh$. Hence

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}m2gh = mgh$$

This last expression depends only on the initial position of the ram, and expresses its potential energy in that position.

By similar reasoning it can be shown that any mass m raised a distance h above a plane acquires potential energy mgh with reference to that plane.

Potential Energy of a Stretched Spring.—It is a matter of common knowledge that potential energy can be stored up in springs. Thus, when a watch is wound up, its mainspring is bent, and energy is stored up sufficient to keep the watch going for the next day. In the same way in a spring gun it is the potential energy of a compressed spring which is converted into the kinetic energy of the bullet.

The potential energy of a stretched spiral spring is given by the formula "half tension by extension." Suppose that the spring is pulled out 2 cms. and that it then exerts a force of 100,000 dynes. As it is pulled out its resistance gradually increases from zero to this value. We substitute for the tension 100,000 dynes and for the extension 2 cms. Then the potential energy is

$$\tfrac{1}{2} \times 100{,}000 \times 2 = 100{,}000 \text{ ergs}$$

This result is derived simply from the formula F*s*. The distance moved by the point of application of the stretching force is *s*, in this case 2 cms. F, the average value of the stretching force, is half its final value, *i.e.* $\frac{1}{2} \times 100{,}000$ dynes, since the force increases uniformly from zero to this final value.

The Principle of the Conservation of Energy.—Suppose a stone is carried up a cliff h units high. Then the work done on it is mgh, and this is equal to its potential energy when placed on the edge at the top. Let us suppose now that it is pushed over. When it has fallen a distance x, its velocity is given by $v^2 = 2gx$, and its total energy is

$$\begin{aligned} \tfrac{1}{2}mv^2 + mg(h - x) &= mgx + mg(h - x) \\ &= mgh \end{aligned}$$

the value which it had at the top. As it falls the energy is gradually changed from potential energy into kinetic, but its total amount remains the same.

This is a particular case of a very general principle, namely, the principle of the conservation of energy, which may be stated as follows: Energy can never be created or destroyed. It may be communicated from one body to another, and may be transformed from the potential to the kinetic form, or into other forms, but the total quantity inside a system not acted on by external forces remains always the same.

The principle appears to be contradicted by the example we have chosen, because when the stone strikes the ground at the foot of the cliff both its kinetic and potential energy disappear. This loss is, however, only apparent. The energy reappears again as heat, which is one of its other forms.

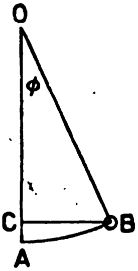


FIG. 36.

We shall here apply the principle of the conservation of energy to the determination of the velocity of the bob of a simple pendulum at its lowest point. Let l be the length of the thread, m the mass of the bob, and ϕ the angle which it makes with the vertical at the extremity of a swing. Then, as has been shown on p. 34, the work done in raising the bob against gravity from A to B is $mgl(1 - \cos\phi)$.

This is its potential energy at B, when of course the kinetic energy is zero. If now the pendulum is let go, when the bob reaches A its potential energy is converted wholly into kinetic energy. Let v be the velocity at A. Then

$$\tfrac{1}{2}mv^2 = mgl(1 - \cos\phi)$$

or
$$v^2 = 2gl(1 - \cos\phi) = 4gl\sin^2\frac{\phi}{2}$$

and
$$v = 2\sqrt{gl}\sin\frac{\phi}{2}$$

EXAMPLES 9

1. A man whose weight is 10 stone climbs a mountain 3800 ft. high in 3 hours. Find in H.P. the average rate at which he works against gravity.

Total work done in ft.-lbs. $= 10 \times 14 \times 3800$

$$\text{Activity} = \frac{10 \times 14 \times 3800}{3 \times 60 \times 60}\text{ ft.-lbs./sec.}$$
$$= \frac{10 \times 14 \times 3800}{3 \times 60 \times 60 \times 550}\text{ H.P.} = {\cdot}090\text{ H.P.}$$

2. A ladder 20 ft. long leans against a vertical wall, and is inclined at an angle of 30° to it. How much work, measured in ft.-lbs., does a man who weighs 12 stone do against gravity in climbing the ladder?

3. How many ergs are there in the ft.-lb.?

4. Find in foot-tons the work done in drawing a truck weighing 10 tons up an incline a mile long, which rises 1 in 120, the coefficient of friction being $\frac{1}{150}$.

5. Find the work done by
   (i) a force of 220 dynes acting through 80 cms.,
   (ii) a force of 16 gms. acting through 2 metres,
   (iii) a force of 18 lbs. acting through 6 inches.

6. The Kinlochleven reservoir is situated 935 ft. above the level of the works; what is the potential energy of the water in ft.-lbs./cub. ft.?

7. Find in ft.-lbs. the kinetic energy of a body of mass 20 lbs. moving with a velocity of 33 ft./sec.

8. A spiral spring is pulled out by the fingers a distance of 10 cms., and when extended this length exerts a force of 250 gms. What is the potential energy stored in the spring in ergs?

9. Find in ergs the kinetic energy of a mass of 17 gms. moving with a velocity of 22 cms./sec.

10. The length of a simple pendulum is 80 cms., and the bob weighs 100 gms. Initially it makes an angle of 20° with the vertical. Find the kinetic energy and hence the velocity of the bob, when (i) it is at its lowest point, (ii) it is 10° beyond its lowest point.

11. A bullet travelling at the speed of 400 metres/sec. is just able to pierce a beam 16 cms. thick. Its mass is 80 gms. Find the resistance of the beam to the bullet in dynes, on the assumption that it is constant. What velocity would enable the bullet to pierce a plank of the same wood 3 cms. thick?

Let F dynes be the resistance offered by the wood.

Then $\quad Fs = \tfrac{1}{2}mv^2$

$$F = \frac{mv^2}{2s} = \frac{80 \times 40000^2}{2 \times 16} = 4 \times 10^9\text{ dynes}$$

Let v be the required velocity. Then

$$v = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2 \times 4 \times 10^9 \times 3}{80}} \text{ cms./sec.} = 173{\cdot}2 \text{ metres/sec.}$$

12. A 10 H.P. engine is employed to pump water from the bottom of a mine 300 ft. deep. How many cubic ft. of water will it raise in an hour?

13. The mass of a train is 300 tons and its velocity is 30 mls./hr. What H.P. would be necessary to give it this velocity in 1 minute? The frictional resistances are to be neglected.

CHAPTER II

HYDROSTATICS

10. THE PROPERTIES OF MATTER

States of Matter.—Matter exists in three states, the solid, the liquid, and the gaseous. A solid body has both a definite shape and a definite volume. A liquid has a definite volume, but no definite shape; it takes the form of the vessel into which it is poured. A gas has neither a definite shape nor a definite volume, but tends to expand indefinitely, so as always to fill the vessel which contains it. Both liquids and gases are included under the name of fluids.

The dividing lines between solid and liquid and between liquid and gas are not sharply defined. Cobbler's wax, for example, occupies a peculiar position. It has a glassy fracture, and can be cast in the form of a bell, which will ring, but if given sufficient time—months or even years—it will flow, and take up the shape of the vessel in which it is placed. Liquids like treacle, which flow very slowly, and which, if dropped upon the floor, take a long time to spread out, are said to be viscous. Viscosity is due to friction between the particles of the fluid. As regards long-continued forces cobbler's wax is a highly viscous liquid, but as regards sound vibrations it is a solid.

The distinction between a solid and a very viscous liquid is, that if an attempt is made to alter the shape of a solid in any way, if, for example, a metal bar is bent, it springs back to its original shape, as soon as the deforming force is removed, and resists the bending with a force which increases with the amount of bending produced. The viscous liquid, on the other hand, once its shape is altered, does not show the slightest tendency to return to the original shape, and resists change with a force quite independent of the change already produced.

Elasticity.—In physics a meaning is attached to the word "elasticity" quite different from its use in ordinary life; it is defined as a property of matter in virtue of which it resists change of shape or change of volume with a force depending on the amount of change produced. There are two kinds of elasticity, elasticity of shape, which in physics is synonymous with rigidity, and elasticity of volume, which is synonymous with incompressibility. Substances like steel or iron possess rigidity in a very high degree, glass in a slightly less degree, and india-rubber in a very much smaller degree. Liquids do not possess rigidity at all. Steel and iron also possess incompressibility in a very high degree; in liquids the force resisting change of volume is smaller, and in gases it is much smaller still.

The nature of an elastic change is best understood by consideration of a special case. If the upper end of a metal wire is clamped, and a scale pan is attached to its lower end, and if weights are put in the scale pan, it is found that the length of the wire is increased, and that the increase in the length of the wire is proportional to the weight put in the pan. Also, when the weights are removed, the wire returns to its original length. The change of form, the increase in length, is referred to as the strain, and the force producing it as the stress. The increase in length is, of course, a very small one; for example, a 5-kg. weight increases the length of a copper wire 5 metres long and 1 mm. diameter only by 2·6 mm. The fact that in this and all similar cases the stress is proportional to the strain was discovered by Robert Hooke in 1676, and is consequently known as Hooke's law.

If, however, the weights are increased beyond a certain limit, after their removal the wire no longer returns to its original length. It is then said to be overstrained, and the weight beyond which recovery is no longer complete is called its elastic limit. If the weight is increased much beyond the elastic limit the wire breaks.

Molecules.—It is now universally assumed that matter is composed of very small particles called molecules. Spaces exist between these particles, and the particles are in constant motion. The reasons for this assumption are chemical as well as physical. In order to explain chemical change it is necessary to assume that the molecules of any one substance are exactly the same, and are made up of smaller particles called atoms. Thus the molecule of common salt is made up of one atom of sodium and one atom of chlorine. The atom is defined as the smallest part of a substance that can take part in a chemical change, and the molecule as the smallest part that can exist in a free state. If we divide a substance into small pieces, and were to divide each of these pieces down, until we came to one that consisted only of a single molecule, the molecule would have essentially the same properties as the substance itself, but the atoms constituting the molecule would have different properties altogether.

In gases the molecules are far apart, and travelling in all possible directions with high velocities. The pressure which a gas exerts on the vessel containing it is due to the bombardment of the side of the vessel by the molecules. In solids and liquids the molecules are much closer together, and thus the greater density of solids and liquids is explained; also they are held together by an attractive force, molecular attraction, which is operative only at very small distances, and about which little is known. Consequently they cannot move far without knocking against one another. When a solid is dissolved in a liquid, its molecules become separated from one another and scattered throughout the liquid. When 50 c.cs. water and 50 c.cs. alcohol are mixed together, the resultant mixture has a volume of only 97 c.cs., the spaces existing between the molecules in the mixture being smaller than in the component liquids.

Capillarity.—Let MN be a water surface, and let A be a molecule situated some distance below the surface. Then it is attracted by all the molecules near it. The molecular attraction between two molecules decreases rapidly as the distance between them increases, and can be neglected altogether when this distance exceeds a certain value, which is called the range of molecular attraction. With the range of molecular attraction as radius describe a sphere round A. Then the molecule at A is attracted by all the molecules within this sphere, and, as they are distributed equally all round it, there are as many pulling one way as another, and the resultant is zero.

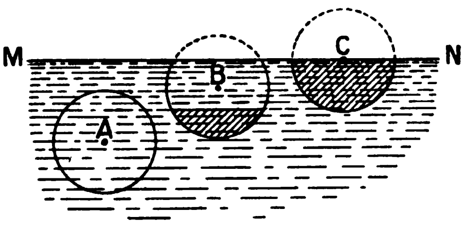


FIG. 37.

Consider now a molecule situated near the surface at B, and with B as centre and the range of molecular attraction as radius describe a sphere. The density of the air molecules is so small that their attraction can be neglected; hence the force due to the shaded part of the sphere is unbalanced, and there is a resultant downward force on B. Consider now a molecule situated on the surface at C, and describe a sphere with the same radius round C as centre. The downward attraction of the whole lower hemisphere is unbalanced, and hence there is in this case a greater resultant downward force on C.

There is a similar force on every particle on the surface tending to pull it into the body of the liquid, so that the surface tends to become as small as possible, and the effect on the liquid is the same as if the surface were surrounded by an elastic skin.

A raindrop or a small drop of mercury is spherical, because, owing to the surface behaving as if it had this elastic skin, it tends to contract until it has the smallest possible surface compatible with its volume, and the geometrical figure with the smallest area for a given volume is the sphere. The force in the surface tending to make it contract is referred to as surface tension or capillarity.

If water is contained in a glass vessel, the surface of the water does not meet the side at right angles, but curves up and meets it tangentially as in Fig. 38. This is because the glass attracts the molecules at C more than the water does. But if the vessel contains mercury instead of water, the surface bends down and meets the glass at an angle of about 52°, as in Fig. 39, because the molecules of mercury on the surface at C are attracted more by the mercury than by the glass. The fact that the molecular attraction of glass for water is greater than that of water for water, and that the molecular attraction of glass for

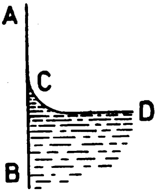


FIG. 38.

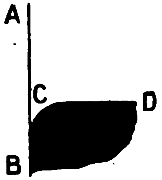


FIG. 39.

mercury is less than that of mercury for mercury, can be shown by dipping glass rods into these two liquids and then withdrawing them; for the rod withdrawn from the water is wet, and the rod withdrawn from the mercury is dry.

Fig. 40 represents a section of a glass tube with a narrow bore, or capillary tube as it is called, dipped into water. Owing to the surface rising to meet the glass it is concave upwards inside the tube, and the surface tension pulls the water up the tube so that the level is higher inside than outside. Fig. 41 represents a section of a similar tube dipped into mercury; here the surface inside the tube is convex upwards, and surface tension depresses the level of the mercury inside the tube. The elevation or depression of the liquid in the tube is inversely proportional to the diameter of the tube, so the effect can be produced only with very narrow tubes.

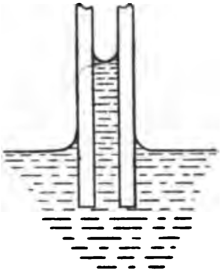


FIG. 40.

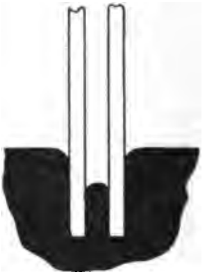


FIG. 41.

Diffusion.—If a glass jar is half filled with water, and then a concentrated solution of copper sulphate is carefully introduced into the bottom of the jar by means of a thistle funnel, the copper sulphate being heavier than water remains at the bottom, and the plane of separation of the copper sulphate and the water is sharply defined; below the plane the solution has an intense blue colour, but above the plane it is quite clear. In the course of a few hours, however, even though the jar is kept perfectly quiet, the blue colour of the copper sulphate has begun to spread into the clear solution, showing that the copper sulphate molecules are moving into the water. To this phenomenon the name of diffusion has been given.

Diffusion takes place also in the case of solids and gases. If a layer of lead is placed upon a layer of gold, molecules of gold may, after a considerable time, be detected throughout the lead. The effect is, however, extremely small and difficult to observe. If a little strong ammonia is introduced into a room, its odour can soon be detected in all parts of the room, even though the air is perfectly still; this is due to the molecules of ammonia diffusing through the air in all directions.

11. FLUID PRESSURE

If several different vessels are connected as shown in Fig. 42 and filled with water, then, when the water comes to rest, its surface is at the same level in the different vessels. This is also the case with two reservoirs miles apart connected by a water main, and is popularly expressed by the phrase that "water maintains its level." The free surface of a liquid at rest is always horizontal, always at right angles to the plumb line. For, if it were not, and one part of the surface were higher than another, gravity would cause the liquid to move from the higher part of the surface to the lower part.

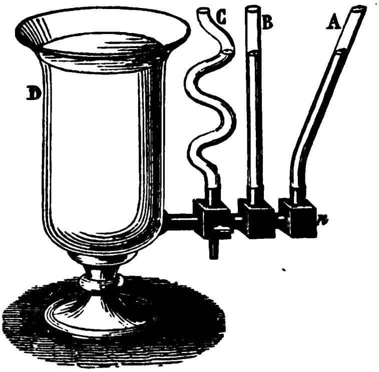


Fig. 42.

From Ganot's Physics.

This fact is taken advantage of in the construction of the spirit level, which consists in its most usual form of a slightly curved glass tube filled with alcohol, except for one small air bubble. Alcohol is used as it is more mobile than water. The tube is fixed in a brass mount. The air bubble forms the free surface of the liquid, and always takes up a position at the highest part of the tube. Two points on the tube are marked, the distance between them being equal to the usual length of the bubble. The instrument is adjusted so that, when it is resting on a horizontal surface, the air bubble comes to rest between these two marks.

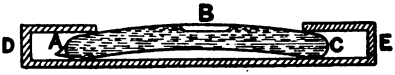


Fig. 43.

If a vessel or a pipe containing liquid at rest is pierced by a small hole, a thin jet squirts out. This jet is always at right angles to the surface of the containing vessel or pipe. Consequently the liquid must press against every part of the surface of the containing vessel with a force at right angles to that part, and the surface must press back with an equal and opposite force.

The density of a substance is its mass per unit volume.

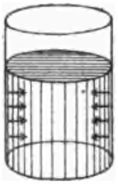


Fig. 44.

Pressure.—Now consider a liquid of density ρ filling a cylindrical vessel to a depth h, as shown in Fig. 44. Let A be the area of the base of the vessel. Then the whole mass of the liquid contained is $Ah\rho$ and its weight $Ahg\rho$. The forces exerted by the sides neutralize one another, as shown by the arrows. The whole weight of the liquid acts, therefore, on the base, equally all over it.

The force per unit area on the base (*i.e.* $Ahg\rho/A$, or $hg\rho$) is said to be the pressure on the base. Thus the pressure on the base depends only on the density of the liquid and the depth of the base below the surface. This was verified by an experiment due to Pascal and known under the name of Pascal's Vases.

Pascal's Vases.—In this experiment three vases A, B, and C with equal bases are taken, A cylindrical, B tapering upwards, and C widened upwards. These vessels have no bottoms. In the figure A is shown held by a clamp. D is a movable disc held against its lower edge by a string attached to one of the scales of a balance. Weights are placed in the other pan to keep the disc against the edge. The vase is filled with water until the pressure is just sufficient to detach the disc from the edge, and the depth of the water noted. The experiment is then repeated with the other two vases, the weights in the other pan being kept the same. It is found that the disc is detached, when the depth of the water is the same in each case.

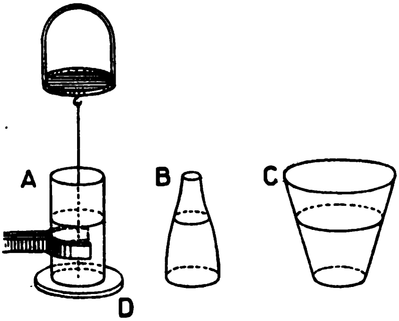


FIG. 45.

The force exerted on the disc must be the same in each case, and the area on which it acts is the same. Hence the pressure on that area is the same, or, in other words, the pressure on the bottom of a vessel is given by the formula $hg\rho$, irrespective of the shape of the vessel.

This result is at first puzzling, because in the experiment the vase B holds less water than A, and the vase C more water than A, and one might expect that for the same depth the pressure on the base would be less in B than in A, and greater in C than in A. But part of the weight of the water in C is supported by the side, and in B the side exerts a downward force against the base.

Pressure at a Point inside a Liquid.—Let us suppose, now, that it is required to determine the pressure at a point P inside the liquid at a depth h below the surface. Imagine a horizontal plane drawn through the point. Then the liquid above the plane presses down on the liquid below it, and the liquid below it presses up on the liquid above it. By considering the weight of the liquid above the plane and proceeding in the same way as on p. 45, we find again that the downward pressure at P is $hg\rho$.

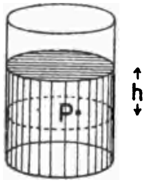


FIG. 46.

Now isolate in imagination a small quantity of the fluid bounded by a horizontal plane EBCF, three vertical planes, namely, FED, FCAD, and CBA, and a plane ABED inclined at any angle to the horizontal, and suppose that the point P is inside this figure. It is clear that, if the liquid were solidified without change of volume, its equilibrium would not be disturbed. Let us therefore suppose it solidified. It is then in equilibrium under the action of the pressure of the liquid on its five faces and its weight acting vertically downwards.

It is clear from symmetry that the forces due to the pressure, or the thrusts as they are called, on the faces FED and CBA balance one another, and do not require to be considered. Let p denote the pressure on the horizontal face, let p_1 denote the pressure on the inclined face, and let p_2 denote the pressure on the vertical faces. The thrust on the upper face is $p\text{FC} \times \text{CB}$. The weight is $g\rho\frac{1}{2}\text{FC} \times \text{CB} \times \text{CA}$. The ratio of the weight to the thrust is, therefore,

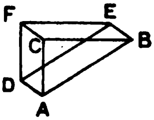


FIG. 47.

$$\frac{g\rho\frac{1}{2}\text{FC} \times \text{CB} \times \text{CA}}{p\text{FC} \times \text{CB}} = \frac{g\rho}{p}\frac{1}{2}\text{CA}$$

Let us suppose that the figure is very small; then CA becomes very small, and the weight can be neglected in comparison with the thrust on the upper face.

The figure is now in equilibrium under the action of three thrusts, one $p_1\text{DA} \times \text{AB}$ perpendicular to AB, one $p\text{FC} \times \text{BC}$ perpendicular to CB, and one $p_2\text{FC} \times \text{CA}$ perpendicular to CA. These three thrusts must obey the triangle of forces. Draw ab, bc, and ca to represent these three thrusts respectively. Then

$$\begin{aligned} ab : bc : ca &= p_1\text{DA} \times \text{AB} : p\text{FC} \times \text{BC} : p_2\text{FC} \times \text{CA} \\ &= p_1\text{AB} : p\text{BC} : p_2\text{CA} \end{aligned}$$

But $\triangle abc$ is similar to $\triangle\text{ABC}$. Therefore

$$ab : bc : ca = \text{AB} : \text{BC} : \text{CA}$$

Thus $\quad \text{AB} : \text{BC} : \text{CA} = p_1\text{AB} : p\text{BC} : p_2\text{CA}$

which is only possible if $p = p_1 = p_2$.

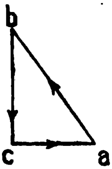


FIG. 48.

Thus the pressure on the plane ABED is equal to p. But the plane ABED was inclined in any direction. When the figure is made infinitely small, the plane passes through the point P. Hence the pressure at P in any direction is $hg\rho$.

U-Tube.—Let us suppose that two liquids which do not mix occupy the arms of a U-shaped tube as shown in the diagram. For the sake of clearness let the liquid DC be oil and the liquid AC be water. Draw a horizontal plane through C to cut the other branch in B. Then the pressure is the same at the two points B and C in the same horizontal. Consequently the pressure due to the column AB of the heavier liquid is equal to the pressure due to the column CD of the lighter liquid. Let h and h' be the heights of the columns of the two liquids respectively and ρ and ρ' their densities; then

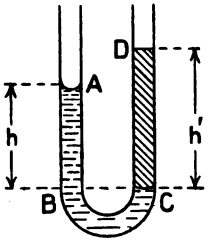


FIG. 49.

$$h\rho = h'\rho' \quad \text{or} \quad \frac{\rho}{\rho'} = \frac{h'}{h}$$

that is, the densities of the liquids are inversely as the heights of the columns.

EXAMPLES II

1. A cubical vessel of 5 cms. edge is filled with mercury to a depth of 2 cms. and then to the top with water. Find (i) the pressure on the bottom, (ii) the total downward thrust on the bottom.

$$\text{Pressure on bottom} = 2 \times 13{\cdot}6 + 3 = 30{\cdot}2 \text{ gms./cm.}^2$$
$$\text{Thrust on bottom} = 25 \times 30{\cdot}2 = 755 \text{ gms.}$$

2. What is the pressure in kgs./cm.^{2} at a depth of 1 kilometre below the surface of the sea, the density of sea water being 1·025 gms./c.c.?

3. Express in lbs./ft.^{2} the pressure at the bottom of a lake 200 ft. deep.

4. A uniform U-tube is about half filled with water. How many c.cs. of oil of density ·85 gm./c.c. must be poured into one limb to make the surface of the water rise 6 cms. in the other. The diameter of the tube is 1 cm.

5. A glass tube open at the top and containing water is gradually tilted, none of the water being spilt, until the pressure at the foot of the tube is diminished by half. What is then the inclination of the tube to the vertical?

12. ATMOSPHERIC PRESSURE

Torricelli's Experiment.—In 1643 Torricelli, a pupil of Galileo, took a glass tube closed at one end, about a yard in length and a third of an inch in diameter, filled it with mercury, stopped the open end with his finger, and inverted the tube with the open end below the surface in a vessel containing mercury. He then removed his finger, and found that the mercury descended in the tube, and after a few oscillations remained stationary with its surface at about a height of 30 inches above the surface of the mercury in the dish. If the glass tube was lowered into the mercury, or inclined to the vertical, the difference in level between the two surfaces remained the same. Obviously the space between the closed end of the tube and the upper surface of the mercury column was a vacuum. If the tube is lowered or inclined, so that the head of the mercury column hits the closed end of the tube, it does so with a characteristic click; there is no air between to act as a cushion. Torricelli concluded from the experiment, that the mercury column was supported in the tube by the pressure of the atmosphere acting outside upon the surface of the mercury in the dish.

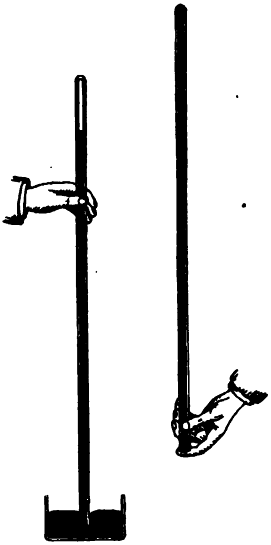


FIG. 50.

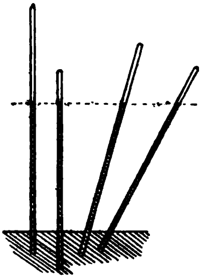


FIG. 51.

Pascal's Experiments.—According to Torricelli's experiment the atmospheric pressure is given by $hg\rho$, where h is the height of the mercury column and ρ is the density of mercury, 13·59 gms./c.c. If the mercury were replaced by water, the column should be 13·59 times as high, or about 34 ft. in height. Reasoning in this way Pascal had a glass tube longer than 34 ft. filled with water and set up vertically. He found his conclusion verified.

Also, since the pressure becomes less in a liquid as we ascend in it, it occurred to Pascal that the atmospheric pressure should be less on the top of a mountain. He accordingly got his brother-in-law Perrier, who lived near Puy-de-Dôme, a mountain near the centre of France, to carry a barometer up this mountain. The latter found the height of the column diminished 8 cms. for a rise of 1000 metres. This decisive experiment was performed in 1648.

The Mercury Barometer.—The pressure of the atmosphere is not constant, but varies from place to place, and at any one place varies from day to day. In all its variations it seldom departs more than five per cent. from its average value. The mercury barometer is an instrument founded on Torricelli's experiment devised to measure these variations. It is made in two forms, the siphon barometer and the Fortin barometer.

The siphon barometer consists simply of a U-shaped tube, the longer limb of which is closed and is about 90 cms. long, and the shorter limb of which is open to the atmosphere. The atmospheric pressure is given by the difference in level of the mercury surfaces in the two limbs, and is expressed usually as a pressure of so many inches or centimetres of mercury. The positions of the mercury surfaces are given by two scales. When the mercury rises in the one limb it falls in the other, and if the diameter of both limbs of the tube is the same, the increase in pressure is equal to twice the rise of the mercury in the closed limb.

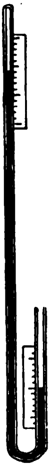


FIG. 52.

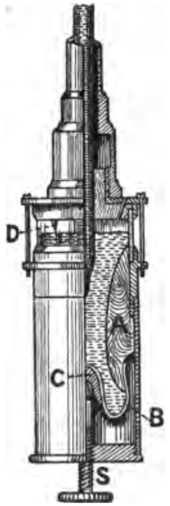


FIG. 53.
From Watson's Physics.

Sometimes the open limb ends in a kind of bulb, and the free surface of the mercury has consequently a much greater diameter than the diameter of the closed tube. In this case the variations of level in the open limb are very small and are neglected.

In the Fortin barometer the tube is straight, and dips into a cistern containing mercury. The upper part of the cistern is of glass, and through the glass the surface of the mercury can be seen. The lower part of the cistern is a leather sack, which carries a wooden button C. This wooden button rests on a screw, and by turning the screw the bottom of the sack can be raised or lowered, and consequently the level of the surface of the mercury in the cistern altered. A small ivory pin D is fixed to the top of the cistern. Before taking a reading the surface of the mercury in the cistern is adjusted until it just touches the point of this pin. The pin is usually mirrored in the surface of the mercury, and, when the level is right, the pin and its image just touch. The vertical scale, which is usually engraved on a metal tube surrounding the glass tube of the barometer, has its zero at the point of the pin.

The Aneroid Barometer.—The mercury barometer, on account of its size and the risk of spilling the mercury, cannot easily be carried about from place to place. So, when a portable barometer is required, another type of instrument, the aneroid barometer, is used instead of it.

The aneroid barometer consists of a shallow cylindrical box, the upper surface of which is corrugated. It is exhausted of air. The atmosphere presses on the corrugated surface, and the latter rises and falls as the atmospheric pressure alters from day to day. This motion is magnified by a system of levers, and communicated to an index which moves round a dial. This dial is calibrated by comparison with a mercury barometer, and so the pressure can be read on it directly in cms. of mercury.

Aneroid barometers are often made as small as watches, and this type is used for determining the heights of mountains. For this purpose the dial is furnished with another scale, which reads heights directly in feet. This scale rotates over the scale of pressures, and is clamped in position at the foot of the mountain with its zero opposite the index. Then, if the atmospheric pressure at the foot remains constant, as the mountain is ascended the index always registers the height climbed. The rough rule is that for every 900 ft. the barometer falls an inch.

Standard Atmospheric Pressure.—By the words "standard atmospheric pressure" is understood a pressure equal to that given by a column of mercury 76 cms. high at a temperature of 0° C. at a place where g is 980·6 cms./sec.^{2} To make the pressure definite it is, of course, necessary to state the temperature, since in the expression $hg\rho$ the density ρ varies with the temperature, and it is necessary to state the value of g, since the latter varies from place to place. As mentioned on p. 49, the atmospheric pressure varies about its standard value. Standard atmospheric pressure $= hg\rho = 76 \times 980{\cdot}6 \times 13{\cdot}596 = 1{,}013{,}250$ dynes per sq. cm., or approximately one million dynes per sq. cm. On the ft.-lb.-sec. system this is approximately 15 lbs. per sq. inch.

The Magdeburg Hemispheres.—If the atmosphere is pressing down on us with a pressure of 15 lbs. per sq. inch, the question arises as to why hollow bodies are not crushed by such a pressure. Why is the lid of a cardboard box, for example, not crushed in by this pressure? The answer is, because the air inside the box has the same pressure as the air outside it. But for this the box would collapse at once. In a celebrated experiment made at Magdeburg by Otto von Guericke, the discoverer of the air pump, two metal hemispheres of 22 inches diameter with flanged edges were fitted together, and the air pumped out. The force thrusting them together due to external atmospheric pressure was then so great, that it required teams of horses to pull them apart.

In finding the pressure on the base of the cylindrical vessel on p. 45, the atmospheric pressure was neglected. This is correct, because the latter acts on both sides of the base. The pressure inside the vessel at A is that due to the column of liquid AB plus that due to the air column BC extending from B to the upper limit of the atmosphere. The pressure outside the vessel at A is equal to that at D, which is on the same level, that is, it is equal to the pressure given by the air column DF extending to the upper limit of the atmosphere. Since the weight of the air in the portion DE can be neglected, the pressures due to the two air columns are the same. Hence the excess of pressure or resultant downward pressure inside the vessel at A is that due to the column of liquid AB.

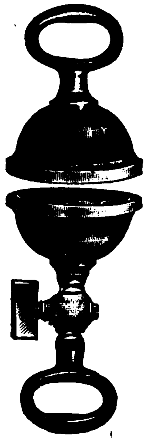


FIG. 54.

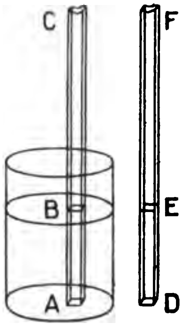


FIG. 55.

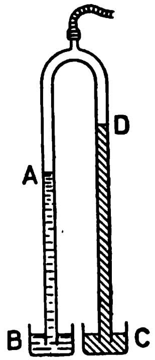


FIG. 56.

Watt's Hydrometer.—Fig. 56 represents Watt's hydrometer, which consists of an inverted U-tube, the arms of which stand in beakers containing different liquids. At the top of the bend a piece of rubber tubing is attached, by which air can be sucked out of the tube. When this is done the liquids rise above their levels in the beakers. Let the heights reached above these levels be h in AB and h' in CD, and let the densities of the liquids in B and C be ρ and ρ' respectively. Let P be the atmospheric pressure. Then, since the pressure is the same inside the tube at A as inside the tube at D, we have

$$P - hg\rho = P - h'g\rho'$$

which gives

$$hg\rho = h'g\rho'$$

or

$$\frac{\rho}{\rho'} = \frac{h'}{h}$$

the same result as was obtained with the U-tube on p. 47, namely, that the densities of the liquids are inversely as the heights of the columns.

If a U-tube is set up in an erect position and half filled with mercury, the liquid comes to the same level in both arms of the tube. If the end of one arm has a piece of rubber tubing attached and we blow into it, the mercury sinks in this arm, the difference of level in the two arms in cms. of mercury being equal to the excess of the pressure of the lungs over the atmospheric pressure. Also, if the rubber tubing is connected to the gas supply, the difference in level gives the excess of the pressure of the gas above atmospheric pressure.

EXAMPLES 12

1. Taking the density of mercury as 13·6 gms./c.c., what would be the height of an oil barometer, the density of the oil being ·845 gm./c.c., when the height of the mercury barometer is 752 mms.?

2. A vertical tube closed at the top is full of water and its lower end stands in water, the surface of the water being 10 ft. below the closed end of the tube. What is the pressure of the water against the top of the tube, given that the water barometer stands at 33 ft.?

3. The tube of a siphon barometer is 7 mm. in diameter, and the open limb terminates in a bulb, so that the free surface of the mercury has a diameter of 20 mm. In taking readings of the daily variations of the atmospheric pressure the change of level of the open surface is ignored. What is the percentage of the error involved?

Let x cms. = change in level of open limb when level in closed limb changes 1 cm. Then

$$7^2 = x20^2, \quad i.e.\ x = \cdot 122$$

The correct change in pressure is therefore 1·122 cms. and the percentage error involved is

$$\frac{\cdot 122}{1\cdot 122} \times 100 = 10\cdot 9 \text{ per cent.}$$

4. A man takes a pocket aneroid up a mountain. At the foot it registers 29·0 inches, at the top 26·8 inches, but on descending to the foot again he finds it gives 28·7 inches. He takes 3 hours to go up and 1½ to come down. What is the height of the mountain?

5. A "sucker" consisting of a disc of leather 3 inches in diameter with a string attached to its middle is moistened and stuck on the pavement. With how many lbs. wt. does the atmosphere press down on it?

6. What would the height of the atmosphere be if its density had the uniform value of 1·293 gms./litre?

7. A U-tube is half filled with mercury and a piece of rubber tubing attached to one end of it. A man blows into the tubing, and creates a difference of level in the two limbs of the tube of 4 cms. What pressure is he exerting in dynes/cm.^{2}?

13. BOYLE'S LAW AND THE AIR PUMP

In 1662 Boyle discovered the law, known by his name, which is stated as follows:—

> The volume of a given quantity of gas varies inversely as its pressure, provided that the temperature remains constant.

The law was discovered seventeen years afterwards on the Continent by Mariotte, and is consequently referred to there as Mariotte's law. Boyle verified it only for small pressures, and it has been investigated since his time by other investigators, notably Amagat, who used pressures as great as 3000 atmospheres. The result of these researches has been to show that the law is not exactly true for wide ranges of pressure.

Fig. 57 represents a simple apparatus used for verifying it, which can be used for pressures above and below atmospheric pressure. AB is a glass tube closed at the end A, which can be fixed in different positions alongside a vertical scale. CD is a glass tube open to the atmosphere at the end C, which can be fixed in different positions on the other side of the same scale. The two tubes are connected at their lower ends by an indiarubber tube, and are filled with mercury to the level E in the closed tube and to the level F in the open one, a quantity of air being imprisoned in the space AE. The diameter of the tube AB is constant; hence the volume of the air enclosed can be taken as proportional to the length of the air column AE. The pressure on the air enclosed is equal to the atmospheric pressure increased by the pressure due to a column of mercury equal in height to the height of F above E. If, as is usual, the height of the barometer is read in cms. and the height of F above E is in cms., the sum gives the pressure of the air enclosed in AE in cms. The verification of the law consists in fixing the tubes in different positions varying between that in which F is as far above E as possible to that in which it is as far below E as possible, and showing that the product of AE and the pressure of the enclosed air is constant.

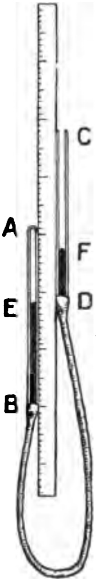


Fig. 57.

The "Short Mercury Column."—This is the name given to a simple experiment which is often performed as an illustration of Boyle's law.

A thin tube of soft glass of about $2\frac{1}{2}$ mm. internal diameter is taken, and a thread of mercury drawn into it. One end of the tube is then sealed off in the bunsen flame. The tube is next held erect in the two positions shown in Fig. 58, first with the closed end A up, and then with it down. The tube is narrow enough for capillarity to prevent the column from breaking, but wide enough to permit of the easy motion of the column as a whole up and down it. Let P be the atmospheric pressure. When the closed end is up, the pressure on the enclosed air is equal to P less the pressure due to a column of mercury BC high, and the enclosed air consequently expands. When the closed end is down, the pressure on the enclosed air is equal to P increased by the pressure due to a column of mercury EF high, and the enclosed air contracts. Its volume

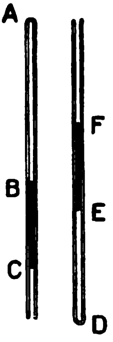


Fig. 58.

is proportional to the length of the air column. Hence, applying Boyle's law,

$$(P - BC)AB = (P + EF)ED$$

which gives

$$P = \frac{BC \times AB + EF \times ED}{AB - ED}$$

$$= \frac{BC(AB + ED)}{AB - ED}$$

since EF = BC. Hence, if the lengths AB, BC, and ED are measured, P can be calculated to about ½ cm. mercury.

The Air Pump.—The air pump is an instrument used for withdrawing air from a vessel. It was invented by Otto von Guericke in 1650. In its simplest form it is represented in Fig. 59. R is the receiver or glass vessel to be exhausted. It consists of a bell jar with a carefully ground edge, which is smeared with lard, and rests on the flat metal plate P, making an air-tight contact. T is the tube connecting the receiver to the pump; S is a stopcock. The pump itself consists of a cylinder in which a piston fits air-tight. There is a hole in the piston, closed by the valve A which opens upward. The end of the tube from the receiver is closed by a valve B, which also opens upward. When the piston descends, the valve A opens, the valve B closes, and all the air in the cylinder is forced out into the atmosphere. On the upstroke the valve A closes, the valve B opens, and some air is sucked out of the receiver through the tube into the barrel of the air pump. On the next downstroke this air is forced out into the atmosphere. And so on; every double stroke of the piston removes some more air, until finally the pressure of the air left in the receiver is not sufficient to lift the valve B.

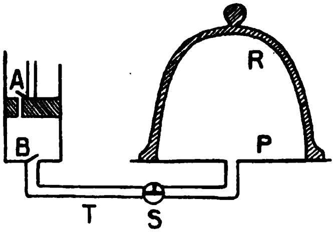


FIG. 59.

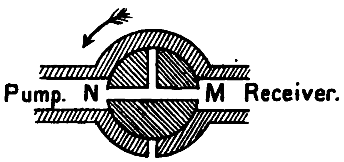


FIG. 60.

It is easy to calculate the theoretical value of the pressure of the gas left in the receiver after n double strokes. For let p be its initial value, let V be the volume of the receiver and the tube connecting it to the pump, and let v be the volume of that part of the barrel swept through by the piston during a stroke. If we start with the piston at the foot of a stroke, the volume of the gas is V; after an upstroke it becomes $V + v$. During the downstroke the volume v is expelled. Hence the fraction

$$\frac{V}{V + v}$$

of the original mass is left. At the end of the next upstroke the gas has again expanded to $V + v$; the same volume v is again expelled in the downstroke, and consequently after two complete strokes the fraction

$$\left(\frac{V}{V+v}\right)^2$$

of the original mass is left. Similarly after n complete strokes the fraction

$$\left(\frac{V}{V+v}\right)^n$$

of the original mass is left, and since by Boyle's law the pressure varies as the density, which in this case varies as the mass, we find that after n complete strokes the pressure is

$$\left(\frac{V}{V+v}\right)^n p$$

Theoretically an absolute vacuum is impossible for $(V/(V + v))^n$ does not become zero, though it becomes very small for large values of n. But when it becomes small, owing to leakage and to the piston not descending accurately to the foot of the barrel, the expression ceases to represent the pressure accurately.

The stopcock S is shown on a large scale beside the diagram of the pump. As shown in the diagram it is set for exhausting the receiver. When the vacuum is as good as can be obtained, the stopcock is turned through 90° in the direction of the arrow. This closes the receiver and prevents air leaking into it through the pump; at the same time it puts the barrel of the pump into communication with the atmosphere. If the cock is turned through a further 90°, the receiver is put into communication with the atmosphere and the vacuum destroyed.

For measuring the pressure in the receiver a gauge depicted in Fig. 61 is used. This consists of a U-tube closed at the end G, and filled with mercury to the closed end. The sides of the tube are about three inches long. It acts as a kind of siphon barometer, the pressure at G being less than the pressure at F by about $2\frac{1}{2}$ inches of mercury. The gauge is placed inside the receiver or, more usually, in a cover of its own connected with the receiver. As long as the pressure in the receiver is more than $2\frac{1}{2}$ inches, the gauge shows no change. But, when the pressure falls below this value, the mercury column comes away from the closed end leaving a vacuum there, and the difference of level in the two arms of the tube gives the pressure in the receiver. For a perfect vacuum the mercury should stand at the same level in each branch.

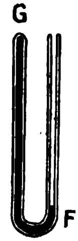


FIG. 61.

The Töpler Pump.—The mechanical pump just described does not give a good vacuum. For better work very many different patterns have been made. One of the most useful, the Töpler pump, is shown in Fig. 62. It can reduce the pressure to ·000012 millimetre of mercury.

It consists of a glass vessel A to which there are attached vertical tubes EF and DG, both about 32 inches long. F is connected by a piece of rubber tubing with a reservoir L containing mercury, and DG dips into a trough containing mercury. The vessel to be exhausted is connected on to the side branch CH. To work the pump the vessel L is raised above the level of D; the mercury consequently rises inside the pump, sealing up the side tube CH and driving all the air in A before it down DG and out into the atmosphere. It is then lowered; a vacuum forms at D, the level of the mercury sinks to the lower end of B, and immediately some of the gas to be exhausted rushes in from HC and fills the vacuum. L is then raised, so as to fill A with mercury and drive the gas out through DG. The mercury here plays the part of the piston and valves in the mechanical pump. The side branch B is not absolutely essential to the pump, but it prevents the gas rushing into A too violently, and diminishes risk of breakage. The disadvantage of the Töpler pump is that it is slow in its action.

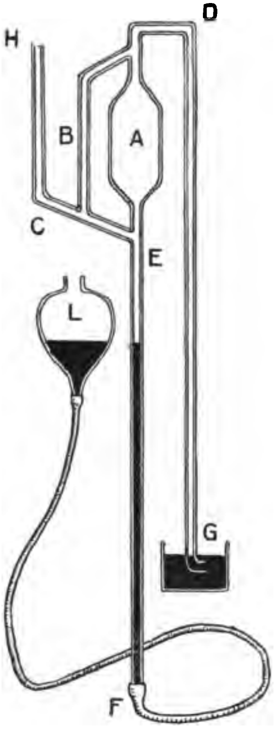


FIG. 62.

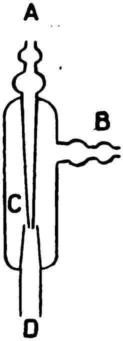


FIG. 63.

The Filter Pump.—Fig. 63 represents another type of pump, the filter pump or water pump. It is usually made of glass. The tube A is connected by a piece of rubber tubing to a water pipe, and the side tube B by a piece of rubber tubing to the vessel to be exhausted. The water issues from the glass nozzle C at a very high speed, and escapes by the tube D carrying out some of the air with it. Consequently air is sucked in through the opening B. The filter pump does not give a good vacuum, but is very convenient, working without any attention whatever.

Compression Pumps.—A compression pump, the commonest example of which is the ordinary bicycle pump, is used for increasing the pressure of the air inside a certain space. The simple pump represented in Fig. 59 acts as a compression pump, if the valves A and B are arranged so as to work the other way, *i.e.* so that A closes on the downstroke, and B closes on the upstroke. In the bicycle pump the piston consists of a cup-shaped disc of leather clamped to a piece of metal of slightly smaller diameter than the barrel. When the piston forces the air into the tyre, the leather cup is pressed against the side of the barrel, and so the air does not pass it. When the piston is drawn back, the leather leaves the side of the barrel, allowing the air to pass. The arrangement is called a cup valve. There is no other valve in the pump, the other valve necessary to the working of the arrangement being on the tyre. The pump is not an efficient one; in addition to leakage in the valve and the connecting tube there is the loss due to "untraversed space," owing to the piston not sweeping out the air in the connecting tube.

Examples 13

1. A quantity of oxygen was found to measure 146·8 c.cs. when the barometer stood at 78 cms. On the next day the volume had increased to 157·3 c.cs.; what was now the barometric height?

2. The volume of a quantity of gas is 159 c.cs. when the barometer stands at 72 cms. What is it at normal pressure? At what pressure would it have a volume of 200 c.cs.?

3. A piston is situated in the middle of a cylinder 12 inches long, and there are equal quantities of air on each side of it. The piston is pushed down until it is an inch from one end; what is then the ratio of the pressures on the two sides?

4. A tube 150 cms. long, closed at one end, is half filled with mercury and is then inverted with its open end just dipping into a mercury trough. No air escapes from the tube, and the barometer stands at 75 cms. What is the height of the mercury inside the tube?

Let x cms. = height of mercury inside the tube. Then the initial volume is 75 cms. long and the initial pressure 75 cms., the final volume $150 - x$ cms. long, and the final pressure is $75 - x$.

$$\begin{aligned} 75 \times 75 &= (150 - x)(75 - x) \\ 75^2 &= 150 \times 75 - 225x + x^2 \\ 0 &= x^2 - 225x + 75^2 \end{aligned}$$

or $x = 28{\cdot}6$ cms., the other root being inadmissible.

5. The tube of a barometer has a cross-sectional area of 1 cm.^{2}, and when the column stands at 74 cms. 2 c.cs. air at atmospheric pressure are passed up the tube. How far is the surface of the mercury depressed, if initially it was 10 cms. below the closed end of the tube?

6. A barometer has a little air in the space at the top of the mercury column, and as a result reads 28 inches when it ought to read 29 inches, and reads 29 inches when it ought to read 30·2 inches. What is the real value of the atmospheric pressure when it reads 29·4 inches?

7. A column of mercury 8 cms. long is introduced into a thin tube, and one of the ends of the tube sealed off in a bunsen flame enclosing a quantity of air between this end and the mercury. After the tube has cooled to the temperature of the room, it is suspended with its closed end up, and the length of the air column is found to be 20·0 cms.; when it is suspended with the closed end down, the length of the air column is 16·1 cms. Find the atmospheric pressure.

8. If in the previous question the atmospheric pressure had been 73 cms., the length of the mercury column 12 cms., and the length of the air column 15 cms. when the closed end was down, what would it have been when the closed end was up?

9. To what fraction of its original value has the pressure fallen in a theoretically perfect air pump after 10 strokes, if the volume of the receiver is three times that of the barrel?

$$\text{Result} = \left(\frac{V}{V + v}\right)^n = \left(\frac{3}{4}\right)^{10} = {\cdot}0575$$

since $10(\log 3 - \log 4) = \log {\cdot}0563$

10. After five complete strokes the density of the air in the receiver of an air pump is $\frac{2}{5}$ of its original value. What is the ratio of the volume of the receiver to the volume of the barrel?

11. The receiver of an air pump has a volume of 1400 c.cs., and it contains air of density ·001293 gm./c.c. at normal pressure. The pump is worked until the density of the air is reduced 30 per cent. Find the mass of the air removed.

12. Air is forced into a vessel by a compression pump, the barrel of which has $\frac{1}{10}$ the volume of the vessel. Calculate the density of the air in the vessel after 15 complete strokes.

14. WATER PUMPS AND THE HYDRAULIC PRESS

The Common Suction Pump.—Fig. 64 represents the common suction pump which has been used from ancient times for raising water out of wells, etc. A represents the level of the water in the well. From A a vertical pipe AB leads to the barrel of the pump. At B there is a valve opening upwards; at C there is a valve in the piston also opening upwards.

Initially the water stands at the same level at A both inside and outside the pipe. During the first strokes therefore the pump acts as an air pump, removing the air from the pipe AB and sucking up the water. Then, when the water reaches the barrel of the pump, on the downstroke the valve B closes, and C opens; consequently the water passes from below to above the piston. On the upstroke the valve B opens and the barrel fills again with water, while the valve C closes, and the water already above the piston is lifted up and escapes by the outlet at D.

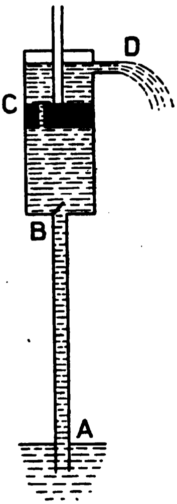


FIG. 64.

Since the pump acts at first by creating a vacuum in the barrel and so letting the atmospheric pressure force the water up the pipe AB, the pump should not work if AB is greater than 34 ft., the height of the water barometer. Owing to mechanical imperfections the limit is less than this, about 28 ft. or thereabouts. Pouring in a little water at the top makes the valve C and the contact between the piston and the side of the barrel more air-tight, and assists greatly in starting the pump.

The Force Pump.—Fig. 65 depicts the force pump. It differs from the suction pump in having no valve in the piston, in having the outlet at the foot of the barrel, and in having a valve D opening outwards at the outlet.

Initially the pump works as an air pump until the level of the water reaches the barrel. Consequently it is subject to the same limitation as the suction pump, *i.e.* AB must be less than the water barometer. Then when the water reaches the barrel, on the downstroke the valve B closes and D opens, and the piston forces the water up the tube DF; on the upstroke D closes, B opens, and the barrel fills with water.

The force pump is used when the water has to be delivered at a point higher than the position of the pump itself, as for example in the case of the fire engine. The latter consists of a double-barrelled force pump, the downstroke in the one barrel corresponding to the upstroke in the other. The height to which it forces the water is the height above the engine of the highest point, which the jet of water played upon the flames can reach.

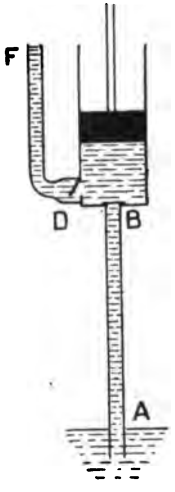


Fig. 65.

The Siphon.—The siphon is an arrangement for draining off the upper layers of liquid in a vessel without disturbing the lower layers, or for emptying a vessel that cannot be overturned. It consists of a bent tube, one arm of which is longer than the other.

Let us suppose that it is to be used for removing water from the vessel in Fig. 66. Then it is first filled with water, the ends being closed with the fingers, and placed in the position shown in the figure. The ends are then opened by removing the fingers, and a steady stream of water flows through the tube from A to D.

In order to understand the action of the siphon consider two points B and C in the tube at the top of the bend. Let h be the height of B above the level of the surface at A, let h' be the height of C above the level of the surface at D, let H be the height of the water barometer, and suppose the liquid in the tube for an instant at rest. Then the pressure at B calculated from the end A is $H - h$ in cms. of water, while the pressure at C calculated from the end D is $H - h'$ in cms. of water. If we consider the element of liquid between B and C, the pressure on the end B exceeds the pressure on the end C by $(H - h) - (H - h')$ or $h' - h$. Hence the liquid will flow from B to C and down to D, while the atmospheric pressure forces more liquid into the tube at A. Thus a steady stream is maintained.

The siphon works as long as h' is greater than h. Also it is not necessary for the end D to dip below a surface of water; the water may issue from D in a jet. In that case h' is measured from the end of the tube. The siphon would not work if h were greater than H; for in that case a vacuum would form at the top.

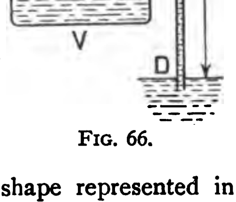


Fig. 66.

Transmission of Fluid Pressure.—Any pressure communicated to any part of a fluid is communicated without diminution to every other part of the fluid. This law was first stated by Pascal.

Let us suppose we have a vessel of the shape represented in Fig. 67, terminating above in a tube and filled to the level A with water. Then the pressure at a point P situated h cms. below A is $hg\rho$, or, since $\rho = 1$, in gravitational units simply h gms. per cm.^{2} If now water is poured into the vessel so as to raise the level 1 cm. from A to B, then the pressure at P becomes $h + 1$ gms. per cm.^{2} Thus at every point inside the vessel the pressure increases by 1 gm. per cm.^{2} The increase of pressure might equally well have been produced by pressing a closely fitting piston down the tube instead of pouring water into it. The result is the same in each case, and it is also obviously independent of the shape of the vessel. We see, therefore, that Pascal's law for the transmission of fluid pressure is an inevitable consequence of the pressure formula.

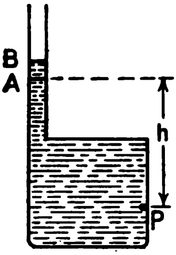


FIG. 67.

The Cartesian Diver.—The transmission of fluid pressure is illustrated by the toy called the Cartesian diver (Fig. 68). This consists of a glass cylinder nearly full of water and closed at the top by a brass cap, through which works a piston. By lowering the piston the pressure of the air and consequently of the water inside the cylinder is increased. Inside the water is a little figure attached to a glass ball *a* which contains water and air; in the lower part of this ball there is a small hole by which water can enter or escape. When the pressure in the water is increased more water is forced into the ball, and it becomes heavier and sinks; when the pressure is diminished, the air expands, forcing the water out, and the ball rises. Thus by taking pressure off and putting pressure on to the piston the diver can be made to move rapidly up and down the cylinder. The apparatus can be simplified by using instead of the cap and piston an ordinary jar with a sheet of indiarubber tied tightly over its mouth. The pressure of the hand on the indiarubber is then sufficient to make the diver sink to the bottom of the jar.

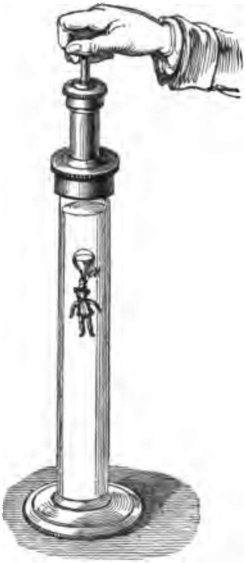


FIG. 68.

From Ganot's Physics.

The Hydrostatic Paradox.—Let us suppose we have two cylinders, one of large cross-sectional area A and one of small cross-sectional area *a*, and that these are fitted with water-tight pistons and filled with water. The pistons are in equilibrium, each resting on the top of the water.

Let us suppose now that a weight w is placed on the smaller piston; then the increase of pressure produced is given by $p = w/a$. The increase of pressure at once exerts an upward force pA on the other piston, and to prevent it rising we have to place on it a weight $W = pA$. Then the weights W and w balance one another. Now

$$\frac{W}{w} = \frac{pA}{pa} = \frac{A}{a}$$

and W is many times greater than w. This result has often appeared paradoxical.

It is, however, quite in accordance with the principle of work. For let the small piston be lowered through a distance d and let the large piston be consequently forced up a distance D. Then the volume of water expelled from the small cylinder is ad, and the volume forced into the large cylinder is AD. These must be equal; therefore $ad = AD$. The work done by the weight w in falling a distance d is $wd = pad$; the work done against the weight W in being raised through D is $WD = pAD$. But $pad = pAD$. Hence the work done by the small weight is exactly equal to the work done against the large weight. The contrivance is, in fact, a "machine" similar to the lever.

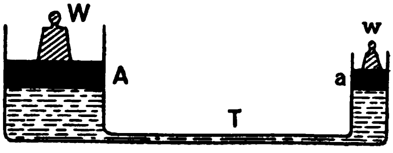


FIG. 69.

Hydraulic Press.—This machine embodies the principle of the hydrostatic paradox, and was invented by Bramah in 1795. It is used for compressing paper and cotton bales, making dies, etc. The object to be compressed is placed between the ram R and the cross-piece T. The ram is connected to a plunger or solid cylinder, which

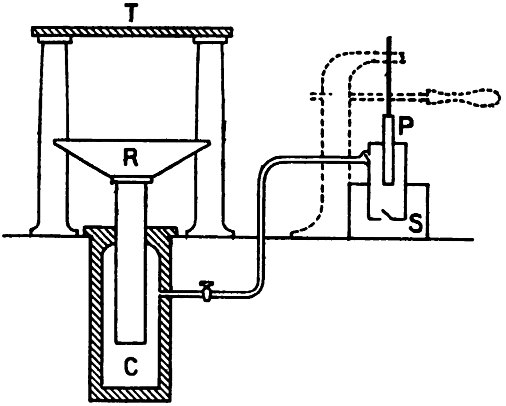


FIG. 70.

works in the large metal cylinder C; the latter has very strong sides. The pressure on the sides of the plunger has no tendency to drive it in or out, so it may be regarded as a piston with the same area as its end. The large cylinder C is connected by a metal pipe with the small cylinder, in which works the plunger P actuated by a lever handle. The small cylinder is provided with two valves, and acts as a force pump, taking water from the reservoir S and pumping it into the large cylinder.

Let F be the force exerted in pushing down the handle, let L be the distance of its line of action from the fulcrum, and let l be the distance of the line of action of the plunger P from the fulcrum. Then the force exerted by the plunger P is LF/l. Let a and A be the cross-sectional areas of the plungers P and R respectively. The pressure in the two cylinders is consequently $LF/(la)$ and the force exerted on the ram

$$\frac{LFA}{la}$$

The total mechanical advantage of the press is consequently $LA/(la)$, or the product of the mechanical advantage of the lever handle by the mechanical advantage due to the inequality of the areas of the plungers.

EXAMPLES 14

1. The radius of the barrel of a suction pump is 3 inches. The piston is 15 ft. above the surface of the water in the well, and the height of the column of water above the piston is 2 ft. Find in lbs. weight the force necessary to sustain the piston in its position.

Let x be the height of the water barometer. The pressure on the under side of the piston is equal to a head of $x - 15$ ft. of water. The pressure on the upper side is equal to a head of $x + 2$ ft. of water. The difference of pressure is $(x + 2) - (x - 15) = 17$ ft. of water. The force is

$$17 \times 62\cdot5 \times \pi(\tfrac{1}{4})^2 = 208\cdot5 \text{ lbs.}$$

2. The tube of a suction pump is 6 yards high. The cross-sectional area of the barrel is 12 times that of the tube. The length of stroke of the piston is 18 inches. To what height will the water be raised in the tube by the first upstroke? The air inside the pump is originally at a pressure of 30 inches of Hg, and the piston is originally at the foot of the barrel.

3. The cross-sectional area of the small plunger in a Bramah press is 1 sq. inch, and that of the large plunger 100 sq. inches. The lever handle gives a mechanical advantage of 5. What is the force exerted by the ram when a force of 1 cwt. is applied to the handle?

4. The two plungers of a hydraulic press have diameters of an inch and a foot respectively. The downward force on the smaller plunger is 56 lbs. What is the force exerted by the ram? If the stroke of the smaller piston is $1\frac{1}{4}$ inches, through what distance has the ram moved after 10 strokes?

15. THE PRINCIPLE OF ARCHIMEDES AND SPECIFIC GRAVITY

The Principle of Archimedes.—Let us suppose that a liquid is at rest and that a certain portion of it, bounded by the closed surface P indicated in the diagram, suddenly becomes rigid without change of density, *i.e.* suddenly becomes a solid. Then this solid is still in equilibrium with the rest of the liquid. Now it is acted on by (i) its own weight vertically downwards; (ii) all the pressure-forces exerted by the rest of the fluid at right angles to its surface and indicated by the arrows in the diagram. Since the solid is in equilibrium, these pressure-forces must have a resultant equal and opposite to the weight.

Let us suppose now that the solidified liquid is removed, and that in its place we have a solid body of exactly the same shape. Then the pressure-forces on its surface are the same as on the surface of the solidified liquid. Consequently the liquid exerts an upward force on it equal to the weight of the liquid it displaces. If it is heavier than the liquid it displaces it will sink, and if it is lighter than the liquid it displaces it will float.

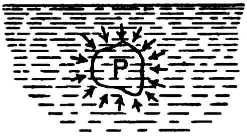


Fig. 71.

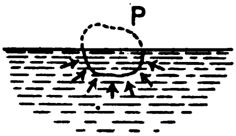


Fig. 72.

Let us suppose now that the portion of liquid which becomes solidified is bounded partly by the surface of the liquid (Fig. 72). The resultant of all the pressure-forces exerted by the rest of the liquid on the solidified part is still equal and opposite to its weight. If we suppose the solidified part replaced by a solid body, which has exactly the same shape below the level of the surface of the liquid as the solidified part has, then the upward force exerted by the liquid on the solid is equal to the weight of the solidified part, *i.e.* to the weight of the liquid it displaces.

We arrive, therefore, at the general result: if any body is wholly or partially immersed in a liquid, it is acted on by an upward force equal to the weight of the liquid displaced. This result is known as the principle of Archimedes.

Experimental Illustration.—The principle is illustrated by a well-known class experiment. In this experiment two cylinders A and B are used; B is of solid brass, and A is hollow and of such a size, that B can just fit inside it. B is attached by a hook to the bottom of A, and A is suspended from one of the arms of a balance. Weights are then placed in the other pan until the balance is in equilibrium. A vessel containing water is then raised from below, so as to immerse B completely, and the other side of the balance goes down at once. When water is poured into A so as to fill it to the top, equilibrium is once more restored.

Thus the loss of weight experienced by B when immersed in water is equal to the weight of the water contained by A, *i.e.* to the weight of the water displaced by B.

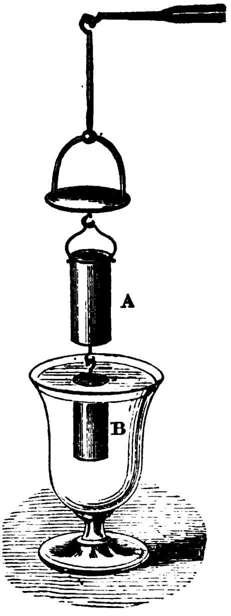


Fig. 73.

The principle of Archimedes applies to gases as well as to liquids. Thus all bodies immersed in the atmosphere experience an upthrust equal to the weight of air they displace. As the density of air is small, being only 1·293 gms. per litre when dry, at atmospheric pressure and at 0° C., this upthrust can usually be neglected in comparison with the weight of the body. The weight of large volumes of air is, however, by no means negligible. Thus a room 14 ft. long, 9 ft. broad and 10 ft. high contains under standard conditions about 102 lbs. of air. It is the upthrust due to the weight of the displaced air that causes balloons and airships to float.

In very accurate weighings it is necessary to take account of the weight of the air displaced by the body. The weighing is then said to be reduced to vacuo. Let W be the true weight of the body, *i.e.* its weight in vacuo, and let w be the true weight of the counterpoising weights. Let D be the density of the body, d the density of the weights, and σ the density of the air. Then the volume of the body is W/D, the volume of the weights is w/d, the weight of air displaced by the body is $\sigma W/D$, and the weight of air displaced by the weights is $\sigma w/d$. For equilibrium

$$W - \frac{\sigma W}{D} = w - \frac{\sigma w}{d}$$

Hence

$$W = \frac{w\left(1 - \frac{\sigma}{d}\right)}{1 - \frac{\sigma}{D}}$$

$$= w + w\sigma\left(\frac{1}{D} - \frac{1}{d}\right)$$

since σ is small compared with D and d.

If, for example, a wooden cube of side 3 cms. is weighed in air and found to be 20·50 gms., and the weights are of brass of density 8·4, then $w = 20{\cdot}50$ gms., $d = 8{\cdot}4$ gms. per c.c., $D = \frac{20{\cdot}5}{27}$, *i.e.* ·759 gm. per c.c., and $\sigma = {\cdot}001293$ gm. per c.c.

Thus

$$w\sigma\left(\frac{1}{D} - \frac{1}{d}\right) = 20{\cdot}50 \times {\cdot}001293\left(\frac{1}{{\cdot}759} - \frac{1}{8{\cdot}4}\right)$$
$$= {\cdot}0265(1{\cdot}32 - {\cdot}12)$$
$$= {\cdot}0265 \times 1{\cdot}20$$
$$= {\cdot}032 \text{ gm.}$$

and the true weight of the body is ·032 gm. greater than the uncorrected result. The density of the air at the time of weighing would probably differ somewhat from the value taken above, but the difference would not affect the result appreciably.

The density of a body is defined as its mass per unit volume. The specific gravity or the relative density of a body is defined as the ratio of its density to the density of some standard substance. Water at a temperature of 4° C. is usually taken as the standard; its density is then 1·000 gm. per c.c. Thus the specific gravity of a body is numerically equal to its density on the c.g.s. system. It must, however, be remembered that density is a physical quantity measured in mass per unit volume, while specific gravity is merely a number, the ratio of two quantities. The specific gravity of a substance is consequently the same, no matter what system of units is used.

It is useful to remember that the density of water on the ft.-lb.-sec. system is 998·8, almost 1000 ozs. per cubic ft., or 62·43 lbs. per cubic ft.

The Hydrostatic Balance.—The most obvious way of determining the specific gravity of a solid is by means of the hydrostatic balance. This is an ordinary balance, over one of the pans of which is placed a small wooden bridge or stool carrying a beaker filled with water. The stool and beaker are arranged so that the balance swings without touching either of them.

If the solid sinks in water, the beaker is removed, and the solid is suspended by a thread from the hook that carries the scalepan, so that it hangs above the stool, and its weight in air, a, determined. The beaker is then inserted so that the solid is suspended in the water, clearing the side of the beaker, and the weight of the solid in water, w, determined. The weight of the water displaced is consequently $a - w$, and the specific gravity of the solid is

$$\frac{a}{a - w}$$

If the solid floats in water, it is first weighed in air. Let the result be a. It is then tied to a piece of metal sufficient to sink it, and they are weighed together in water. Let the result be W. The sinker is next weighed alone in water. Let the result be s; then s is greater than W. It is not necessary to know the weight of the sinker in air. The solid alone has an apparent negative weight in water equal to $W - s$. Thus the weight of the water it displaces is $a + s - W$, and its specific gravity is

$$\frac{a}{a + s - W}$$

If the solid dissolves in water, it can be weighed in another liquid of known specific gravity, and from its loss of weight in this liquid, its loss of weight in water can be calculated.

The hydrostatic balance can also be used to determine the specific gravity of a liquid. In order to do this we find the weight of a solid first in air (a), then in water (w), and finally in the liquid (l). The weight of water displaced is $a - w$, and the weight of the liquid displaced $a - l$. The specific gravity of the liquid is thus

$$\frac{a - l}{a - w}$$

The solid employed should not have a high density, otherwise a, w, and l do not differ much, and consequently the differences cannot be obtained with such accuracy. The methods of the hydrostatic balance can, of course, be used with the spring balance. It is faster, but not nearly so accurate as the ordinary arrangement.

The Specific Gravity Bottle or Pyknometer.—The specific gravity bottle, which is shown in Fig. 74, consists of a small bottle with a carefully ground-in stopper, which has a fine hole passing through it. It is used for determining the specific gravity of a liquid, or of a solid which is in the form of small pieces or a powder, and so is unable to be used with the hydrostatic balance.

In determining the specific gravity of a liquid the bottle is first dried and weighed empty, then filled with the liquid, and the excess which escapes through the hole in the stopper wiped off; it is then weighed again. The difference of the two weighings gives *l*, the weight of the liquid contained. The bottle is then emptied, dried, filled with water and weighed. The difference of this weighing and the first one gives *w*, the weight of water contained. The specific gravity of the liquid is then simply l/w.

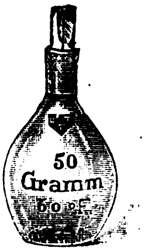


FIG. 74.

In determining the specific gravity of a powder the bottle is first weighed empty, then full of water, and the difference gives *w*, the weight of the water contained. It is next emptied, dried, filled about three-quarters full of the powder and weighed again. The difference of this weighing and the first one gives *a*, the weight of the solid in air. The bottle is then filled up with water, care being taken to get rid of the air bubbles clinging to the powder, and weighed again. The difference of this weighing and the second one gives *e*, the excess of the weight of the powder over the weight of the same volume of water. Hence $a - e$ gives the weight of the water displaced by the solid, and consequently

$$\frac{a}{a-e}$$

the specific gravity of the solid.

Variable Immersion Hydrometers (Baumé's, Twaddell's).—Instruments of this type are used in commerce for measuring the specific gravity of alcohol, milk, acids and sugar solutions. One is shown in Fig. 75. It consists of a glass tube with a bulb in the middle, which is loaded at the foot with mercury. The stem is hollow and has a scale marked on it. Owing to its being loaded with the mercury the instrument floats erect. The greater the specific gravity of the liquid, the higher the hydrometer floats out of the liquid. The specific gravity is given by the reading on the stem at B, where the level of the surface cuts it.

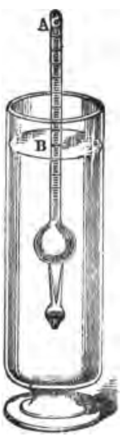


FIG. 75.

Instruments of this type are graduated by immersion in liquids of known specific gravity, so they are not "absolute" instruments, *i.e.* one cannot be constructed without the help of one of the other methods of determining specific gravity. The specific gravity of a solution is usually determined for some special purpose, for example, for finding the percentage of alcohol in spirit. Hence, to save trouble, the hydrometer may be graduated in percentage of alcohol, and not in specific gravity. In order to obtain great accuracy the stem of a variable immersion hydrometer would have to be inconveniently long. Hence such hydrometers are usually supplied in sets; for example, a set might consist of seven instruments, one for each of the ranges ·700–·850, ·850–1·000, 1·000–1·200, 1·200–1·400, 1·400–1·600, 1·600–1·800, 1·800–2·000.

Nicholson's Hydrometer.—This instrument, which is not very accurate, and is used only for instructional purposes, is shown in Fig. 76. It consists of a hollow cylinder of metal B with conical ends, terminated at its upper end by a thin rod carrying a scalepan A and carrying at its lower end another pan C loaded with lead. The instrument thus always floats vertical. In the thin rod there is a fixed mark.

Let h be the weight of the hydrometer. Then, in using it to determine the specific gravity of a liquid, it is floated in the liquid and weights l are added to the upper pan to sink it to the mark. It is then floated in water and weights w added to the upper pan to sink it to the mark. The weight of the liquid displaced is $h + l$ and the weight of water displaced $h + w$. The volume is the same in each case. Hence the specific gravity of the liquid is

$$\frac{h + l}{h + w}$$

In order to determine the specific gravity of a solid, the hydrometer is floated in water and weights w added to the upper pan to sink it to the mark. The weights are then removed, the solid placed in the upper pan, and weights a added to sink the instrument to the mark again. The difference $w - a$ is obviously the weight of the solid in air. The upper pan is again emptied, and the solid next placed in the lower, as shown in the fig. If it floats, it is tied down. Weights b are then placed in the upper pan to sink the instrument to the mark again. The difference $w - b$ consequently gives the weight of the body in water, $(w - a) - (w - b)$, i.e. $b - a$ the weight of the liquid displaced by the solid, and

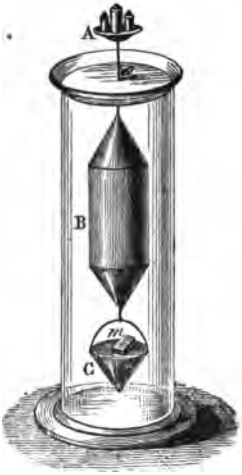


FIG. 76.

$$\frac{w - a}{b - a}$$

its specific gravity.

The various formulæ for the determination of specific gravity should never be memorized, but when a numerical result is wanted it should be worked out from first principles.

Density of a Mixture.—Let volumes v_1, v_2 of different materials be mixed together without swelling or shrinking. Then the volume of the mixture is $v_1 + v_2$. Let d_1 and d_2 be the densities of the two materials respectively. The mass of the mixture is consequently $v_1d_1 + v_2d_2$, and its density is

$$\frac{v_1d_1 + v_2d}{v_1 + v_2}$$

Suppose that on mixing there is an n per cent. contraction in the volume, *i.e.* 100 c.cs. become 100 − n c.cs. The density of the mixture is then

$$\frac{100(v_1d_1+v_2d_2)}{(100-n)(v_1+v_2)}$$

EXAMPLES 15

1. A wooden sphere of diameter 4 cms. is weighed in air and the result found to be 22·57 gms. The weights are of brass of density 8·4 gms./c.c. Find the weight of the sphere in vacuo.

2. According to the newspapers the Zeppelin L 33 was 680 ft. long, her envelope was filled with 2,000,000 cub. ft. of gas, and she weighed, together with her crew and equipment, 50 tons. What is the lifting capacity in tons weight of this volume of (i) hydrogen, (ii) coal gas? The density of hydrogen is ·0899 gm./litre and the density of coal gas ·52 gm./litre.

3. The volume of a balloon is 800,000 litres and the weight of envelope and car together 400 kg. If it is filled with coal gas of density ·52 gm./litre and the density of the air is 1·28 gms./litre, what additional weight can the balloon carry in the car?

4. A cylindrical piece of wood of height h, which is floating with its axis vertical in a beaker of water, is placed together with this beaker in the receiver of an air pump, and the latter exhausted. Show that it sinks a distance $hd(1-\mathrm{D})/(1-d)$, where D is the relative density of the wood and d the relative density of air.

5. Calculate in kgs. wt. the upward force on a spherical balloon of oiled silk, which is filled with impure hydrogen of density ·100 gm./litre. The total weight of the silk envelope is 55·3 kgs., and it weighs 200 gms./metre^2.

6. A solid is weighed in air and in water, and the results are found to be 32·680 gms. and 18·213 gms. respectively. The weights are of brass of density 8·4 gms./c.c. Find the weight of the solid in vacuo.

7. A sphere of 3 cms. diameter weighs 40·032 gms. and a sphere of 5 cms. diameter 40·066 gms. in vacuo. They are placed in the pans of a balance under the receiver of an air pump, and the air gradually exhausted. At what pressure will they balance?

Let p be the required pressure in cms. of mercury. Then ρ, the density of the air displaced, is

$$p \times {\cdot}001293/76 = p \times {\cdot}00001701 \text{ gms./c.c.}$$

By Archimedes' principle

$$40{\cdot}032 - \tfrac{4}{3}\pi(\tfrac{3}{2})^3\rho = 40{\cdot}066 - \tfrac{4}{3}\pi(\tfrac{5}{2})^3\rho$$
$$\tfrac{4}{3}\pi\tfrac{98}{8}\rho = {\cdot}034$$
$$\rho = {\cdot}0006627$$

and
$$p = 39 \text{ cms.}$$

8. A piece of metal weighs 31·57 gms. in air and 28·8 gms. in water. What is its specific gravity?

9. A piece of metal weighs 26·35 gms. in air and 23·4 gms. in water. What is its specific gravity?

10. A piece of rubber weighs 23·32 gms. in air and 9·92 gms. in water. What is its specific gravity?

11. The weight of a solid in air is 29·05 gms., in water 18·72 gms., and in a salt solution 18·01 gms. What is the specific gravity of the salt solution?

12. A piece of wax weighs 24·90 gms. in air. It is tied to a sinker and together with the sinker weighs 26·31 gms. in water. The sinker alone weighs 28·80 gms. in water. What is the specific gravity of the wax?

$$\begin{aligned} \text{Weight of wax in water} &= -2{\cdot}49 \text{ gms.} \\ \text{Weight of water displaced} &= 24{\cdot}90 + 2{\cdot}49 \\ &= 27{\cdot}39 \text{ gms.} \\ \text{Specific gravity of wax} &= \frac{24{\cdot}90}{27{\cdot}39} = {\cdot}909 \end{aligned}$$

13. A piece of wax weighs 18·03 gms. in air. A piece of metal is found to weigh 17·03 gms. in water. It is tied to the wax, and both together weigh 15·23 gms. in water. What is the specific gravity of the wax?

14. A piece of wood weighs 32·07 gms. in air. It is tied to a sinker, and both together are found to weigh 33·12 gms. in water. The sinker alone weighs 48·32 gms. in water. What is the specific gravity of the wood?

15. When 48·84 gms. are placed in the upper pan of a Nicholson's hydrometer, it sinks to the mark. When a piece of metal and 34·40 gms. are placed in the upper pan, it sinks to the mark. When the metal is placed in the lower pan, and 36·01 gms. are placed in the upper pan, it sinks again to the mark. Find the specific gravity of the metal.

16. A piece of wood is placed in the upper pan of the same hydrometer, and 36·29 gms. have to be placed in the upper pan to sink the instrument to the mark. When the wood is tied to the lower pan, 55·5 gms. must be placed in the upper pan to sink the instrument to the mark. What is the specific gravity of the wood?

17. A piece of metal is placed in the upper pan of a Nicholson's hydrometer, and it is necessary to add 21·90 gms. to sink it to the mark. If the metal is taken out, 46·63 gms. must be placed in the upper pan to sink the hydrometer to the mark. If the metal is placed in the lower pan, 24·85 gms. must be placed in the upper pan to bring the hydrometer to the mark. What is the specific gravity of the metal?

18. A Nicholson's hydrometer weighs 226·60 gms. To sink it to the mark in water a load of 67·36 gms. is necessary in the upper pan; to sink it to the mark in a salt solution a load of 83·72 gms. is necessary. Find the specific gravity of the salt solution.

19. A specific gravity bottle weighs 21·57 gms. empty, 69·90 gms. when full of water, and 74·17 gms. when full of a certain solution. What is the specific gravity of this solution?

20. A specific gravity bottle weighed 21·57 gms. empty. Some lead shot was introduced and the bottle weighed again. The weight was found to be 226·35 gms. The bottle was then filled up with water and weighed again. The result was this time found to be 256·64 gms. The weight of the bottle when full of water was 69·90 gms. Find the specific gravity of the shot.

21. A specific gravity bottle was found to weigh 40·12 gms. when filled with water. Some iron nails weighing 8·88 gms. were introduced, and the bottle was again filled up with water. The weight was now 47·83 gms. Find the specific gravity of the nails.

22. A piece of iron weighing 235 gms. floats in mercury of density 13·59 with $\frac{5}{9}$ of its volume immersed. Determine the volume and density of the iron.

The density of the iron is $\frac{5}{9} \times 13{\cdot}59 = 7{\cdot}55$ gms./c.c.
Its volume is 31·13 c.cs.

23. How much lead will a kilogram of cork keep from sinking in sea water? The specific gravities of lead, cork, and sea water are respectively 11·34, ·25, and 1·028.

24. A rectangular block of wood of density ·6 floats in water, on the surface of which is a layer of oil 1 cm. thick. The specific gravity of the oil is ·9. The vertical edges of the block are 9 cms. high. Find how much of this is immersed.

25. The tube of a barometer is 1·5 cms. in diameter. An iron sphere of 1 cm. diameter is allowed to ascend the tube into the vacuum. What effect will it have on the reading of the instrument? The densities of mercury and iron are 13·6 and 7·86.

26. The specific gravity of ice is ·918 and of sea water 1·03. An iceberg floats with 1000 cubic yards of its volume above the surface. What is its total volume?

27. A piece of metal weighing 18·00 gms. and a piece of glass have equal apparent weights when suspended from the pans of a balance and immersed in water. When the water is replaced by alcohol of density ·90, ·86 gm. must be added to the pan from which the metal is suspended in order to restore equilibrium. Determine the weight of the glass.

Let m = mass of the glass, g = specific gravity of glass, and s = specific gravity of the metal.

For equilibrium in water

$$18{\cdot}00 - \frac{18{\cdot}00}{s} = m - \frac{m}{g} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

For equilibrium in alcohol

$$18{\cdot}00 - \frac{18{\cdot}00}{s} \times {\cdot}9 + {\cdot}86 = m - \frac{m}{g} \times {\cdot}9 \quad . \; . \; . \; . \quad (2)$$

Multiply equation (1) by ·9, subtract the result from (2), and we obtain

$$18{\cdot}00 \times {\cdot}1 + {\cdot}86 = m \times {\cdot}1$$

or

$$m = 26{\cdot}6 \text{ gms.}$$

28. The specific gravity of milk is 1·030. If 2 parts by weight of water are added to 11 parts by weight of milk, what is the specific gravity of the mixture? If 2 parts by volume of water are added to 11 parts by volume of milk, what is the specific gravity of the mixture? Assume that the volume of the mixture is the sum of the volumes of its components.

29. Two parts by volume of a liquid of density ·80 gm./c.c. are mixed with 7 of water, and the mixture shrinks in the ratio of 31 to 30. Find its density.

30 A ship sailing from the sea into a river sinks q inches, and on discharging her cargo she rises r inches; on proceeding again to sea she rises another p inches. Show that the specific gravity of sea-water is

$$\frac{r}{p - q + r}$$

The ship's sides must be assumed vertical at the water-line.

CHAPTER III

HEAT

16. TEMPERATURE AND THERMOMETERS

Temperature.—The temperature of a body measures its "degree of hotness." Two bodies are said to have the same temperature, if, when they are put into contact, neither of them gains heat from the other. We may regard temperature as "level of heat"; if two bodies at different temperatures are put into contact, heat flows from the body at the higher temperature to the body at the lower temperature just as water flows from a high to a low level. If we touch a substance which has a lower temperature than the hand, heat flows from the hand to the substance and the substance feels cold; if the substance is at a higher temperature than the hand, heat flows from the substance to the hand, and the substance feels warm. But our feelings are an unreliable guide. If, for example, the hand has been immersed in hot water, tepid water feels cold; if it has been immersed in cold water, tepid water feels warm.

Thermometers.—Bodies generally expand when heated, and the method of measuring temperature most commonly employed is to read the increase in length of a column of mercury in a glass thermometer. Such a thermometer consists of a fine tube of glass of uniform bore enlarged at one end into a bulb containing mercury, and sealed off at the other end. The portion of the tube not occupied by the mercury column is usually vacuous, the sealing having been done when the tube was heated and full of mercury vapour. But in the case of thermometers to be used at high temperatures, 250° to 550° C., in order to prevent the mercury distilling and condensing on the upper part of the tube, the space above the mercury is filled with carbon dioxide or nitrogen, at a pressure which may be as high as 20 atmospheres. In instruments intended for accurate work the scale is engraved on the outside of the stem. In some cheap foreign instruments the stem is made very thin, and the scale is engraved on a piece of opal glass, or merely printed on a piece of paper, and fixed to the back of the stem, the whole being encased in a wide glass tube to prevent relative displacement of scale and stem. This type is known as the sleeve thermometer in contradistinction to the solid-stem thermometer.

Three different scales are at present in use on mercury thermometers, namely, the Fahrenheit scale, the Réaumur scale, and the Centigrade scale, sometimes called the Celsius scale, because it was first devised by Celsius of Upsala in 1742. The mercury thermometer itself was invented by Fahrenheit in 1714. Previous to that other liquids had been used; Newton, for example, experimented with oils, but they smeared the glass. The Fahrenheit thermometer is the common household thermometer of this country and America. The Réaumur thermometer is used in Germany, while the Centigrade scale is the standard scale for scientific purposes. The three scales are all based on the same two fixed points, the freezing point of water and the "boiling point of steam" under normal pressure. These are taken respectively as 0° and 100° on the Centigrade scale, 0° and 80° on the Réaumur scale, and 32° and 212° on the Fahrenheit scale; Fahrenheit in his earlier work took 96° as the temperature of human blood and 0° as the lowest temperature reached at Dantzig in the winter of 1709. He was able to reproduce this temperature with a mixture of ice, salt and water, and thought it was the lowest temperature which could possibly be obtained. The letters C., R., and F. are commonly used as contractions for Centigrade, Réaumur and Fahrenheit; thus, for example, 10° C. is read ten degrees Centigrade. Temperatures below zero on any scale have a minus sign prefixed.

Mercury is chosen as a thermometric substance on account of its cleanness and its high boiling point, 356° C. Also owing to its great conductivity mercury responds quicker, and a fine mercury column is less liable to break than columns of other liquids. Alcohol is sometimes used as the thermometric substance instead of mercury. It has the advantage of a much lower freezing point, $-130°$ C. instead of $-39°$ C., and so can be used for lower temperatures; it is also cheaper, and so is used in instruments of very large size. Alcohol expands about five times as much as mercury.

Sometimes when the thread of mercury is descending rapidly it breaks, leaving an isolated portion near the top of the tube. This portion can be joined on again by heating the bulb to make the mercury ascend again to touch it, or by jerking the instrument, holding it vertically in the hand; and jerking the arm in somewhat the same manner as when cracking a whip.

Verification of Thermometer Fixed Points.—If, when it is first constructed, a mercury thermometer reads correct, it will read too high, perhaps about $\frac{3}{10}°$ C., after half a year. This is due to a gradual recovery of the glass from the effects of the heating it was subjected to, when it was made; this recovery is rapid at first but slower afterwards. Also if a thermometer has been heated and then is cooled, the glass does not take up its proper volume immediately but "lags" above it; consequently there is a temporary depression in the readings, which may be as much as $\frac{1}{10}°$ C. The lag can be neglected at high temperatures. For this reason the freezing point of a thermometer should always be verified before its boiling point.

To verify the freezing point of a Centigrade thermometer it is held in a vertical position with its bulb surrounded up to the zero mark by pure melting snow or pounded ice and water, and the reading taken as soon as the temperature becomes stationary. The mixture must be free from any impurities such as salt, otherwise the temperature will be too low. It is found, that if a thermometer is surrounded with pure melting ice the mercury stops at the same point under whatever circumstances the experiment is performed, and remains stationary there until all the ice is melted. Variations of atmospheric pressure have no appreciable effect on the freezing point.

To determine the boiling point of a thermometer it is placed in the arrangement shown in Fig. 77, which is called a hypsometer, because, as will be immediately evident, it can be used for determining the heights of mountains. The temperature is then read as soon as it becomes stationary. Steam from boiling water passes up an inner cylinder A, round the stem of the thermometer, down between the inner cylinder and outer cylinder, and then escapes by the outlet at B. The outlet must be wide enough to prevent an excess of pressure inside the hypsometer. The boiling point of the thermometer should just show above the cork.

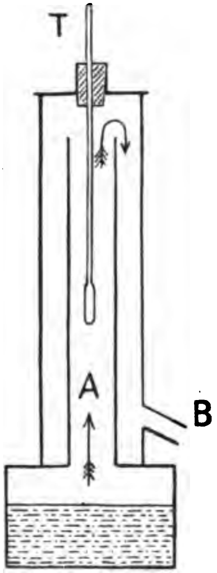


FIG. 77.

The thermometer is placed in steam and not in boiling water, because slight impurities in the form of dissolved salts affect the temperature of the water considerably, but not the temperature of the steam given off by the water. Also the temperature of water at the boiling point depends slightly on the nature of the containing vessel; water if contained in a clean glass vessel sometimes rises above the theoretical boiling point, then there is a violent burst of ebullition, after which the temperature falls to the correct value.

The temperature of the steam depends on the atmospheric pressure under which the water boils according to the following table :—

Pressure.	Temperature.
70 cms.	97·71° C.
72	98·49
74	99·25
76	100·00
78	100·73
80	101·44

Thus, after the boiling point has been determined, the barometer must be read, and the correct value of the boiling point calculated according to the table for the pressure in question.

When the error of the thermometer has been determined for the two fixed points, a graph may be drawn giving it for all intermediate values. Thus, if, for example, in the case of a Centigrade thermometer the reading at the freezing point is ·3° too low, and at boiling point ·8° too high, the graph over the page would give the correction to be added at all intermediate values. Thus no correction would be required at 27°.

Let v be the volume of the part of the tube between the two fixed points of a Centigrade thermometer, and let v be divided into 100 equal parts, and a mark put on the stem at the termination of each of these parts, the temperature of the tube being kept constant; then, if at a subsequent time the mercury column occupies t of these divisions, the temperature on the mercury-in-glass Centigrade thermometer is defined to be $t°$. The temperatures as defined in this way, and read on mercury-in-glass thermometers made of different kinds of glass, do not agree, the maximum divergence on the range 0° to 100° being about $\frac{1}{10}$° C.

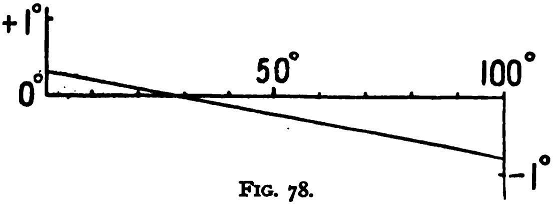


Fig. 78.

Stem Correction.—For a thermometer to read correctly it must be at the same temperature up to the division in question. Thus in determining the boiling point the stem should be immersed in steam up to 100° C. If only the bulb is immersed in a hot liquid, and the stem is cold, the mercury in the stem does not expand, and the reading is consequently too low. This error may be allowed for by adding on what is known as the stem correction, which may be as much as 1·5° C. if the bulb is at 100° C. and the stem at 0° C. throughout, but stem corrections are always unsatisfactory owing to the impossibility of knowing definitely what the temperature of the stem is.

Conversion of Temperatures.—Since 180° F. = 100° C., *i.e.* 9° F. = 5° C., and the Fahrenheit scale starts at 32°, to convert a temperature from the Fahrenheit to the Centigrade scale, subtract 32 and multiply by $\frac{5}{9}$. To convert a temperature from the Centigrade to the Fahrenheit scale, multiply by $\frac{9}{5}$ and add 32. To convert a temperature from the Centigrade scale to the Réaumur scale, multiply by $\frac{4}{5}$; to convert a temperature from the Réaumur scale to the Centigrade one, multiply by $\frac{5}{4}$.

EXAMPLES 16

1. Change blood heat, 98°, summer heat, 76°, and temperate, 56°, all from the Fahrenheit to the Centigrade scale.

$$\text{Blood heat} = \tfrac{5}{9}(98 - 32) = 36{\cdot}7° \text{ C.}$$
$$\text{Summer heat} = \tfrac{5}{9}(76 - 32) = 24{\cdot}4° \text{ C.}$$
$$\text{Temperate} = \tfrac{5}{9}(56 - 32) = 13{\cdot}3° \text{ C.}$$

2. Convert −4°, 22°, 88°, and 112° C. to (i) the Fahrenheit scale, (ii) the Réaumur scale.

3. Convert −22°, 6°, 47°, 112° F. to the Centigrade scale.

4. At what temperature has the Fahrenheit scale the same numerical value as (i) the Centigrade scale, (ii) the Réaumur scale?

5. At what temperature does water boil when the barometric pressure is (i) 71 cms., (ii) 72·8 cms., (iii) 78·5 cms.?

6. Water boils at 100° C. at the foot of a mountain and at 97·71° C. at its top. What is the height of the mountain?

The barometric pressure at the foot is 76 cms. and at the top (c.f. table on p. 73) 70 cms. The difference = 6 cms. = 2·36 inches. Using the rule 1 inch for 900 ft. this gives 2126 ft.

7. About what temperature will water boil on the top of (i) Ben Nevis, (ii) Mont Blanc? Ben Nevis is 4406 ft. and Mont Blanc 15,781 ft. high. Water boils at 75·9° C. when the pressure is 30 cms., and at 93·5° C. when it is 60 cms.

8. When the fixed points of a Centigrade thermometer are verified, it reads ·5° at the freezing point and 99·2° at the boiling point. The barometric pressure is 75 cms. What is the correct reading when it reads 15°, and at what temperature is it exactly right?

9. Two thermometers are made of the same glass; the bulbs are spherical, the internal diameter of the first being 7·5 mm. and of the second 6·2 mm. The diameter of the tube of the first is 2·5 mm. and of the second 1·8 mm. What is the ratio of the length of a degree of the first thermometer to a degree of the second?

17. EXPANSION OF SOLIDS AND LIQUIDS

Solid bodies in general expand when heated. This can be demonstrated by a well-known apparatus known as Gravesande's ring. This apparatus consists of a metal ring mounted on a stand, and when a metal sphere is lowered by a chain it can just pass through it. If, however, the sphere is heated by holding it over a flame, it will no longer pass through, as it has expanded and become too large. But if the ring is also heated so as to make it expand also, then the sphere can again pass through.

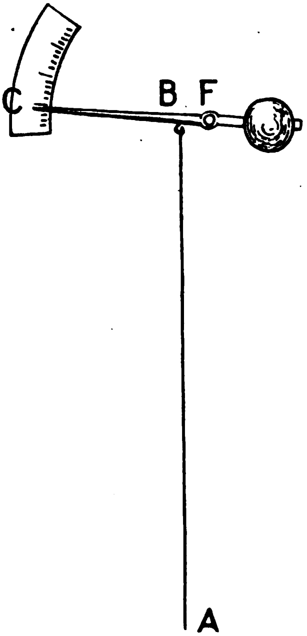


FIG. 79.

Or if we have a vertical wire AB (Fig. 79), the upper end of which is attached to a horizontal lever FC, the end C of which passes over a vertical scale, and if the wire is heated by bringing a flame near it, then the point C travels up the scale, showing that the wire has increased in length. The motion of C can be increased by increasing the ratio of the arms FC and FB of the lever.

Linear Expansion.—The expansion of a solid body is specified by its coefficient of linear expansion. If a bar is heated from 0° to t° C., its mean coefficient of linear expansion for this range is the increase in length per unit length divided by t. Thus if l_0 is the length of the bar at 0° C., a its coefficient of linear expansion, and l its length at t° C., then

$$l = l_0(1 + at)$$

a has practically the same value for all ordinary ranges of temperature, and is always very small, for example, in the case of copper it is ·000017.

Let us suppose that a copper rod is 10·000 cms. long at 10° C., and that its length is required at 18° C. Then from the above equation we have

$$l_{10} = l_0(1 + 10a)$$
$$l_{18} = l_0(1 + 18a)$$
$$l = l_{10}\frac{1 + 18a}{1 + 10a} = l_{10}(1 + 8a - 80a^2 \ldots)$$

Now $8a = \cdot 000136$ and $80a^2 = \cdot 0000000231$; hence $80a^2$ can be neglected in comparison with $8a$, and we can write simply

$$l_{18} = l_{10}(1 + 8a)$$

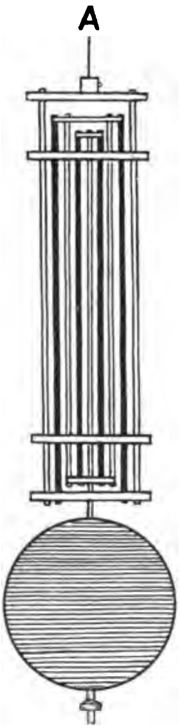


Fig. 80.

Thus, if l is the length at t° C. and the bar is heated to t_1° C., the increase of length is $la(t_1 - t)$ within experimental error. It is not necessary to substitute l_0 for l.

One of the best methods of measuring the coefficient of linear expansion of a rod is to rest it horizontally on rollers in a bath of water, and have two vertical needle points fixed to it near its ends sticking up above the side of the bath. Two measuring microscopes with object glasses of long focal length are focussed on the needles. The water in the bath is then heated and the displacements of the needle points measured by the microscopes. Their sum gives the increase in length. For high temperatures an oil bath or sand bath must be used instead of the water bath.

In railways a small space is left between the rails to allow for expansion in the hot days of summer. The force with which metals expand is very great indeed, and, if there were no air space, the rails would curve and twist from their positions. Similar provision has to be made for the expansion and contraction which takes place in metal structures, such as large bridges, etc. In fitting the iron tyre to a cart wheel it is made very hot, and consequently expands; it is then placed in position. When it cools it embraces the wheel with great force, keeping itself in position and pressing home the spokes into their joints.

When the temperature increases, the length of the pendulum shown in Fig. 25 increases. Consequently the period increases and the clock goes slower. It is thus a problem of practical importance to design a pendulum, the length of which is the same at all temperatures. One solution, the so-called gridiron pendulum, is represented in Fig. 80; A is the point of attachment of the pendulum, which is made of steel rods and brass rods. The steel rods are shown white, and the brass rods black, in the diagram. It will be observed from the diagram that the effect of an expansion of the steel rods is to lower the bob, and the effect of an expansion of the brass rods is to raise the bob. The coefficients of linear expansion of steel and brass are respectively $\cdot 11 \times 10^{-4}$ and $\cdot 187 \times 10^{-4}$. Hence, if the total lengths of steel and brass rod are inversely in this ratio, the bob remains at the same distance below the point of suspension, and the period is unaltered.

Cubical Expansion.—In the case of crystals such as quartz or Iceland spar the coefficient of linear expansion is different according as the direction of expansion is parallel or perpendicular to the axis of the crystal. In the case of woods it is different along and across the grain.

Let us suppose that a rectangular block, the lengths of the sides of which are a, b, and c, is heated through a range of temperature $t°$ C., and that the coefficients of linear expansion in the directions of the three sides are α, β, and γ. Then the lengths of the sides become $a(1 + \alpha t)$, $b(1 + \beta t)$, and $c(1 + \gamma t)$, and the volume of the block becomes

$$abc(1 + \alpha t)(1 + \beta t)(1 + \gamma t)$$
$$= v\,\{1 + (\alpha + \beta + \gamma)t + (\alpha\beta + \beta\gamma + \gamma\alpha)t^2 + \alpha\beta\gamma t^3\}$$

where v is its original volume. Since α, β, and γ are very small, only their first powers require to be retained, and the volume becomes

$$v\{1 + (\alpha + \beta + \gamma)t\}$$

Suppose now that the block is isotropic, *i.e.* the same in all directions, then $\gamma = \beta = \alpha$, and the volume becomes

$$v(1 + 3\alpha t)$$

3α is defined to be the coefficient of cubical expansion of the block. It is clear that the above expression gives the volume of the solid after heating, whether the original form is rectangular or not, because an irregular form can be divided up into a great number of little rectangular blocks.

Since a liquid has a definite volume but no definite shape, when we refer to its coefficient of expansion it is always the coefficient of cubical expansion that is meant.

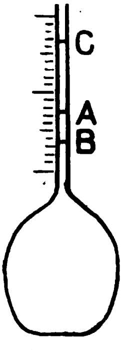


FIG. 81.

The Dilatometer.—Let Fig. 81 represent a dilatometer, *i.e.* a glass vessel with a long straight graduated neck. Let b be the coefficient of cubical expansion of the glass. Let the dilatometer contain liquid up to the level A at a temperature of 0° C., let the coefficient of cubical expansion of the liquid be a, and let the volume of the liquid be V. Let the vessel and liquid be heated through a range of $t°$ C. Consider their expansions separately.

Suppose the vessel were filled with glass up to the level A instead of with liquid. Then it would expand as a solid mass. The volume of glass enclosed would be V. Consequently the volume of the vessel up to A at the higher temperature would be $V(1 + bt)$. If the liquid did not expand, its level would sink to B, the volume of AB at the higher temperature being Vbt. But owing to the expansion of the liquid its level rises to C and its volume becomes $V(1 + at)$, the volume of BC at the higher temperature being Vat.

If we had not allowed for the expansion of the glass, we would have taken AC for the expansion of the liquid. The volume AC is called its apparent expansion. The coefficient of apparent cubical expansion of a liquid is defined as the apparent increase in volume per unit volume divided by the increase in temperature. We have thus

$$\left.\begin{array}{l}\text{Co-eff. of}\\ \text{App. Exp.}\end{array}\right\} = \frac{AC}{Vt} = \frac{BC - BA}{Vt} = \frac{Vat - Vbt}{Vt} = a - b$$

$$= \text{Coeff. of True Exp.} - \text{Coeff. of Cub. Exp. of Glass.}$$

The coefficient of true expansion of a liquid is often referred to as its coefficient of absolute expansion. In order to make a determination of the coefficient of apparent expansion by the dilatometer it is necessary to know (i) its volume to the foot of the scale, (ii) the volume of a division on the scale. This can be obtained by weighing the vessel empty, full of mercury to the foot of the scale, and full of mercury to the top of the scale, all at the same temperature. The volumes can then be obtained from the density of mercury.

The Weight Thermometer.—Another method of measuring the coefficient of apparent expansion of a liquid is by means of the weight thermometer, a little glass vessel with a piece of bent capillary tube as neck, as shown in Fig. 82.

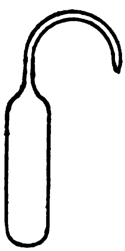


FIG. 82.

It is weighed first empty, then full to the end of the capillary with the liquid in question. The difference of these two weighings gives w, the weight of the liquid contained. It is next heated through a range of temperature $t°$ C.; the liquid expands more than the glass does, and some is expelled from the capillary. The vessel is then weighed a third time and w_1, the weight of the liquid left in it, determined. The weight expelled is $w - w_1$. The apparent increase in volume is to the original volume as $w - w_1$ to w_1. Hence the coefficient of apparent cubical expansion is

$$\frac{w - w_1}{w_1 t}$$

A small specific gravity bottle may be used instead of a weight thermometer.

Absolute Expansion of Mercury.—A direct method of determining the absolute coefficient of expansion of mercury was devised and carried out by Dulong and Petit. This method is based on the principle, that, if two columns produce equal pressures, their heights are inversely as their densities.

It is represented in Fig. 83. A and B are two vertical glass tubes connected by a horizontal capillary C. Each tube is surrounded by a metal case indicated by dotted lines; the case surrounding A is filled with ice, and that surrounding B contains oil heated to temperature $t°$ C.

Let h_0, ρ_0, v_0 be the height, density, and volume of unit mass for column A, and h, ρ, v the height, density, and volume of unit mass for column B.

Then $$h_0\rho_0 = h\rho$$

and $$\frac{\rho}{\rho_0} = \frac{h}{h}$$

But $$\rho = \frac{1}{v}, \ \rho_0 = \frac{1}{v_0} \text{ and } v = v_0(1 + at)$$

Hence $$1 + at = \frac{h}{h}$$

and $$a = \frac{h - h_0}{h_0 t}$$

For the range 0° – 100° C., Dulong and Petit found $a = \frac{1}{5550}$. The experiment was repeated by Regnault with an improved apparatus of the same type, and Dulong and Petit's result was confirmed.

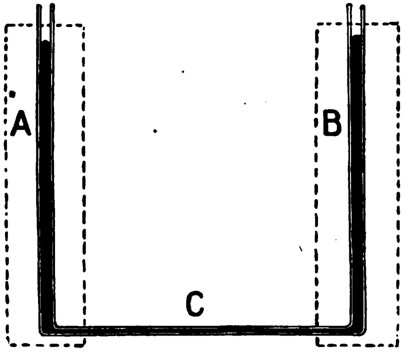


FIG. 83.

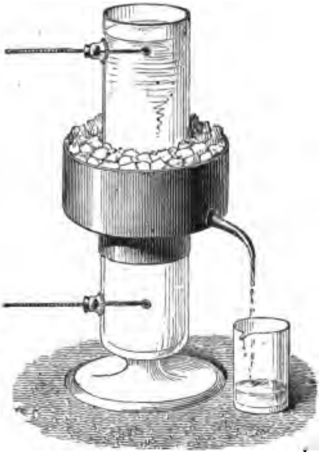


FIG. 84.

From Ganot's Physics.

Maximum Density of Water.—The expansion of water is anomalous; when it is heated from 0° C. to 100° C., it contracts from 0° C. to 4° C., then expands to 100° C., the coefficient of expansion increasing with the temperature. At 4° C. its density is a maximum and equal to 1·00000.

The temperature of maximum density can be determined by Hope's apparatus, which is shown in Fig. 84, and consists of a glass jar surrounded at its middle by an ice jacket and carrying two thermometers, one above and one below the ice. If the temperature of the water is initially 15° C., as the water is cooled by the ice it sinks, and so the lower thermometer gives a lower temperature than the upper one until 4° C. is reached; then the colder water begins to ascend, and the upper thermometer gives the lower temperature.

The fact that, when the cold air causes the temperature of a lake to fall below 4° C., the colder water remains on the surface, is of great importance in nature, for it prevents convection currents downward, and the water below remains at 4° C. even in the most severe winter.

EXAMPLES 17

1. A metal metre scale was carefully measured at 5° C. and its length found to be 99·977 cms. At 35° C. its length was found to be 100·011 cms.; calculate the coefficient of expansion of the metal, and find the temperature at which the scale is exactly 1 metre long.

$$\text{Coefficient of expansion} = \frac{100{\cdot}011 - 99{\cdot}977}{99{\cdot}977(35 - 5)}$$
$$= {\cdot}0000113$$

To become exactly right scale has to expand $100 - 99{\cdot}977$
$$= {\cdot}023 \text{ cm.}$$

The required rise of temperature is

$$\frac{{\cdot}023}{{\cdot}0000113 \times 99{\cdot}977} = 20{\cdot}4^\circ \text{ C.}$$

The required temperature is consequently 25·4° C.

2. A brass rod and a steel rod are both measured at 0° C. and their lengths found to be 150·00 and 150·20 cms. respectively. At what temperature are their lengths equal, and what is their common length at this temperature?

3. Find the least distance that must be distributed over 20 miles of railway, which is laid at 37° F., in order to allow for expansion to 120° F. in summer. The coefficient of expansion of wrought iron is ·0000122 per degree Centigrade.

4. A pendulum which is made wholly of brass beats seconds when the temperature is 15° C. Find how many seconds it will gain or lose in a day, if the temperature rises to 20° C. The coefficient of linear expansion for brass is ·0000189. (You may assume for purposes of calculation, that the pendulum consists of an infinitely light brass rod with a weight at its end; the form of the pendulum does not affect the influence of temperature on its period.)

5. The height of the barometer is found to be 78·25 cms. at 22° C. Find at what height the column would stand, if its temperature were 0° C. Coefficient of absolute expansion of mercury = ·000182.

6. The area of a brass plate is 123·02 sq. cms. at 5° C. What will it be at 122° C., given that the coefficient of linear expansion of brass is ·0000189?

7. By how many inches is the circumference of an iron wheel increased, when its temperature is raised 400° C.? The diameter of the wheel is 3 ft., and the coefficient of expansion of iron is ·0000122.

8. The coefficient of linear expansion of Iceland spar is ·0000251 parallel to the axis and $-{\cdot}0000056$ at right angles to the axis. The crystal contracts at right angles to the axis, when it is heated. The volume of a piece of Iceland spar is 30·20 c.cs. at 15° C. By how much has it increased at 40° C.?

The increase is $v(\alpha+\beta+\gamma)t$. Here $t = 40 - 15 = 25$, $\alpha = {\cdot}0000251$, and $\beta = \gamma = -{\cdot}0000056$. Hence $\alpha+\beta+\gamma = 0{\cdot}0000139$, and the result is—

$$30{\cdot}20 \times {\cdot}0000139 \times 25 = {\cdot}0105 \text{ c.c.}$$

9. The coefficient of linear expansion of pine is about ·000005 along the grain and ·000034 across the grain. Find how much a sphere of 6 cms. diameter made of pine will expand on being heated from 10° to 40° C.

10. The density of mercury is 13·5955 gms./c.c. at 0° C. Its coefficient of cubical expansion is ·000182. What is its density at 60° C.?

11. The density of water is 1·00000 at 4° C. and ·9718 at 80° C. A cube of glass of 3 cms. edge is weighed in water at both temperatures. Find the difference of the two results, given that the coefficient of linear expansion of the glass is ·000009.

12. A thermometer is sunk to the 25° mark in a liquid and reads 85° C. The mean temperature of the rest of the stem is 30°. Find the true temperature of the liquid, the coefficient of expansion of mercury-in-glass being ·00015.

The exposed part of the stem is 60 divisions. It should have expanded another 60 × ·00015 × (85 − 30) = ·495 division. The true temperature is therefore 85·5° C.

13. An accurate Centigrade thermometer is immersed to its −10° mark in steam at 100° C., *i.e.* its bulb is wholly immersed, but none of the stem is, and the stem is exposed to a cold draught, so that its mean temperature is 15° C. What does the instrument read, given that the coefficient of apparent expansion of mercury in glass is ·00015?

14. The bulb of a thermometer, together with as much of the stem as is below 0° C., contains at that temperature 2·83 gms. of mercury. Each degree on the stem is 1 mm. long. Calculate the diameter of the stem on the assumption that the apparent expansion of mercury in glass is ·000155.

15. A specific gravity bottle weighs 21·57 gms. empty and 69·90 gms. when full of water at 20° C. When full of water at 90° C. it weighs 68·39 gms. The coefficient of cubical expansion of the bottle is ·000025. What is the average coefficient of cubical expansion of water over the range 20° C. to 90° C.?

16. A weight thermometer, which contains 100 gms. of mercury at 10° C., is placed in an oil bath, and the mercury expelled is found to weigh 1·89 gms. Find the temperature of the bath, the coefficient of apparent expansion of mercury in glass being ·000155.

17. A weight thermometer contains 122·70 gms. of mercury at 15° C. It is heated to 82° C. What weight of mercury is expelled, given that the coefficient of apparent expansion of mercury relative to glass is ·000155?

18. EXPANSION OF GASES

When gases are heated they expand much more than solids or liquids. This can be shown by the arrangement depicted in Fig. 85, which was used by Galileo as a thermometer, and is often used for demonstration purposes. It consists of a glass bulb containing air, with a long, straight neck dipping into water usually coloured blue or black to make it visible. The level of the water stands halfway down the stem. When the temperature in the bulb is increased, for example, by clasping the fingers round it, the air in it expands, and the top of the water column travels down the stem.

When measuring the coefficient of expansion of a liquid or solid it is not necessary for the atmospheric pressure on it to be constant throughout the operation. Any change in its volume produced by an alteration of pressure is so small in comparison with the change produced by the increase in temperature, that it can be ignored. But it is quite otherwise in the case of gases; they are so compressible, that in defining their coefficient of cubical expansion we must specify that the pressure of the gas remains unaltered during the expansion. Thus, if v_0 is the volume of a gas at 0° C., v its volume under the same pressure at t° C., and α its mean coefficient of cubical expansion for the range 0° to t° C. measured on the mercury-in-glass thermometer,

$$v = v_0(1 + \alpha t)$$

Let us suppose that p_0 is the pressure of the gas at 0° C. Now v is its volume at t° C. when the pressure is p_0; let p be its pressure at t° C. when the volume is v_0. Then, by Boyle's law

$$pv_0 = p_0v = p_0v_0(1 + \alpha t)$$

or

$$p = p_0(1 + \alpha t)$$

Thus the coefficient of increase of pressure at constant volume is the same as the coefficient of cubical expansion at constant pressure.

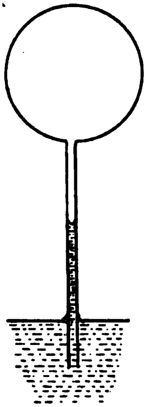


FIG. 85.

It was discovered by Charles that α is the same for all gases as for air. This fact is consequently referred to as Charles's law. The law was verified by Gay-Lussac, who determined α, and showed that it was independent of the pressure. His result for air was ·00366 or $\frac{1}{273}$. Charles's law holds with about the same degree of exactitude as Boyle's law does. α is practically constant for the permanent gases for all ranges of temperature recorded by a mercury-in-glass thermometer.

It should be noted, that if v is the volume of a gas at t° C. and v_1 its volume at t_1° C. under the same pressure, we cannot use the approximation made on p. 76 in the case of solids, and write

$$v_1 = v\{1 + \alpha(t_1 - t)\}$$

α is too large in the case of gases for its square to be neglected. v_1 must be obtained by the rigorous equation

$$v_1 = v\frac{1 + \alpha t_1}{1 + \alpha t}$$

Experimental Determination of α.—The easier method of determining α is not to keep p constant and measure the increase of v, but to keep v constant and measure the increase of p. For this purpose the simple apparatus shown in Fig. 86 can be used. A is a glass bulb containing the gas to be experimented on; it is made in one piece with the glass tube BCD, the whole being kept fixed in position. The glass tube CD is connected by the rubber tubing DE with the vertical glass tube EG. The lower parts of both tubes and the rubber tubing contain mercury. The bulb A is surrounded by a can W containing water, the position of which is indicated by the dotted lines. A mark is engraved on the right-hand tube at C. The level of the left-hand tube is adjusted until the surface of the mercury in the right-hand tube just rises to the mark, then the difference of level of the mercury in the two tubes and the temperature t of the water in the can are noted. The water is then heated to temperature t_1, the left-hand tube raised up the vertical scale until the surface of the mercury in the right-hand tube is brought back to the mark at C, and the difference of level in the two tubes noted. The volume of the gas enclosed by the mercury is the same in each case. The pressure is obtained by adding the barometric pressure to the difference of level of the mercury in the two tubes; let p be its value in the first case, and p_1 its value in the second case. Then

$$\frac{p_1}{p} = \frac{1 + at_1}{1 + at}$$

whence

$$a = \frac{p_1 - p}{pt_1 - p_1t}$$

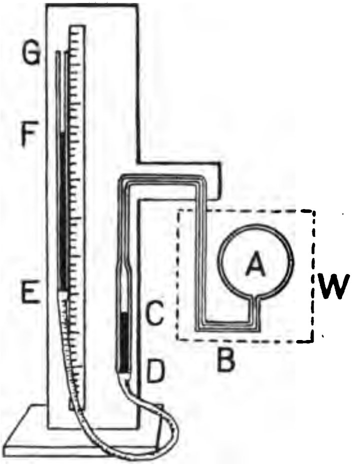


FIG. 86.

The volume of the gas in the part of the tube protruding out of the water should be small in comparison with the volume of the bulb. If the left-hand tube is left in too high a position, when the gas is cooled the mercury sometimes runs over into the bulb, but there is no difficulty in getting it out again when the tube enters from below, as in the figure.

Air Thermometers.—So far we have measured temperature on the mercury-in-glass scale. The facts that the different gases have all the same expansion coefficient, that a gas can be used as thermometric substance a long way below the freezing point and above the boiling point of mercury, and that the expansion of gases is so much greater than the expansion of their containing vessels that the latter is comparatively unimportant, are all strong reasons for using a gas as thermometric substance instead of mercury.

Accordingly the International Committee of Weights and Measures (1887) has taken as its standard thermometric scale that of the constant volume hydrogen thermometer. The latter instrument is in principle like the apparatus shown in Fig. 86, only it has the top of the left-hand tube closed and the space FG a vacuum, so the pressure on the gas is obtained directly from the difference in level of the two mercury surfaces without the use of a barometer. The bulb is filled with hydrogen. The surface of the mercury is always brought back to the mark C. To calibrate the instrument the bulb is immersed in melting ice and the level of F marked on the scale; it is then immersed in steam given off by water boiling under standard pressure and the level of F again marked. These two marks are taken as the 0° and 100° C. graduations, and the vertical scale is divided into 100 equal parts between the marks, and into parts of the same length above the 100° C. mark and below the 0° C.

The hydrogen thermometer on account of its shape is seldom used directly, but forms a standard of reference for correcting mercury thermometers. The maximum divergence between the mercury thermometer and the hydrogen thermometer is less than ·2° C. for the range 0° to 100° C.

Absolute Temperature.—The pressure of the hydrogen in the hydrogen thermometer is given by the formula

$$p = p_0(1 + at)$$

t being the temperature on the instrument's own scale. Suppose t is negative; then as the temperature decreases p decreases, and finally when $t = -\frac{1}{a}$, or $-273°$ C., the pressure becomes zero. As it is impossible to think of the pressure of a gas being less than zero, this temperature is considered to be the lowest temperature possible and to denote entire absence of heat; it is termed the "absolute zero." When temperature is measured in degrees Centigrade from the absolute zero it is referred to as "absolute temperature," and is said to be measured in degrees absolute; thus 100° C. is 373° abs., −10° C. is 263° abs.

The lowest temperature that has ever been reached is that of the liquefaction of helium, namely −271·3° C. Helium was liquefied by Kamerlingh Onnes in 1911.

The Gas Equation.—Let us suppose that at temperature t the pressure and volume of a mass of gas are given by p and v. Keep the pressure constant, but decrease the temperature to 0° C. and suppose the volume decreases to v_1. Then by the equation on p. 82

$$pv = pv_1(1 + at)$$

Now change the pressure p to its standard value and at the same time let v_1 become v_0. Then by Boyle's law

$$pv_1 = p_0v_0$$

Hence $$pv = p_0v_0(1 + at)$$

Let T denote the absolute temperature of the gas. Then

$$T = t + \frac{1}{a}$$

Consequently $$\frac{pv}{T} = ap_0v_0 = \text{const.} = R$$

The equation $$\frac{pv}{T} = R$$

is referred to as the gas equation, and is of great use in solving numerical examples.

EXAMPLES 18

1. Given that the volume of a gas is 2300 c.cs. at 0° C. and a pressure of 76 cms., what is it at 15° C. and a pressure of 78 cms.?

The required volume is $2300 \frac{273 + 15}{273} \cdot \frac{76}{78}$
$= 2364$ c.cs.

2. Gas is contained in a vessel of 2000 litres capacity at 0° C. and under a pressure of 74 cms. What does the pressure become when the temperature rises to 24° C.?

3. Compare the densities of the air at the top and bottom of a mine shaft. The temperature and pressure are 10° C. and 30 inches at the top and 14°C. and 30·5 inches at the foot.

4. At what temperature will a litre of air weigh a gram when under a pressure of 74 cms.? A litre of air weighs 1·293 gms. at 0° C. and 76 cms.

5. An empty flask with a capacity of 1 litre is immersed in water at 80° C. with only the neck above the surface of the water, until the air inside it takes the temperature of the water. Its mouth is then closed with the thumb, it is suddenly inverted and completely immersed in water at 14° C., and the thumb is removed. What weight of water enters the flask?

6. A balloon at its utmost capacity can hold 1000 cubic ft. of gas. 900 cubic ft. are let into it at a pressure of 29 inches and a temperature of 40° F. The valve then chokes, and the balloon rises into a region where the pressure is 27 inches and the sun's rays heat it to 70° F. Will it burst?

7. Ten cubic feet of dry air measured at 32° F., when the barometric height is 28 inches, are pumped into a globe whose diameter is 8 inches. The globe is then heated to 200° F. Find the pressure.

19. CALORIMETRY

Quantity of Heat.—It is a well-known fact that when a kettle is full of water, it takes longer to boil than when it is only one-quarter full. In the former case a greater *quantity of heat* is required to heat the water up to boiling point.

The unit quantity of heat used in scientific work is called the calorie or gram-degree, and is defined as the quantity of heat necessary to raise the temperature of one gram of water 1° C. The quantity of heat necessary to raise the temperature of one gram of water 1° C. is not rigorously the same at all temperatures, but decreases from 0° C. to a minimum at 37·5° C., and then increases to 100° C., the extreme variation over the range 0° to 100° C. being less than 1 per cent. Hence in very accurate work it is necessary to specify for what particular temperature the calorie is defined.

The thermal capacity of a substance at any temperature is the quantity of heat required to raise the temperature of the substance one degree at that temperature. The specific heat of a substance is the thermal capacity of its unit mass. The specific heat of copper, for example, is ·093; thus the amount of heat required to raise the temperature of 12·2 grams of copper from 10·2° to 15·3° C. is $\cdot093 \times 12\cdot2 \times 5\cdot1 = 5\cdot79$ cals.

Sometimes in engineering a unit of heat called the British thermal unit, and defined as the quantity of heat required to raise the temperature of 1 lb. of water 1° F., is employed. The specific heat has the same numerical value whether measured in calories or in British thermal units; it is independent of the particular units of mass and temperature employed, since the latter are also involved in the definition of the unit of heat.

Determination of Water Equivalent of a Calorimeter.—The most straightforward method of determining the specific heat of a substance is by the method of mixtures. In applying this method a piece of apparatus called a calorimeter is used. This consists of a cylindrical copper vessel suspended on three threads inside another cylindrical copper vessel, so that it touches nothing else but the three threads. Consequently, if the inner vessel is filled with warm water, heat cannot escape from it except by conduction along the threads or by radiation and convection, and the outer surface of the inner vessel and inner surface of the outer vessel are polished, the radiation loss being less from a polished surface. The inner vessel is provided with a copper stirrer.

Before using a calorimeter for specific heat measurements we must determine its water equivalent, *i.e.* the mass of water with the same thermal capacity as it has. To do this the inner vessel of the calorimeter is first weighed empty, then rather more than half full of cold water, and so the mass m of the water is determined. It is then replaced in the outer vessel, and the temperature of the water t noted. Next some water at a higher temperature T is poured in, sufficient to raise the temperature of the water in the calorimeter about 3°; the mixture is stirred well, and θ, the exact value of the highest temperature reached, noted. The inner vessel is then weighed again, and a determination made of M, the mass of the hot water poured into it.

When the mixture is stirred, the hot water loses heat, and the cold water and the metal of the calorimeter gain heat. The heat lost by the hot water is

$$M(T-\theta)$$

The heat gained by the cold water is

$$m(\theta-t)$$

Let w be the water equivalent of the calorimeter. Then the heat gained by the calorimeter is

$$w(\theta-t)$$

Equating heat lost and heat gained we obtain

$$M(T-\theta)=m(\theta-t)+w(\theta-t)$$

whence

$$w=\frac{M(T-\theta)}{\theta-t}-m$$

The value of the water equivalent may also be found by calculation, by multiplying the mass of the stirrer and of the inner vessel of the calorimeter by the specific heat of the metal of which they are composed.

Determination of Specific Heat.—In determining the specific heat of a solid the inner vessel of the calorimeter is first weighed empty, then rather more than half full of cold water and m, the mass of the cold water, determined. It is next replaced in its outer vessel and t, the temperature of the water, is taken. Then the specimen, which has meantime been heated to a high temperature T, is dropped into the water, the water is stirred and θ, the highest temperature reached, is noted. The inner vessel is next weighed and M, the mass of the specimen, determined. Let w be, as before, the water equivalent of the calorimeter, and let s be the specific heat of the specimen.

The heat lost by the specimen is

$$Ms(T - \theta)$$

The heat gained by the water and calorimeter is

$$(m + w)(\theta - t)$$

On equating we obtain

$$Ms(T - \theta) = (m + w)(\theta - t)$$

whence

$$s = \frac{(m + w)(\theta - t)}{M(T - \theta)}$$

In performing the experiment it is very important that the temperature of the specimen should be accurately determined, and that it should be dropped rapidly into the water without having time to lose heat to the air. For this purpose special pieces of apparatus have been devised, and Fig. 87 represents a section of a simple and very efficient type. It consists of a fixed metal tube AB bent at an obtuse angle, and surrounded by a steam or water jacket. Telescoping into the vertical part of the tube is another movable one, which is closed at the top by a cork carrying a thermometer C. The cork is removed and the specimen dropped into this tube. When the temperature indicated by the thermometer has been steady for five minutes, the upper tube is raised, and the specimen falls out of the heater in the direction of the arrow into the calorimeter.

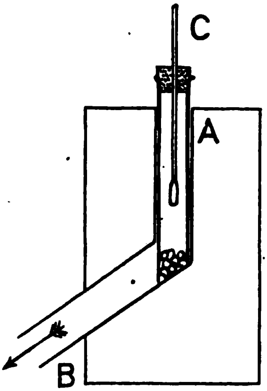


FIG. 87.

The specific heat of a liquid can be determined by half filling the calorimeter with it, instead of with water, and dropping in a hot solid of known specific heat. The reasoning at the top of the page then still holds, if *m* be taken to denote the thermal capacity of the liquid.

A small tin without any outer vessel, such as a ¼-lb. cocoa tin, may be used instead of a calorimeter. But it has the disadvantage of losing heat through its bottom to the table on which it rests, and so does not give such accurate results.

In solving problems on calorimetry it is better not to memorize formulæ but to work out each case from first principles, equating heat gained to heat lost.

Law of Dulong and Petit.—The product of the atomic weight of an element and its specific heat is constant, and equal to about 6·4. This law was given by Dulong and Petit, and amounts to stating that the thermal capacity of every atom is the same. It is not strictly true, boron, carbon, and silicon being exceptions at ordinary temperatures. But these elements obey the law at high temperatures, and it has recently been shown that the specific heats of all the elements probably decrease to zero at the absolute zero of temperature.

Latent Heat of Water.—When a vessel containing a mixture of ice and water is warmed, it is found that the temperature remains at 0° C. as long as any ice remains unmelted, provided that uniformity of temperature is maintained throughout the vessel by stirring. Since heat is being supplied, and the temperature does not rise, it follows that the heat must be used for another purpose, namely for melting the ice. The heat required to melt unit mass of ice without change of temperature is called the latent heat of water. The same quantity of heat is given out again without change of temperature, when unit mass of water is frozen.

The latent heat of water can be determined with an ordinary calorimeter. Let the calorimeter be more than half filled with m grams of warm water, and let w be its water equivalent. Let t be the temperature of the water. Dry two or three lumps of ice about the size of walnuts on blotting paper, drop them into the calorimeter, stir, note the lowest temperature reached (θ), and then weigh the inner vessel so as to find M, the mass of the ice dissolved. Then, if L denotes the latent heat of water, the amount of heat gained by the ice in melting is ML, and in being raised to temperature θ is $M\theta$. The amount of heat lost by the water in being cooled from t to θ is $m(t-\theta)$, and the amount of heat lost by the calorimeter in being cooled through the same range is $w(t-\theta)$. Equating heat gained to heat lost we obtain

$$M(L+\theta)=(m+w)(t-\theta)$$

or

$$L=\frac{(m+w)(t-\theta)}{M}-\theta$$

whence L can be calculated. The correct value is 80 calories.

The passage from the solid to the liquid state is generally abrupt, though not always so. When glass, for example, is heated, it passes through an intermediate viscous state before reaching the state of perfect liquefaction. To convert unit mass of any substance from the solid to the liquid state without change of temperature a definite quantity of heat is required; this is known as its latent heat of fusion.

Latent Heat of Vaporization.—In the same way, when unit mass of a liquid is changed to the state of vapour without change of temperature, a certain quantity of heat is absorbed, which is known as its latent heat of vaporization, or, in the case of water, as the latent heat of steam. The latent heat of steam varies with the temperature at which vaporization takes place. If Q, the total heat of steam, is defined as the quantity of heat necessary to raise one gram of water from 0° C. to a temperature of t° C., and evaporate it at that temperature, then, according to Regnault,

$$Q=606{\cdot}5+{\cdot}305t$$

If we put $t=100$,

$$Q=606{\cdot}5+30{\cdot}5=637$$

Hence if we subtract 100, the quantity of heat necessary to raise one gram of water from 0° to 100° C., we find that the latent heat of steam at 100° C. is 537 calories.

Fig. 88 shows a simple arrangement sometimes adopted for determining the latent heat of steam at atmospheric pressure. As calorimeter a rectangular copper box is used. This is filled to the level indicated with water, and contains a copper condenser with entrance and exit tubes A and B showing above the surface of the water. In the neck of the flask there is a cork into which a short piece of glass tube fits; to this glass tube is attached a short piece of rubber tubing. The experiment is performed by waiting until the water in the flask is boiling vigorously, then inserting the rubber tube in the entrance tube of the condenser for a minute or two. The steam which does not condense escapes by the exit tube. After the rubber tube is disconnected, the water in the calorimeter is stirred, and the highest temperature reached is noted.

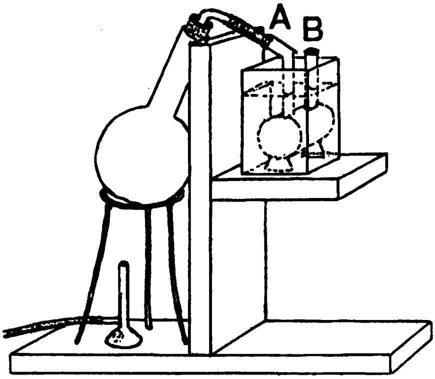


FIG. 88.

Let L be the latent heat of steam, w the water equivalent of the calorimeter and condenser, m the original mass of water, M the mass of steam condensed, t the initial temperature of the water, and θ its final temperature. Then it may be shown that

$$L = \frac{(m + w)(\theta - t)}{M} + \theta - 100$$

Convection Correction.—It has hitherto been assumed, that no heat escapes from, or to the calorimeter, during the course of the experiment. This is not strictly true, and for accurate work the radiation loss, or convection loss, as it ought to be called, for convection plays a larger part in it than radiation, must be corrected for.

Fig. 89 shows how to do this in the case of a determination of the latent heat of steam. The crosses represent temperature readings of the water in the calorimeter plotted as a function of the time. The temperature of the room was 15·0° C. At 0 mins. the temperature of the water was 16·0° C. At 1 min. it was still 16·0° C., and the steam was connected on. At 2 mins. it was 25·0° C., and the steam was disconnected. At 3 mins. it was 29·0° C. At $3\frac{1}{2}$ mins. the temperature reached its highest value, namely 29·5°; thereafter the temperature began to fall. Now the rise of temperature took $2\frac{1}{2}$ mins., from the vertical A to the dotted vertical B. In the equal time from B to C the fall of temperature was ·7° C. The fall of temperature is roughly proportional to the difference of temperature between the calorimeter and the room. The average difference of temperature in the interval AB is 9·9°, and the average difference of temperature in the interval BC is 14·2°. Hence we are justified in supposing that there was a loss of $\frac{99}{142} \times \cdot 7^\circ$ during the interval AB, and in adding this as a correction to the highest temperature reached in the expression for the heat gained by the water and calorimeter; θ in this expression should consequently be $29{\cdot}5 + {\cdot}5 = 30^\circ$ C. But in the expression for the heat lost by the steam θ should be unaffected, for the temperature of the condensed steam falls to $29{\cdot}5^\circ$ C., not 30° C.

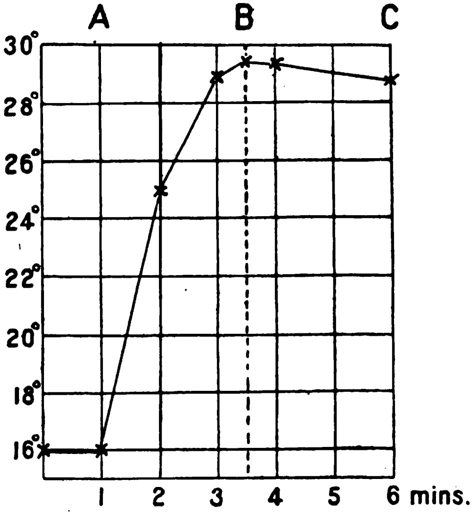


Fig. 89.

Other cases may be treated similarly. Convection and radiation corrections should be used with extreme caution, and the extent to which they affect a result should always be clearly indicated.

Examples 19

1. A calorimeter contains water at 40° C., the equivalent mass of water being 100 gms. In order to cool the water (*a*) 30 gms. of water are withdrawn and replaced by 30 gms. of water at 10° C.; or (*b*) 15 gms. of water are withdrawn and replaced by 15 gms. of water at 10° C., and, after stirring, this process is repeated. Assuming there is no cooling due to other causes, determine the final temperature in the two cases.

2. In determining the latent heat of water the mass of the calorimeter (inner vessel) was 39·31 gms., the mass of calorimeter and warm water 99·90 gms., the initial and final temperatures 36·0° C. and 16·2° C., and the mass of calorimeter, water, and ice melted 113·4 gms. Work out the result on the assumption that the water equivalent is $\frac{1}{10}$ the mass of the inner vessel of the calorimeter.

3. In a second attempt with the same calorimeter the mass of the calorimeter and warm water was 102·02 gms., the initial and final temperatures were 34·6° and 18·0° C., the mass of calorimeter, water, and ice melted 113·00 gms. Work out the result.

4. In determining the specific heat of lead the mass of calorimeter and stirrer is 46·50 gms., the mass of calorimeter, stirrer, and cold water

103·15 gms., the lead is heated to a temperature of 93·3° C., the initial and final temperatures of the water are 15·7° and 28·1° C., and the mass of the lead is 359·0 gms. Work out the result. Assume that the water equivalent of the calorimeter is $\frac{1}{10}$ the mass of the inner vessel and stirrer.

5. The following data were obtained in a determination of the latent heat of steam: Mass of calorimeter and condenser 304·95 gms., mass of cold water 305·25 gms., initial temperature 21·0° C., final temperature 32·0° C., mass of steam condensed 6·3 gms. The water equivalent of the calorimeter and condenser can be taken as $\frac{1}{10}$ their mass. Work out the result.

6. The following data were obtained in a determination of the specific heat of a metal: Mass of calorimeter 49·22 gms., of calorimeter and water 117·92 gms., temperature of heater 97·5° C., initial temperature of water 15·5 C., final temperature of water 19·4° C., mass of calorimeter, water, and metal 172·9 gms. Work out the result on the usual assumption as to the water equivalent of the calorimeter.

7. A calorimeter contains 80·2 gms. of water at 14·9° C. To this 42·9 gms. at 30·2° C. are added, and, when the mixture is stirred, its temperature is found to be 20·1° C. Find the water equivalent of the calorimeter.

Let w = water equivalent.

$$\text{Then heat gained} = (80{\cdot}2 + w)(20{\cdot}1 - 14{\cdot}9) = 417{\cdot}0 + 5{\cdot}2w$$
$$\text{Heat lost} = 42{\cdot}9(30{\cdot}2 - 20{\cdot}1) = 433{\cdot}0$$
$$\text{Hence} \quad 5{\cdot}2w = 433{\cdot}0 - 417{\cdot}0 = 16{\cdot}0$$
$$\text{and} \quad w = 3{\cdot}1 \text{ gms.}$$

8. A calorimeter contains 129·4 gms. of water at 16·2° C. To this 38·7 gms. are added at 37·3° C., and, after stirring, the temperature of the mixture is found to be 21·0° C. What is the water equivalent of the calorimeter?

9. What is the temperature of a piece of iron weighing 5·2 lbs., which, when immersed in 11·0 lbs. of water, raises its temperature from 14·0° to 26·4° C.? The specific heat of iron is ·119.

10. A piece of silver weighing 12·6 gms. was heated to 101·4° C. and dropped into 68·2 gms. of turpentine at 11·2° C. The final temperature was 13·0° C. The water equivalent of the calorimeter was 3·1 gms. The specific heat of silver is ·056. Find the specific heat of turpentine.

11. An aluminium calorimeter of mass 26·2 gms. contains 83·8 gms. ot water at 14·6° C. A piece of aluminium is heated to 100·1° C. and dropped into the water. What is its mass, given that the final temperature of the water is 19·7° C. and the specific heat of aluminium is ·21?

Let M be the mass.

$$\text{Then heat gained} = (83{\cdot}8 + 26{\cdot}2 \times {\cdot}21)(19{\cdot}7 - 14{\cdot}6)$$
$$= 89{\cdot}3 \times 5{\cdot}1 = 455{\cdot}4$$
$$\text{Heat lost} = {\cdot}21\text{M}(100{\cdot}1 - 19{\cdot}7)$$
$$= {\cdot}21\text{M}80{\cdot}4 = 16{\cdot}88\text{M}$$
$$\text{Hence} \quad 16{\cdot}88\text{M} = 455{\cdot}4 \quad \text{and} \quad \text{M} = 26{\cdot}98 \text{ gms.}$$

12. Three lbs. of water are cooled down 2° C. below the freezing point without freezing. An ice crystal is then dropped in. How much water instantly freezes?

13. A glass vessel of weight 3 oz. contains 1 lb. of water, the glass and water being at 20° C.; 4 lbs. of mercury at 45° C. are added. What is the common temperature when the mercury and water have been stirred together? The specific heats of mercury and glass are ·033 and ·2 respectively.

14. The following figures were obtained with a latent heat of steam apparatus: Water equivalent of calorimeter and condenser 30·2 gms., weight of cold water 288·1 gms., initial temperature 16·3° C. Steam was then

passed into the condenser, the increase in weight being 7·2 gms., but ·5 gm. of this condensed in the neck of the flask, and was blown into the condenser as water. What ought the final temperature of the water in the calorimeter to be?

15. Equal volumes of alcohol at 5° C. and turpentine at 60° C. are shaken together; find the temperature of the mixture. The specific gravities of alcohol and turpentine are ·80 and ·87, and their specific heats are ·60 and ·42.

16. The specific heat of ice is ·5. What quantity of heat will be required to change 10·2 gms. of ice at −7·6° C. to water at 14·0° C.?

17. What is the result of mixing 1 lb. of ice at −10·0° C. with 2 lbs. of water at 25·0° C.?

The numerical result will be the same if we use grams instead of lbs. Let us assume that the final temperature is 0° C., and that x grams melt.

$$\text{Then heat gained} = 10 \times \cdot 5 + 80x$$

$$\text{Heat lost} = 2 \times 25$$

Hence $$80x = 50 - 5 = 45$$

and $$x = \tfrac{9}{16}$$

i.e. 9 ounces of the ice are melted.

18. At Quito (2808 metres above sea-level) water boils at 90·1° C., and at Madrid (610 metres above sea-level) water boils at 97·8° C. What is the value of the latent heat of steam at these two places?

19. How many calories are there in the British thermal unit?

20. CHANGE OF STATE

Evaporation and Boiling.—If a kettle is left on the fire too long, it boils dry, all the water passing away as steam. If the water in a pool on the road is exposed to the action of the wind, it evaporates away, although its temperature may not be as much as 15° C. It is interesting to consider these phenomena from the molecular standpoint.

The molecules in a liquid are all in a state of motion in different directions and with different velocities, some moving much faster than others. It was shown on p. 43, that if a molecule reaches the surface it is acted on in a downward direction by molecular attraction, and consequently it usually flies back into the liquid again. If, however, its velocity is very great, the molecular attraction is not sufficient to stop it, and it passes out into the atmosphere above. The number of molecules with a velocity greater than the average is continually being reinforced as a result of the impacts and jostlings of the molecules with one another. Consequently there is a steady stream of molecules through the surface. This is the mechanism by which evaporation goes on. As a result of the abstraction of the molecules with high velocities, the average kinetic energy of the molecules diminishes. But it will be shown in the next section that the heat of a body is simply the kinetic energy of the motion of its molecules. Consequently, as a result of evaporation, the liquid cools.

If a kettle is half full of water, the space above the water contains a mixture of water vapour and air. The resultant pressure exerted by this mixture is the sum of the pressures exerted by the vapour and air separately. Let us suppose that the lid of the kettle is off. Then the sum of the pressures of the air and vapour is equal to the atmospheric pressure.

Suppose, now, that the temperature of the water is gradually raised. The rate of evaporation from its surface increases owing to the velocities of the water molecules increasing, and the mixture of air and water vapour in the upper part of the kettle contains more vapour and less air. When the boiling point is reached, the vapour expels the air, and takes exclusive possession. Its pressure alone is then equal to the pressure of the external air. The boiling point is consequently the temperature at which the pressure of the aqueous vapour is equal to the pressure of the atmosphere.

The essential difference between evaporation and boiling lies in the fact, that molecules evaporate only at the free surface of a liquid, but, when boiling takes place, they also pass into the vaporous condition at bubbles inside the body of the liquid. Consequently at boiling the rate of formation of vapour molecules is very greatly increased.

Vapour Pressure.—Let us suppose we have two barometric tubes (Fig. 90), and that by means of a curved tube a few drops of water are introduced into the right-hand one. They at once pass up the mercury column into the vacuum and vaporize there. Consequently the level of the surface is depressed to B, the height AB being a measure of the pressure exerted by the vapour. If the temperature of the room is 15° C., this depression, AB, increases to a maximum of 13 mm. It may be less than this if insufficient water has been introduced into the vacuum, but it refuses to increase beyond this. If more water is introduced after the maximum value of the depression has been attained, it does not vaporize, but lies simply as a liquid on the top of the mercury. If the barometer tube AC is then raised slightly, more water vaporizes, but the difference in level of the two columns always remains the same. If the tube AC is pushed down into the mercury, the vapour condenses on the surface at B, the difference in level of D and B again remaining constant. If the tube AC is pushed down until its end reaches B, all the vapour condenses.

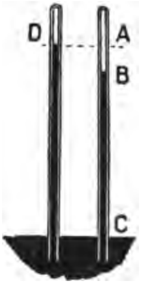


FIG. 90.

It is clear from these experiments, that when the vapour is in contact with its liquid it has a definite pressure. It is then said to be saturated, and the value of its pressure is referred to as the saturation pressure.

If now the barometer tube were surrounded with a water-jacket, so that its temperature could be increased, it would be found that as the temperature was gradually increased, the saturation pressure would increase also, and that when the temperature became 100° C., the saturation pressure would be 76 cms.

The experiment of Fig. 90 is not very striking in the case of water, as its vapour pressure is so small at 15° C. If, however, ethyl ether is used instead of water, a depression of 35·6 cms. is obtained at 15° C.

Measurement of Vapour Pressure.—A barometric tube surrounded by a water-jacket is not a very satisfactory way of determining the saturation pressure for different temperatures, since it is difficult to read the level of the mercury through the water-jacket, and, in any case, it does not go above pressures of one atmosphere. Fig. 91 shows an arrangement devised and used by Regnault, which is suitable for all pressures.

A is an air-tight vessel in which the liquid is boiled. The temperature of the liquid and the vapour above it are given by thermometers T, which dip into closed iron tubes containing mercury. The vapour rises out of A into the condenser C, a long inclined tube surrounded by a jacket in which a stream of cold water is flowing. It is all condensed before it reaches the end of the tube, and flows back into the boiler. B is a large reservoir containing air surrounded by water, so as to maintain its temperature constant. The pressure of the air in the reservoir is read by the manometer at the side.

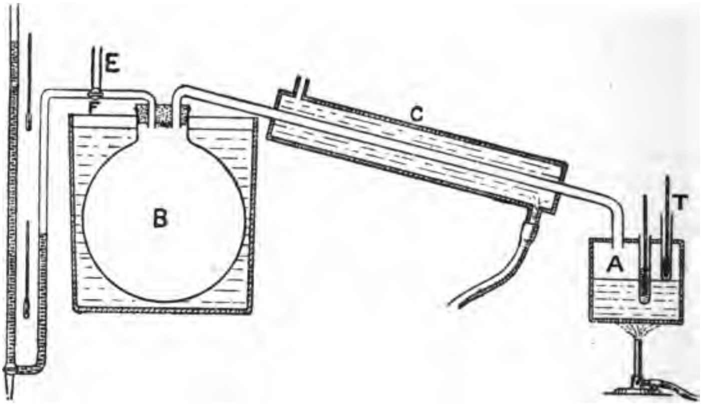


FIG. 91.
From Watson's Physics.

In making measurements with this apparatus, the pressure in B was first adjusted to a definite value by connecting a compression or exhaustion pump to the three-way cock at E. The liquid was then boiled, and the temperature of the vapour read as soon as it became constant; the pressure of the air in the reservoir was then equal to the saturation pressure of the liquid for this temperature. The pressure was then altered to a new value, and after the temperature had become steady again, its value for the new pressure was determined. By this means Regnault was able to determine the saturation pressure of water up to 230° C., the value for that temperature being 27½ atmospheres.

The Cryophorus.—Whenever some of a liquid evaporates, heat is absorbed, and the temperature of the liquid left behind falls. The cryophorus is an instrument which illustrates this fact. It consists of a bent glass tube with a bulb at each end (Fig. 92), and contains water and water vapour, but no air. When the water falls down the tube, it hits the wall at the foot with a metallic click owing to the absence of an air cushion. The water is introduced into the bulb A, and the other bulb immersed in a freezing mixture. This causes the vapour in the tube to condense, and produces rapid evaporation of the water in A. After a short time the temperature of the water in A is so much reduced that it freezes.

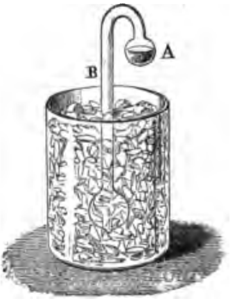


FIG. 92.

Distinction between a Vapour and a Gas.—Up to a temperature of 365° C. water vapour can be liquefied by the application of pressure, the pressure necessary at this temperature being 194·6 atmospheres. Above 365° C. it cannot be liquefied by pressure, and is then called a gas; below this temperature it is said to be a vapour, 365° being known as its critical temperature. A vapour is simply a gas below its critical temperature.

All gases have their critical temperatures, below which they can be liquefied by the action of pressure alone. The critical temperatures of the gases formerly known as permanent gases are very low, the values for hydrogen, oxygen, and nitrogen being $-234{\cdot}5°$ C., $-118°$ C., and $-146°$ C. The pressure necessary to liquefy a gas at its critical temperature is known as its critical pressure.

Vapours obey the gas equation, provided that the pressure is not too great. The density of aqueous vapour is $\frac{5}{8}$ of the density of dry air under the same conditions of temperature and pressure.

Solids can pass directly to the state of vapour without passing through the intermediate liquid state. They are then said to sublime. Iodine crystals form a good example of a case of sublimation. When gently warmed, they pass into a dark vapour. Snow and ice also evaporate away, even though they are kept below the freezing point.

If some water is left inside an air-tight vessel, a portion evaporates and fills the space above the surface. The quantity of vapour formed depends only on the temperature of the enclosed space, and practically not at all upon the pressure of the air inside the enclosed space. But the presence or absence of air has a great influence on the speed at which the vapour is formed; the whole evaporation takes place at once in a vacuum, but may require several hours if the space is already full of air. This result, that just as much liquid evaporates into a space which is full of air as into a vacuum, was first given by Dalton.

Relative Humidity.—The atmosphere always contains aqueous vapour. In general it is not saturated, and the ratio of the quantity of aqueous vapour actually present in the atmosphere to the quantity of aqueous vapour it would contain if it were saturated, the temperature remaining constant, measured in per cent., is called the relative humidity or hygrometric state of the atmosphere.

Since aqueous vapour obeys Boyle's law, its density varies as its pressure if the temperature is kept constant. Hence

$$\frac{\text{actual density}}{\text{saturation density}} = \frac{\text{actual pressure}}{\text{saturation pressure}}$$

It is the relative humidity on which our feeling of the dryness of the air depends, not on the actual density of aqueous vapour present. On a hot day the air may feel dryer than on a cold day, because the relative humidity is less, although on the hot day there may have been much more aqueous vapour in the atmosphere. The greater amount of aqueous vapour present has been more than offset by the higher value of the saturation density. If the relative humidity is 100 per cent., the weather feels oppressive and we perspire easily.

Relative humidity determinations are made in hot-houses to see that the plants receive their proper supply of moisture. They are also made regularly at meteorological stations, and they afford information as to the likelihood of rain or frost.

Hygrometers.—Instruments used for measuring the relative humidity of the atmosphere are termed hygrometers. Here two types will be described, namely Regnault's hygrometer and the wet-and-dry-bulb thermometer.

Regnault's hygrometer consists essentially of a brightly polished cylindrical vessel, mounted on a stand; this vessel is half full of ether, and is closed at the top by a cork, through which passes the stem of a thermometer, the bulb of which dips into the ether. There is an arrangement by which air can be made to bubble through the ether. This causes it to evaporate, and gradually lowers its temperature and the temperature of the vessel containing it. Consequently the air in its neighbourhood becomes cooled, is no longer able to contain its aqueous vapour, and some is deposited as moisture on the brightly polished surface, causing a dimness on the latter. As soon as this dimness is noted, the evaporation of the ether is stopped, and the temperature is read. The temperature of the surface then begins to rise, and the dimness gradually disappears. As soon as it is all gone, the temperature on the thermometer is read again. The mean of the temperatures of appearance and disappearance is the dewpoint, the dewpoint being defined as that temperature for which the actual pressure of the aqueous vapour in the atmosphere becomes the saturation pressure.

After the dewpoint has been determined, the temperature of the air itself is taken. The relative humidity is then simply the ratio of the saturation pressure of aqueous vapour at the dewpoint to its value at the temperature of the air. The following table of vapour pressures is suitable for use with a Regnault's hygrometer:—

Temp.	Press.	Temp.	Press.	Temp.	Press.	Temp.	Press.
1°C.	4·92 mm.	8°C.	8·04 mm.	14°C.	11·98 mm.	20°C.	17·51 mm.
2	5·29	9	8·61	15	12·78	21	18·62
3	5·68	10	9·20	16	13·62	22	19·79
4	6·10	11	9·84	17	14·52	23	21·02
5	6·54	12	10·51	18	15·46	24	22·32
6	7·01	13	11·22	19	16·46	25	23·69
7	7·51						

As an illustration of the use of the above table, let us suppose that the dewpoint is 7·2° C., and the temperature of the air 15·1° C. Then

$$\text{Pressure at } 7{\cdot}2^\circ = 7{\cdot}51 + \tfrac{2}{10}(8{\cdot}04 - 7{\cdot}51) = 7{\cdot}62 \text{ mm.}$$
$$\text{Pressure at } 15{\cdot}1^\circ = 12{\cdot}78 + \tfrac{1}{10}(13{\cdot}62 - 12{\cdot}78) = 12{\cdot}86 \text{ mm.}$$

and the relative humidity is $\dfrac{7{\cdot}62}{12{\cdot}86}$ or 59 per cent.

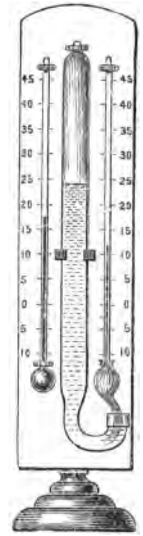


Fig. 93.

The wet-and-dry bulb thermometer is represented in Fig. 93. It consists of two thermometers mounted on the same stand. The left-hand or dry-bulb thermometer gives the temperature of the room. The right-hand thermometer has its bulb covered with a piece of muslin, which is constantly kept moist by being connected with a reservoir of water. Unless the air is saturated with moisture, the right-hand thermometer indicates a lower temperature than the left-hand one; this is due to the cooling effect of the evaporation from the muslin. The dewpoint is obtained by multiplying the difference of temperature of the wet and the dry bulbs by a factor known as Glaisher's factor, and subtracting the result from the temperature of the dry bulb. Glaisher's factor varies with the temperature of the dry bulb, and is given by the following table:—

Dry Bulb Temp.	Factor.	Dry Bulb Temp.	Factor.	Dry Bulb Temp.	Factor.	Dry Bulb Temp.	Factor.
1°C.	2·81	8°C.	2·13	14°C.	1·92	20°C.	1·79
2	2·54	9	2·10	15	1·89	21	1·77
3	2·39	10	2·06	16	1·87	22	1·75
4	2·31	11	2·02	17	1·85	23	1·74
5	2·26	12	1·99	18	1·83	24	1·72
6	2·21	13	1·95	19	1·81	25	1·70
7	2·17						

As an illustration of the table, let us suppose that the dry bulb reads 14·9° C. and the wet bulb 11·7° C. Then the factor is 1·89 and the dewpoint $14{\cdot}9 - 1{\cdot}89(14{\cdot}9 - 11{\cdot}7) = 14{\cdot}9 - 6{\cdot}0 = 8{\cdot}9^\circ$ C. When once the dewpoint is obtained, the relative humidity can be calculated in the same way as for Regnault's hygrometer.

Rain, Hail, Snow, Dew.—If the temperature of the atmosphere falls below the dewpoint, the water vapour which was before perfectly transparent condenses in liquid drops on dust particles floating in the air, forming a mist. If the condensation takes place in the upper regions of the atmosphere, for example, owing to a warm current of air entering a cold region, the mist becomes a cloud. If much vapour condenses, the drops are large and fall as rain. If the falling rain passes through cold regions, it may freeze into hail, but the formation of hail is not thoroughly understood. If the temperature at which the condensation takes place is below the freezing point, the aqueous vapour condenses into snow.

At night, if the blades of grass and other objects on the earth's surface cool more rapidly than the air above them, the air coming into contact with them has its temperature lowered, and consequently moisture is deposited on the grass. Thus dew is formed. Dew is most copiously deposited on the best radiators and conductors, because the temperature of these bodies falls most rapidly. If the temperature of the earth's surface falls below 0° C., the water vapour in the atmosphere is precipitated in the solid form of hoarfrost.

EXAMPLES 20

1. Is the white cloud we see coming out of the spout of a kettle steam?

2. Explain how the sun causes a morning mist to disappear, and why fogs are formed round icebergs.

3. Why does the outside of a jug containing ice cream become moist on a warm day?

4. Would a breeze have a cooling effect on a hot day, if there were no moisture on the face? Would it produce a cooling effect, if there was moisture on the face and the atmosphere was saturated?

5. The temperature of the air is 16·3° C. and the dewpoint is 9·2° C. Find the relative humidity and the quantity of aqueous vapour present in the atmosphere in grams per litre.

6. The temperatures of the wet and dry bulb thermometers stand respectively at 13·5° C. and 15·4° C. Find the dewpoint and the quantity of aqueous vapour present in the atmosphere in grams per litre.

7. A mass of air is at a temperature of 100° C. Its dewpoint is 48° C. If the saturation pressure of aqueous vapour at 48° C. is 8·35 cms., find the relative humidity of the air.

8. Calculate the weight of 15 litres of air saturated with aqueous vapour at 18° C. and 750 mm.

The saturation pressure at 18° C. is 15·46 mm. The pressure of the air alone is consequently 734·54 mm.

$$\begin{aligned}
\text{Weight of air} &= 15 \times 1{\cdot}293 \times \frac{734{\cdot}54}{760} \times \frac{273}{291} \\
&= 17{\cdot}59 \text{ gms.} \\
\text{Weight of aqueous vapour} &= 15 \times \tfrac{5}{8} \times 1{\cdot}293 \times \frac{15{\cdot}46}{760} \times \frac{273}{291} \\
&= {\cdot}23 \text{ gms.} \\
\text{Required result} &= 17{\cdot}59 + {\cdot}23 = 17{\cdot}82 \text{ gms.}
\end{aligned}$$

9. A cubic metre of air at 20° C. is found to contain 10 gms. of aqueous vapour. What is its relative humidity?

10. The quantity of water given out by the lungs and skin may be taken as 30 ounces in 24 hours. How many cubic feet of half-saturated air at 15° C. and 78 cms. will this quantity of water saturate?

11. The temperature of the air at sunset is 10° C. and the temperature of the ground is 4° C. No dew falls. What is the greatest value which the relative humidity of the air can have?

12. It is sometimes stated, as a practical rule, that when the saturation pressure of aqueous vapour is x mms. of mercury, the weight of water present in a cubic metre is x gms. What is the percentage error in this statement for a pressure of 8 mms.?

If the pressure is 8 mms., the temperature is 7·9° C.

Weight of water in cubic metre

$$= 1000 \times \frac{5}{8} \times 1{\cdot}293 \times \frac{8}{760} \times \frac{273}{280{\cdot}9}$$
$$= 8{\cdot}27 \text{ gms.}$$

The error is $\frac{27}{8{\cdot}27} = 3{\cdot}3$ per cent.

13. A mass of air extending over an area of 60,000 square metres to a height of 300 metres has the dewpoint at 15° C., its temperature being 20° C. How much rain will fall if the temperature sinks to 10° C.? Express your answer in mms. The pressure of water vapour is 9·2, 12·8, and 17·5 mms. at 10°, 15°, and 20° C. respectively. The density of aqueous vapour is $\frac{5}{8}$ the density of dry air under the same conditions of temperature and pressure.

21. THE MECHANICAL EQUIVALENT OF HEAT

Two different views have been held as to the nature of heat. According to one of these views it was an imponderable fluid called caloric. When the temperature of a body was increased, the amount of caloric in it increased; when heat flowed from a hot body to a cold one, there was a passage of caloric from the one body to the other. According to the other view, heat is a brisk agitation of the molecules of a body. When a body is heated, the molecules move faster. When a body is cooled, they move slower, and at the absolute zero they come to rest altogether. This view has been universally established for a long time now, and we have already used it in the explanation of evaporation on p. 92.

Count Rumford, an American in the service of the Bavarian government, noticed at the close of the eighteenth century, that an enormous amount of heat was generated when cannon were bored. In one special experiment, when a blunt steel borer was used, a cylinder of gun metal was raised in temperature from 60° F. to 130° F., and only $\frac{1}{950}$ of the mass of the whole cylinder was rubbed off. On the caloric theory the result was explained by assuming that the disintegrated metal could not contain the fluid; consequently it was forced out into the solid cylinder. But it seemed quite improbable that such a small quantity of disintegrated metal could hold so much caloric; so Rumford came to the conclusion, that the heat was a motion of the molecules of the cylinder produced by the friction of the borer.

According to the caloric theory ice contained less caloric than water. Sir Humphrey Davy rubbed two pieces of ice together in vacuo, so that they melted. Since caloric could not be created, this experiment could not be explained on the caloric theory.

Still, in spite of these experiments, the caloric theory held the field until about 1842, when it was shown by Joule and others, that not only was heat motion, but that, when heat was produced by friction, the heat produced was proportional to the work done.

The apparatus for Joule's best-known experiment is shown in Fig. 94. Resting on a wooden grating is a copper calorimeter, which is provided with a brass paddle wheel. The paddle wheel is attached to the roller A, and so rotates about a vertical axis. The calorimeter is fitted with vanes, so that, when the paddle wheel rotates, the water does not rotate as a whole, but there is a maximum of friction and irregular motion. Cords are wound on the roller A, pass over the pulleys C and D, and carry the weights E and F. The pulleys are mounted on friction wheels. The experiment was performed by allowing the weights to fall to the floor. The distance they fell through (h) was given on the scales G and H. The work done by gravity on the weights was $2mgh$, where m is the mass of each weight.

As the weights fell, the cords unwound from the roller A, the paddle wheel rotated, and the temperature of the water in the calorimeter rose. The amount of heat produced in the calorimeter was found to be proportional to the work done on the weights. Hence it was reasonable to conclude that the potential energy lost by the weights reappeared as heat in the calorimeter. Corrections had to be made for the energy lost by the weights striking the floor, etc., but when all the corrections had been applied, and other experiments had been taken into account, Joule found that 772 ft.-lbs. of work produced a rise of 1° F. in a lb. of water.

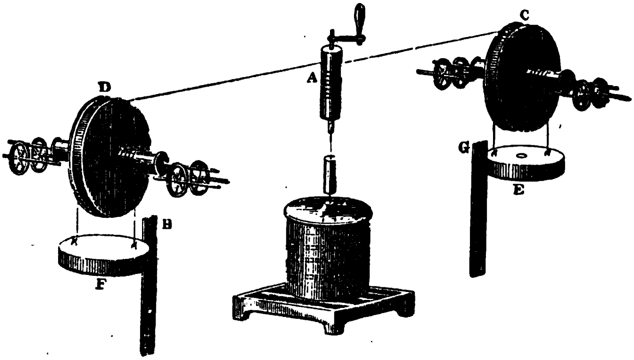


FIG. 94.
From Ganot's Physics.

This result has been confirmed by other experimenters. Whenever by any experimental arrangement energy is apparently destroyed and heat produced, the amount of energy lost always corresponds in the same way to the heat produced. It has been shown in section 9 how potential energy changes into kinetic energy and *vice versâ*. We regard the heat of the body simply as the kinetic energy of the motion of its molecules. Joule's experiment is thus an illustration of the principle of the conservation of energy. As a matter of history it helped very considerably to establish this principle.

When a locomotive pulls a train, it is converting the heat energy in its furnace into kinetic energy of the train. The kinetic energy of the train is itself turned into heat again by the friction of the axles. The heat energy in the furnace came from the energy of constitution of the coal, which was in turn due to the action of the sun's heat on vegetable matter in a previous age of the earth's history. So we have energy converted from one form to another, but never created or destroyed.

Joule's Equivalent.—Joule's result 772 ft.-lbs. = 1 lb.-degree Fahr. gives 772 foot-grams = 1 gram-degree Fahr., or 772 × 30·48 gram.-cms. = $\frac{5}{9}$ cals. Hence

$$\begin{aligned} 1 \text{ cal.} &= \tfrac{9}{5} \times 772 \times 30{\cdot}48 \times 981{\cdot}4 \text{ ergs} \\ &= 4{\cdot}157 \times 10^7 \text{ ergs} \end{aligned}$$

This result is now regarded as somewhat low, $4{\cdot}185 \times 10^7$ being the mean result given by more recent experiments. It is customary, however, in calculations simply to take the round number 42,000,000. This number is denoted by J, just as 3·1416 is denoted by π, and is always referred to as Joule's equivalent.

Since the heat of a body is simply the kinetic energy of the motion of its molecules, heat can be measured in ergs, one calorie being equal to J, or 42,000,000 ergs.

EXAMPLES 21

1. Show that J is not affected by a change in the unit of mass employed.

2. Find the equivalent in ergs of the amount of heat required to raise 1 lb. of water through 1° F.

$$\begin{aligned} 1 \text{ lb.} \times 1^\circ \text{ F.} &= 453{\cdot}6 \text{ gms.} \times \frac{5^\circ}{9} \text{C.} \\ &= 453{\cdot}6 \times \tfrac{5}{9} \times 42 \times 10^6 \text{ ergs} \\ &= 1{\cdot}059 \times 10^{10} \text{ ergs} \end{aligned}$$

3. Mercury falls from a height of 15 metres upon a perfectly non-conducting surface. How much is its temperature raised? The specific heat of mercury is ·033.

4. A waterfall is 80 ft. high. How much warmer should the water be at the foot than at the top of the fall if all the energy acquired is changed into irregular motion in a pool at the foot, and then into heat, none of the heat escaping to the bed of the river?

5. From what height must a raindrop fall to the ground in order that its temperature may be raised 1° C.? (The result will be only of theoretical interest; in practice the resistance of the air cannot be neglected.)

6. A lead bullet of specific heat ·032 strikes an iron target with a velocity of 350 metres per second. If the bullet falls dead, and half the heat produced is lost to the target, find its rise in temperature.

7. Given that the melting point of lead is 327° C. and its latent heat of fusion is 5·4, find with what velocity a lead bullet at 20° C. must strike a target, in order that all the heat produced should be just sufficient to melt it. 357

8. An engine consumes 56 lbs. of coal per hour. The calorific value of coal is $3{\cdot}6 \times 10^6$ calories per lb., and 1 ft.-lb. = $13{\cdot}56 \times 10^6$ ergs. Given that the efficiency of the engine, *i.e.* the ratio between the useful work performed and the work represented by the heat of combustion of the fuel, is 5 per cent., find the horse-power that the engine develops.

Calorific value of coal consumed per minute

$$\begin{aligned} &= \frac{56 \times 3{\cdot}6 \times 10^6}{60} \text{ cals.} \\ &= 56 \times 6 \times 10^4 \text{ cals.} \\ &= \frac{56 \times 6 \times 10^4 \times 42 \times 10^6}{13{\cdot}56 \times 10^6} \text{ ft.-lbs.} = 1{\cdot}041 \times 10^7 \text{ ft.-lbs.} \end{aligned}$$

Useful work done per min. = $5{\cdot}205 \times 10^6$ ft.-lbs.

$$\text{H.P.} = \frac{5{\cdot}205 \times 10^6}{33{,}000} = 15{\cdot}8$$

9. What quantity of coal would produce the same quantity of heat as the work of one horse for one hour? Take the value of the calorific value of coal from the preceding question.

10. A cylindrical copper tube closed at the ends, and of 1 sq. cm. cross-sectional area, has an internal length of 40 cms., weighs 20 gms., contains 40 gms. of lead shot, and is filled up with water. It is tied to a diameter of a vertical wheel, which is then rapidly rotated 300 times. Find the rise in the temperature of the water, neglecting the thermal capacity of the copper and lead and the space occupied by the lead.

11 Work out the preceding problem, taking into consideration the neglected factors.

12. In Joule's classic experiment each weight was 28·5 lbs., the distance fallen through was 5·9 ft., the water equivalent of the calorimeter was ·45 kg., and the mass of the water in the calorimeter was 6 kg. Find by how much the temperature rose when the weights dropped once. (J = 1400 ft.-lbs. per lb.-degree-Centigrade unit of heat.)

22. CONDUCTION, CONVECTION, AND RADIATION

Conduction.—If one end of a small iron rod is held in a bunsen flame, the other end being held in the fingers, the end in the fingers becomes so hot that the rod has to be dropped. Heat is then said to be conducted along the rod from the flame to the fingers. All metals conduct heat well, the best conductor being silver, then copper and gold following in the order named. Wood does not conduct nearly so well. The end of a pencil may be held in the bunsen flame without the fingers at the other end experiencing any discomfort at all.

When the end of an iron rod is held in the flame it becomes red-hot, and the molecules at that end are thrown into violent irregular to-and-fro motion. They collide with the neighbouring molecules, and throw them in turn into motion; these in turn collide with molecules further down the rod, and presently all the molecules in the rod are in a state of violent motion. The molecules themselves do not travel down the rod; they keep in the same place. But the state of motion spreads along the rod. As has been mentioned in the last section, heat itself is simply the kinetic energy of the motion of the molecules.

Cork and felt are bad conductors of heat. As a consequence of this, on a cold day they feel warmer to the touch than a piece of metal does, although, as may be shown by a thermometer, they are actually at the same temperature. Similarly, in a room where there is no fire, the iron fender feels much colder to the touch than a rug beside it. This is simply because it conducts away heat more rapidly from the hand than the rug does.

Ingenhousz's Experiment.—In order to compare roughly the conducting power or conductivity of different solids Ingenhousz used the arrangement shown in Fig. 95. It consists of an iron trough, into the sides of which are fixed rods of the different metals to be compared. These rods are all of the same shape, and extend the same distance into the trough. They are coated with paraffin wax, which melts about 52° C., and the trough is filled up with boiling water. Heat from the water is conducted along the rods, and the wax melts to a certain distance along each rod, and drops off. The wax melts furthest along the rod in the case of the best conductor; indeed, it may be shown by reasoning beyond the scope of this book, that the conductivity of the metal of the rod is proportional to the square of the distance for which the wax melts.

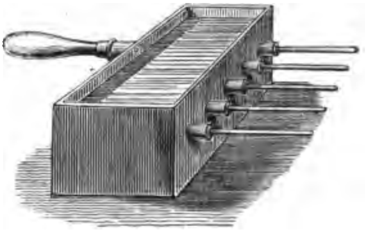


Fig. 95.

It should be noted, that the quickness with which the wax starts to fall off is no guide to the conductivity, but depends as well on the thermal capacity of the rod. The heat has to fill up the rod as it travels along it, and may at first travel faster along a rod with a low thermal capacity. Only when the wax ceases to fall off can a conclusion be drawn as to the conductivity.

Convection.—If a few grains of aniline dye are dropped to the bottom of a flask rather more than half full of water, and the latter heated by a bunsen, the water directly above the bunsen becomes lighter and ascends, its place being taken by colder water which comes in from the sides, and the upward currents are coloured by the dye, and take place as shown in Fig. 96. Such currents are called convection currents. They carry heat from the hotter to the colder parts of the liquid by transferring the molecules themselves bodily together with their heat.

Convection currents can be shown more strikingly by the arrangement of Fig. 97, which illustrates the principle of the hot-water-heating system. The water heated in the lower vessel ascends the tube AB; the colder water in the upper vessel travels down the tube CD to fill its place. The motion can be shown very clearly by dropping some dye into the upper vessel and watching it travel down the tube CD.

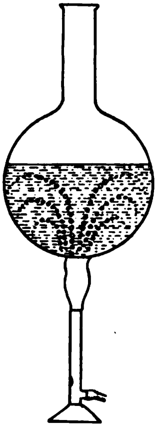


Fig. 96.

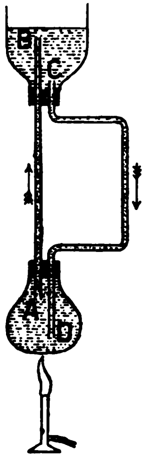


Fig. 97.

The ascent of smoke up a chimney, or of hot air above an electric glow lamp, is a convection current. Winds are convection currents in the atmosphere. If any part of the earth's surface becomes hotter than the surrounding parts, the air there tends to rise and spread outwards, while colder air flows in below to take its place. Thus on a sea-coast we often have sea breezes in the afternoon and land breezes in the early morning. The land heats more rapidly than the sea; consequently in the afternoon, when it is hottest, the air rises and an under-current comes in from the sea. But the land also cools more rapidly, so in the early morning the under-current is outward from the land.

Radiation.—The earth derives its heat and light from the sun. But the earth's atmosphere becomes thinner and thinner as we ascend upwards from its surface, ceasing altogether at a height estimated between 100 and 500 miles. Above this there is merely empty space. Now we cannot have conduction or convection where there is no matter; the heat from the sun must be radiated to the earth, travelling out from the sun in straight lines in the same way as the light does. When the heat rays fall on a body they set its molecules into vibration, but when the heat is on its way from the sun the energy exists in empty space, and does not consist in the motion of any molecules.

Heat rays are reflected by mirrors and concentrated by lenses in the same way as light rays are. This can be shown very well by the experiment represented in Fig. 98. A copper sphere is heated in a bunsen flame; a black heat is sufficient. It is then placed at the focus of a concave metal mirror, and acts as a source of heat. Its rays fall on the mirror, are rendered parallel, are reflected by a plane mirror, fall on another concave mirror, and are brought to a focus on the bulb of a thermometer. The reading of the thermometer rises immediately the copper sphere is placed in position, and falls as soon as it is taken away. In order to make the experiment visible to a large audience a Galilean air thermometer is sometimes used instead of the ordinary mercury one. The mirrors must be of metal, not of glass; glass absorbs most of the rays in question. But the surfaces of the metal mirrors do not require to be highly polished.

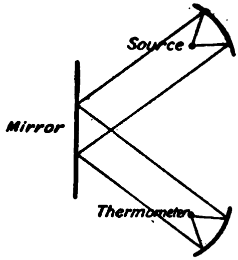


FIG. 98.

The Dewar Flask.—The radiation from a surface depends very much on the nature of the surface, being greatest from black surfaces and least from highly polished ones. Surfaces which radiate out heat well also absorb it well, the one power being proportional to the other.

These facts are taken advantage of in the construction of the Dewar flask. This consists of a glass vessel with double walls; the space between the two walls is a vacuum, and both walls are silvered on the side next the vacuum. The flask was originally designed for storing liquid air, which has, of course, a very low temperature, and, if exposed to the atmosphere, would evaporate away at once. In the Dewar flask, however, heat can get through the vacuum only by radiation, and since both walls are silvered the heat radiated from the outer wall and absorbed by the inner one is as small as possible. Consequently the liquid air is preserved for a long time.

Dewar flasks are now in wide use in ordinary life under the name of Thermos flasks, for the converse purpose of keeping liquids hot. Here the object is to retain the heat in the flask, not to keep it out of it.

The ordinary calorimeter is designed to diminish heat losses from the inner vessel as much as possible. Air is a bad conductor, so the conduction loss is small. The outer vessel keeps off draughts, so the convection loss is small, and, if the inner surface of the outer vessel and outer surface of the inner vessel are roughly polished, the radiation loss is small. The surfaces do not require to be highly polished; surfaces which are rough as regards light rays may be smooth as regards heat rays.

CHAPTER IV

SOUND

23. WAVE MOTION

Transverse Wave.—It was explained in 7, that when a fictitious particle moves round a circle with uniform speed, and when a perpendicular is

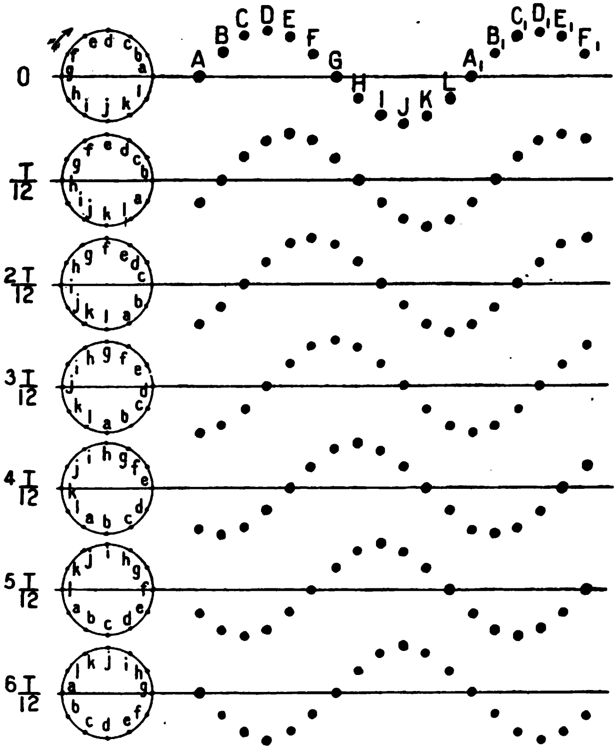


FIG. 99.

drawn from its instantaneous position to a fixed diameter, then the point at the foot of this perpendicular is said to describe a S.H.M.

Let us suppose that the topmost circle in Fig. 99 has twelve points on it, *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *j*, *k*, *l*, each point being distant 30° from the next one, and that these points are all moving round the circle in the direction of the arrow with the same speed. Let A, B, C, etc., be particles which move up and down equidistant vertical lines in such a way, that the distance of each particle from the horizontal line is the same as the distance of the point on the circle with the same letter from the horizontal line; A_1, B_1, C_1, etc., have the same distances from the horizontal line as A, B, C, etc. Consequently, as the points *a*, *b*, *c*, etc., move round the circle, the particles A, B, C, etc., describe S.H.Ms. Let T be the time taken by the points to go round the circle, *i.e.* T is the period of the S.H.Ms. Then the successive diagrams in Fig. 99 give the positions of the particles A, B, C, etc., at times $\frac{1}{12}T$, $\frac{2}{12}T$, $\frac{3}{12}T$, $\frac{4}{12}T$, $\frac{5}{12}T$, and $\frac{6}{12}T$.

If all the particles in each diagram are joined by a curve, it will be noticed that, as the motion goes on, the whole curve appears bodily to move towards the right. If instead of stopping at F_1 we had added other particles, making the displacement of every twelfth particle the same, we should have obtained a curve of the form moving bodily towards the right, or, in other words, we should have obtained a harmonic transverse wave. The wave is said to be transverse, because the particles move at right angles to the direction of propagation. Although the wave moves towards the right, the particles themselves do not move in this direction. The distance between two successive crests or two successive troughs is defined to be the wave-length, and is usually denoted by λ; the velocity with which any crest or trough moves is defined to be the velocity of the wave, and is usually denoted by V, and the time taken by a crest to travel a distance equal to one wave-length is defined to be the period of the wave. It is obvious from Fig. 100, that the period of the wave is the same as the period of the S.H.Ms. of the individual particles which constitute the wave motion; it is hence denoted by T. We have then

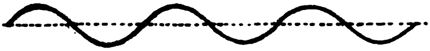


FIG. 100.

$$T = \frac{\lambda}{V} \quad \text{or} \quad \lambda = VT$$

The number of complete wave-lengths which pass a given point in one second is called the frequency, and is denoted by *n*.

$$n = \frac{1}{T} \quad \text{and} \quad V = n\lambda$$

n is also the number of complete oscillations made by each of the individual particles in one second. The maximum displacement of each particle from its equilibrium position, *i.e.* from the horizontal line, is called the amplitude of the wave.

Stationary Waves.—The dotted curves in the first diagram in Fig. 101 represent two transverse harmonic waves moving in opposite directions; the full curve, which is obtained simply by adding the ordinates of the dotted curves, gives the resultant displacement. The second diagram represents conditions a time $\frac{1}{8}$T later, when the dotted waves have moved respectively $\frac{1}{8}\lambda$ to the right and $\frac{1}{8}\lambda$ to the left; the full curve again gives the resultant displacement. The third diagram represents conditions a time $\frac{1}{4}$T later, when the dotted curves have moved respectively $\frac{1}{4}\lambda$ to the right and $\frac{1}{4}\lambda$ to the left; the dotted curves now exactly neutralize one another, and the full curve is a straight line. The fourth and fifth diagrams represent respectively conditions after times $\frac{3}{8}$T and $\frac{1}{2}$T, the full curves giving the resultant of the dotted curves in each case. If we take later times, $\frac{5}{8}$T, $\frac{6}{8}$T, etc., we simply get the same changes in the reverse order.

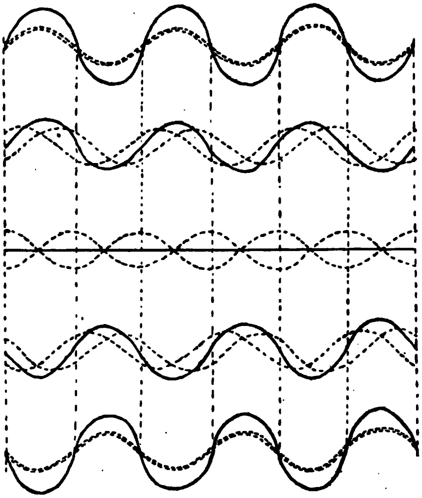


Fig. 101.

Note that the points in which the full curves are intersected by the seven vertical dotted lines are always at rest. Such points are called nodes. The points midway between the nodes are called loops; at the loops the displacement changes from a maximum on the one side to a maximum on the other. The resultant displacement changes simply from the form given by the full curve in Fig. 102 through the straight line to the dotted curve, and then back again through the straight line to the full curve. There is no forward motion of the wave as a whole, and the wave is said to be a stationary one.

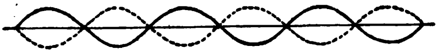


Fig. 102.

Thus two progressive waves of equal amplitude in opposite directions produce a stationary wave with an amplitude twice as great as that of either of its components.

Experimental Production of Transverse Wave.—If a rope about 25 feet long is suspended vertically and the lower end is waggled, a harmonic wave rapidly ascends the rope, is reflected at the top, and comes down again. The ascending and descending waves superimpose, and consequently cannot be seen very clearly. If, however, the lower end of the rope is held with one hand, and the rope struck with the other hand a few inches above this end, an indentation or pulse travels up the rope, is reflected at the top, and can be quite clearly seen coming down the other side of the rope.

If a stretched string or wire is plucked, a stationary transverse wave is produced in it.

Longitudinal Wave.—Let us suppose that in Fig. 99 the displacements of the various particles are marked out along the horizontal line instead of at right angles to it. Then we shall have the following figure:—

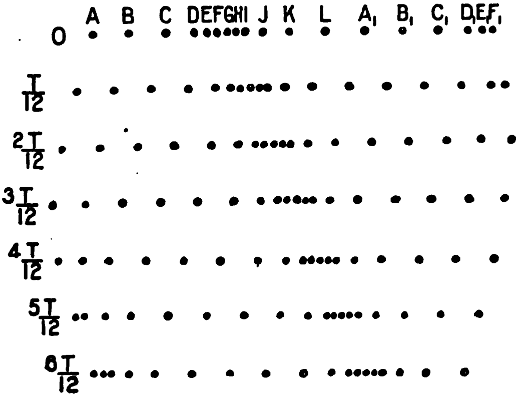


FIG. 103.

In the top row the particles are crowded together at G and beyond F_1; at these two points a condensation or compression is said to take place. At A and at A_1 the particles are farthest apart; here a rarefaction is said to take place. And, as the individual particles perform their S.H.Ms., these condensations and rarefactions move towards the right, just in the same way as the crests and troughs moved towards the right in Fig. 99. A longitudinal wave is then said to pass along the particles from left to right, and the distance between any condensation and the one next to it, or between any rarefaction and the one next to it, is said to be the wave-length of the wave. At condensations and rarefactions the particles are passing through their equilibrium positions; it is evident also, by comparing Figs. 99 and 103, that the crests and troughs of the transverse wave correspond to the points in the longitudinal wave where the density is unaltered.

We can have stationary longitudinal waves just in the same way as we have stationary transverse waves, the density being unaltered at the loops and there being no displacement at the nodes.

Properties of Wave Motion.—The waves produced on the surface of water are partly transverse and partly longitudinal, *i.e.* the water particles on the surface execute at the same time vertical and horizontal S.H.Ms. of the same period, as the wave passes. Water waves illustrate very well some of the properties of wave motion.

First of all, if a stone is thrown into a pool, the waves diverge from the point where it strikes the surface, and travel out in circles to the edge of the pool. The waves carry energy with them, as may be seen by the way they roll pebbles up and down at the edge of the pool. This energy wasted in friction at the edge was part of the energy of the falling stone. Now the circumference of a circle drawn on the surface of the pool with the source of the waves as centre would be $2\pi r$. The energy of the wave passes through this circle, and its total value is always the same, no matter what the value of r is; hence the quantity of energy passing through unit length of the circumference must vary inversely as r. In the same way, if a wave were to diverge from a point in three dimensions, and a sphere of radius r were drawn round this point, the area of the sphere would be $4\pi r^2$; consequently the energy passing through unit area of the sphere would be inversely proportional to r^2.

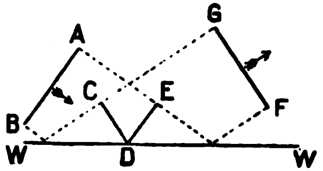


FIG. 104.

The crests of the waves formed by throwing a stone into a pool are circular. Let Fig. 104 represent a piece AB of a solitary plane wave, *i.e.* one with a straight crest, moving in the direction of the arrow towards a wall WW, and let the depth of the water be uniform up to the wall. The figure shows the wave as seen from above. The wave is reflected by the wall, and is shown in one of its positions after reflection by the line GF. The direction of propagation of the wave makes equal angles with the wall before and after reflection. CDE represents the wave in an intermediate position, when only part has undergone reflection. The reflection of water waves can be observed at a breakwater, but, owing to the waves coming on in trains, incident waves are usually superimposed on the reflected waves.

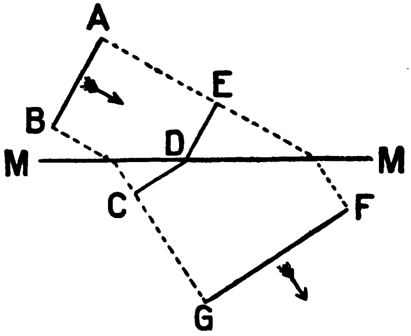


FIG. 105.

The velocity of a water wave varies with the depth of the water, being greatest when the latter is greatest. In Fig. 105 let AB be a plane wave travelling on the surface of a sheet of water of uniform depth. On the other side of MM let the depth suddenly become shallower, but let its value on this side be uniform also. The velocity of the wave becomes less when it crosses MM. Consequently its direction of propagation alters, as is shown by the figure, GF representing the wave after it has crossed MM and CDE giving its position at an intermediate stage. This bending of the wave on entering a medium, where the velocity is different, is known as refraction. It can be seen sometimes in the case of steamer waves when they pass from the dredged out channel of a river to the shallow water at the sides. It can be seen whenever we have waves breaking on a shelving beach, because, although the waves may be approaching the beach obliquely some distance out, when they reach shallower water they go more slowly, and consequently gradually wheel round, finally breaking with their crests parallel to the beach. In this case the refraction takes place gradually, not suddenly on crossing a line.

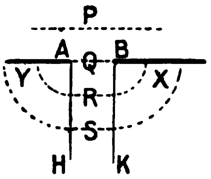


FIG. 106.

Fig. 106 represents a water wave passing perpendicularly through a hole, AB, in a wall. P is a plan of the wave before it reaches the wall, Q shows it actually at the opening, and R and S give two of its positions on the other side. It will be noticed, that after passing through the hole the wave is not restricted to the space between the lines AH and BK, but curves round behind the wall at X and Y. To this bending round of the wave behind an obstacle the name diffraction has been given.

24. THE NATURE AND PROPAGATION OF SOUND

The source of a sound is always in a state of vibration, and, as the vibration dies down, the sound diminishes in intensity. This can be shown by a variety of experiments. If a bicycle bell is touched with the fingers, the ringing sound stops, because the fingers stop the vibrations. If it is clasped firmly in the hand, the vibrations cannot be started, and the hammer of the bell simply makes a rattling noise.

If a noise is caused by dropping a weight on the floor, it and the part of the floor which it strikes are both set into violent vibration by the impact. If a stretched string or wire is plucked, so as to give a musical note, owing to the vibrations taking place too fast for the eye to follow them the string appears drawn out into a ribbon at the middle. Also small V-shaped pieces of paper placed on the string are thrown off by the vibrations. When a tuning fork is sounding, its prongs alternately approach and recede from one another. This is visible in the case of large vibrations; also the vibrations can be felt on lightly touching the prongs with the fingers.

Tuning forks are sometimes mounted on wooden boxes open at one end, called resonance boxes or resonators. This has the effect of making the note much louder. The vibrations of the fork are communicated to the box and to the air inside the latter; this happens very easily, because the air inside the box has the same period as the fork itself. But, though the box makes the sound louder, it causes it to die away faster. What is gained in intensity is lost in duration.

Velocity of Sound.—It is a fact of common observation, that sound takes an appreciable time to travel. Thunder is not heard until some seconds after the lightning flash, and the flash of a cannon at a distance is seen some time before the report is heard.

A famous determination of the velocity of sound in air was made by Arago and other French observers in 1822. Two stations were chosen 18·6 kilometres apart; at each station a cannon was placed, and observers were stationed with chronometers. The cannon at the two stations were fired alternately, and the time noted which elapsed between the flash and the report. Observations were made in both directions, for, if the wind assisted the velocity in the one direction, it would retard it in the other. The temperature was observed at a number of places between the two stations, the mean result being 15·9° C. The value of the velocity found was 340·9 metres per second for this temperature.

The velocity of sound in water was experimentally determined by Colladon and Sturm in 1827 on the Lake of Geneva. Two boats were moored at a distance of 13·5 kms. apart. The first carried a bell below the surface of the water; the second carried an ear trumpet, the mouth of which dipped below the surface of the water, and was closed by an indiarubber membrane. When a hammer struck the bell, some gunpowder was automatically ignited. An observer in the second boat measured the time between the flash of light and the arrival of the sound by the water. In this way it was found, that the velocity of sound in water was four times as great as in air.

The fact that sound travels in solid bodies, such as wood and iron, can be shown very prettily by an experiment with a tuning fork and its resonance box. A rod of wood about 14 feet long and $\frac{3}{4}$ inch thick with sharpened ends is taken, and the one end held lightly against the resonator. If the tuning fork is sounded and held against the other end, the sound travels along the rod to the box, and is audible to a large audience. But if the tuning fork is removed from the other end, the sound becomes very much fainter, and is audible only to the person holding the fork.

Medium of Transmission.—If an electric bell is suspended inside the receiver of an air-pump by the two wires carrying the current, and the air pumped out, the sound of the bell will become less and less as the air is removed, but becomes stronger at once if the air is suddenly readmitted. This shows that sound cannot be propagated in a vacuum.

The experiment would not succeed, if the bell were allowed simply to rest on the bedplate of the receiver, for the vibrations would then be transmitted directly to the plate, and thence to the external air, and the air inside the receiver would not be necessary as an intermediary.

Reflection.—If an observer stands at a distance of about 300 ft. from a cliff or large building, and shouts towards it, his words are echoed back to him. The sound is reflected back by the cliff or building. At this distance the sound would take about half a second to go to the cliff and back, and as the observer would utter only two syllables in this time, only his last two syllables are echoed back clearly to him. The rest of the echo is not heard distinctly, owing to its being superimposed on the sound itself.

The rolling of thunder is simply due to the reflection of the original sound by clouds at different distances from the observer. The sounding boards placed behind pulpits are for the purpose of reflecting towards the audience the sound that otherwise would travel upwards and backwards, and be lost.

Nature of Sound.—Since sound spreads out from a source in all directions it must be propagated either by waves or by the emission of very small particles travelling at a high velocity. The fact, that it cannot cross a vacuum, shows that it is not propagated by particles, for they could cross a vacuum, but by waves travelling in the air. Colladon and Sturm's experiment and the experiment with the wooden rod show that sound waves are also propagated in liquids and solids. The fact, that the source is always in a state of vibration, shows that the waves are started by the vibratory motion of some piece of matter, just as water waves might be started by dipping the end of a stick regularly below the surface.

Sound waves are longitudinal waves. The air particles execute S.H.Ms. in the direction of propagation as the wave passes, not at right angles to it. We know this, because Laplace has proved mathematically, that on this assumption the velocity of sound in air is given by $\sqrt{1\cdot41\,p/\rho}$, where p is the pressure and ρ the density of the air, and this formula agrees accurately with experiment. When air is compressed, it is heated, and when it is rarefied, it is cooled. Laplace assumed that the condensations and rarefactions in a sound wave took place so rapidly, that the temperature had not time to become uniform. Newton had previously obtained the wrong result $\sqrt{p/\rho}$ for the velocity of sound, on the assumption that during the passage of the wave the temperature of the air remained uniform.

Let us consider Laplace's formula in detail. Let V_0 = velocity of sound at 0° C, and V = velocity at t° C. Then, from the gas equation,

$$\frac{p}{\rho}=\frac{p_0}{\rho_0}\left(1+\frac{t}{273}\right)$$

and
$$V=\sqrt{1\cdot41p/\rho}=\sqrt{1\cdot41p_0/\rho_0}\sqrt{1+\frac{t}{273}}$$

$$=V_0\sqrt{1+\frac{t}{273}}=V_0\left(1+\frac{t}{546}\right)\text{ very nearly,}$$

since t is usually 15° C. or thereabouts. Thus the velocity increases with the temperature, and is uninfluenced by changes in the pressure and density, as long as the temperature remains constant. According to the best modern determinations $V_0 = 331\cdot3$ metres/sec. or 1087 ft./sec. Very violent sounds travel faster than this.

The variation of the velocity with temperature sometimes causes refraction of a sound wave. For, in Fig. 107 let EF represent the surface of the earth, and A a wave front proceeding from left to right. Let the air be hotter near the earth's surface. Consequently the lower edge of the wave travels faster, the direction of propagation is gradually altered, and the wave is refracted into the upper regions of the atmosphere.

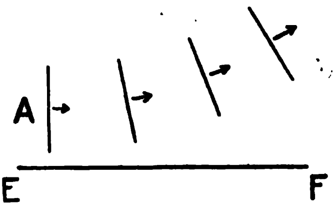


FIG. 107.

The velocity of sound is increased when it is travelling with the wind, and diminished when it is travelling against the wind. If the wave represented in Fig. 107 were opposed by a head wind, the velocity of which increased with the distance above the earth's surface, it would be refracted in exactly the same manner as is represented in that figure.

Speaking tubes increase the distance at which sound is audible by confining within a tube the energy which would otherwise be free to spread in all directions.

Velocity of Sound by Resonance.—Fig. 108 represents a simple method of determining the velocity of sound. A cylindrical glass tube AB is held in a vertical position with its lower end dipping into a vessel containing water, and a tuning fork of frequency n is caused to sound above the upper end of the tube. The tube is then lowered into the water, so as to gradually diminish the length AB of the enclosed air column. For one particular position the air column resounds to the note of the fork. This position is determined as accurately as possible, and the tube clamped in it. The length l of the air column is then measured.

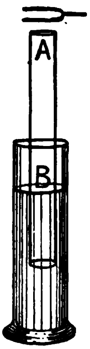


FIG. 108.

Let us suppose that at a certain instant the lower prong of the fork is moving downwards. This causes a wave of compression to move down the tube. The wave is reflected by the water at the foot and travels up again, the interval of time required for the passage down and up the tube being $2l/V$. If this interval is exactly equal to half a period of the fork, the reflected wave will superimpose on the wave of compression produced by the upward motion of the lower prong, reinforce it, and we shall have resonance. The condition for resonance is therefore

$$\frac{2l}{V} = \tfrac{1}{2}T \quad \text{or} \quad V = 4nl$$

whence, since n and l are known, V can be calculated. In practice the formula $V = 4n(l + {\cdot}4d)$ gives somewhat better results, d being the diameter of the tube; the term $\cdot 4d$ takes account of a disturbing effect produced by the end of the tube.

EXAMPLES 24

In order to simplify calculation take V = 340 metres/sec. or 1100 ft./sec., if the temperature is not specified.

1. Given that the velocity of sound is 331·3 metres/sec. at 0° C., find its value at 15° C. in metres/sec. and ft./sec.

2. A peal of thunder is heard 4 secs. after the lightning flash is seen. How far distant was the thunder?

3. A ship enters a fog and blows its siren. The blast is echoed back in $1\frac{1}{4}$ secs. Assuming that the reflection is from an iceberg right ahead, and that the ship's speed is 12 knots, how long would it take to reach the iceberg? (1 knot = 6080 ft./hr.)

4. What numerical value does Newton's formula give for the velocity of sound at 0° C.?

$$\text{Required value} = \sqrt{\frac{p}{\rho}} = \sqrt{\frac{76 \times 13{\cdot}6 \times 981}{{\cdot}001293}}$$

$$= 280 \text{ metres/sec.}$$

5. It is observed that waves pass a fixed point once in every four seconds, and that the distance from crest to crest is 23 ft. Find their velocity in miles per hour.

6. A bullet fired from a rifle with a velocity of 2000 ft./sec. is heard to strike a target 2·5 secs. later. Required the distance of the target from the marksman.

7. A stone is dropped into a well and heard to strike the water 2 secs. later. How deep is the well?

8. A bell is struck below the surface of the sea, and 1½ secs. later its sound is heard reflected from the bottom. How deep is the sea at that place?

9. In determining the velocity of sound by means of the arrangement described on p. 114, tuning forks of frequency 256, 320, 384, and 512 were used, and resonating lengths of 31·3, 23·3, 19·7, and 14·3 cms. were obtained. The diameter of the resonating tube was 6·7 cms. Assume the correct value for the velocity of sound at 0° C., namely, 331·3 metres/sec., and calculate the temperature at which the experiment was performed.

We have

n	l	$\lambda = 4(l + {\cdot}4d)$	V
256	31·3	136·0	34810
320	23·3	104·0	33280
384	19·7	89·6	34410
512	14·3	68·0	34820

Mean value of V = 34,330 cms./sec.

$$\text{We have therefore } 34330 = 33130\left(1 + \frac{t}{546}\right)$$

$$\text{i.e. } \frac{120}{3313} = \frac{t}{546} \quad \text{or} \quad t = 20^\circ \text{ C.}$$

10. In determining the velocity of sound by resonance tuning forks of frequency 256, 320, 384, and 512 were used, and resonating lengths of 31·05, 24·32, 20·0, and 14·20 cms. were obtained. The diameter of the tube was 6·5 cms., and the temperature at which the experiment was performed 17° C. Calculate the velocity of sound at 0° C. from the above data.

11 Show that the velocity of sound in air increases approximately 2 feet per sec. per degree Centigrade.

25. MUSICAL NOTES

The distinction between a noise and a musical note consists in the fact that a musical note is propagated by a regular wave with a definite frequency and wave-length, while a noise has no definite frequency, and is propagated by a wave of an irregular form.

A musical note has three characteristics, pitch, intensity, and timbre or quality. The pitch of a note is determined by the frequency of the sound wave. The intensity depends simply on the amplitude of the vibrations of the air particles, increasing when the amplitude increases. The timbre is that property of the sound which distinguishes a note when sounded on one instrument, *e.g.* the piano, from a note of the same pitch and intensity sounded on another, *e.g.* the flute.

Siren.—The fact that the pitch of a note is determined by its frequency can be shown by an instrument called the siren, which was invented by Cagniard de Latour. Fig. 109 shows its external appearance, and Fig. 111 a vertical section of it. It is made of brass. A is a disc with 18 holes in it, which is free to rotate about a vertical spindle. Fig. 110 gives a plan and section of this disc, one of the holes being shown at *n*. Below this movable disc there is a fixed disc B, also with 18 holes in it; these are placed so as to coincide simultaneously with the 18 holes on the upper disc. One of the holes in B is shown at *m*. The holes in A and in B are inclined obliquely to one another as shown at *n* and *m*. Consequently when wind from a bellows is forced up the tube O and escapes through *m* and *n* and the other holes to the atmosphere, it strikes obliquely against the sides of the holes in A, and imparts a rotary motion to A. As the pressure of the air in the bellows increases, the speed of rotation of the disc A increases. At first, when motion starts, the siren gives only a hissing sound; this changes into a note of very low pitch, which goes higher and higher as the speed of rotation increases. By regulating the pressure of the air it is possible to keep the speed of rotation and consequently the pitch of the note constant.

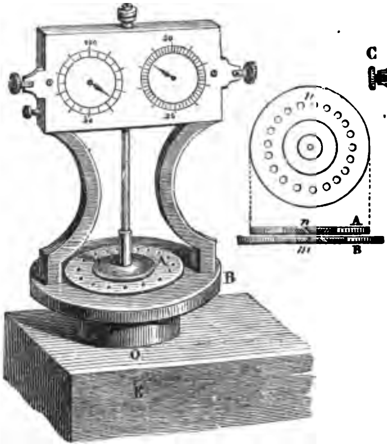


FIG. 109.

FIG. 110.

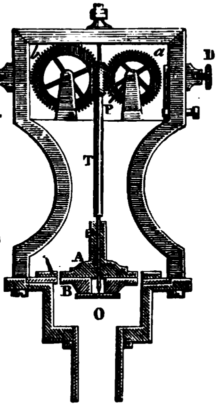


FIG. 111.

From Ganot's Physics.

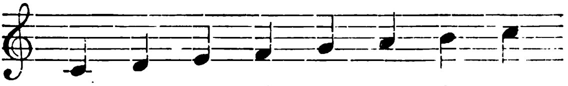
The spindle T is connected to a speed counter with two dials, one of which reads single revolutions and the other hundreds of revolutions. The speed counter is started and stopped by the screws C and D.

Let us suppose that the pitch of the note emitted by the siren is kept constant, that the speed counter is started, and that N revolutions are recorded in an interval of time *t*. In each revolution the holes come opposite one another 18 times. When the holes come opposite one another 18 simultaneous blasts of air are emitted, and a wave is started. Thus in the time t there are 18N waves started, and the frequency of the note emitted is consequently $18N/t$.

If the frequency of a note lies below 30 vibrations per second or above 20,000 or 40,000 vibrations per second, then the ear cannot hear it. It is, however, very difficult to assign these limits with accuracy, since they change with age and the individual.

Interval between Two Notes.—Musical Scale.—When two notes have the same pitch, they are said to be in unison. When they have not the same pitch, the ratio of their frequencies is called the interval between them. Thus, if the frequencies of two notes are 256 and 192, their interval is $\frac{256}{192} = \frac{4}{3}$, and if the frequencies are 512 and 384, the interval is $\frac{512}{384} = \frac{4}{3}$. The ear is able to estimate the interval between two notes. Certain intervals have names; thus 2 is the octave, $\frac{3}{2}$ the fifth, $\frac{4}{3}$ the fourth, $\frac{5}{4}$ the major third, $\frac{6}{5}$ the minor third.

Now it is found, that when the interval between two notes is represented by a simple ratio such as 2 to 1, 3 to 2, etc., the notes produce a pleasant sensation when sounded, either one after the other, or together. Consequently the earliest musical instruments were tuned, so that their intervals were simple ratios. It appears that the most natural interval to start with is a descending fourth; and the notes of the ancient Greek lyre were arranged in tetrachords, each tetrachord consisting of four notes, the interval between the highest and lowest note of each tetrachord being a fourth. This system of tetrachords gradually developed into the modern scale.



c	*d*	*e*	*f*	*g*	*a*	*b*	*c'*
do	*re*	*mi*	*fa*	*so*	*la*	*si*	*do*
24	27	30	32	36	40	45	48

The notes of the major diatonic scale are shown above, first in musical notation in the treble clef for the octave starting on middle *c*, then their names are given, and finally their names on the sol-fa notation. The row of numbers beneath should be memorized, as it gives the easiest way of finding the interval between two notes. Thus the interval between *g* and *c* is $\frac{36}{24} = \frac{3}{2}$, a fifth, and between *c'* and *g* is $\frac{48}{36} = \frac{4}{3}$, a fourth. The intervals between consecutive notes of the scale are $\frac{9}{8}, \frac{10}{9}, \frac{16}{15}, \frac{9}{8}, \frac{10}{9}, \frac{9}{8}, \frac{16}{15}$; of these $\frac{9}{8}$ is called a major tone, $\frac{10}{9}$ a minor tone and $\frac{16}{15}$ a diatonic semitone. The note from which the scale starts is called the tonic.

Standard middle *c* forks used in physical laboratories all have the frequency 256. The octaves above the one represented on this page are denoted in order by the indices ^{i}, ^{ii}, ^{iii}, ^{iv}, and the octaves below by capital letters and capital letters with suffixes $_{i}$, $_{ii}$. Thus we have the series C_{ii}, C_{i}, C, c, c^{i}, c^{ii}, c^{iii}, c^{iv}, for the notes with the frequencies 32, 64, 128, 256, 512, 1024, 2048, 4096.

Equally Tempered Scale.—In the piano and other instruments with fixed notes, if the frequencies of the notes were proportional to the numbers given on p. 117, it would be possible to play music only in one key. For example, if *g* were taken as *do*, the interval between it and the next note above it is $\frac{40}{36} = \frac{10}{9}$, whereas the interval between *do* and *re* should be $\frac{27}{24} = \frac{9}{8}$. Hence such instruments are tuned, not according to the natural scale, but according to the equally tempered scale, which represents a compromise between all possible keynotes. On this scale the frequencies of the notes are in the following ratio :—

c	*d*	*e*	*f*	*g*	*a*	*b*	*c′*
1	$2^{\frac{2}{12}}$	$2^{\frac{4}{12}}$	$2^{\frac{5}{15}}$	$2^{\frac{7}{12}}$	$2^{\frac{9}{12}}$	$2^{\frac{11}{12}}$	2

Thus on the equally tempered scale a major tone is $2^{\frac{1}{6}}$ and a semitone $2^{\frac{1}{12}}$. If the frequency of *c* is 256 the frequencies of the notes on the two scales are as follows :—

	c	*d*	*e*	*f*	*g*	*a*	*b*	*c′*
Natural Scale . .	256	288	320	341⅓	384	426⅔	480	512
Tempered Scale .	256	287·4	322·5	341·7	383·6	430·5	483·3	512

The Doppler Effect.—If an observer is standing by the side of a railway line, and an engine passes at a high speed sounding its whistle, the pitch of the whistle appears to fall suddenly. This apparent change of pitch is referred to as the Doppler effect.

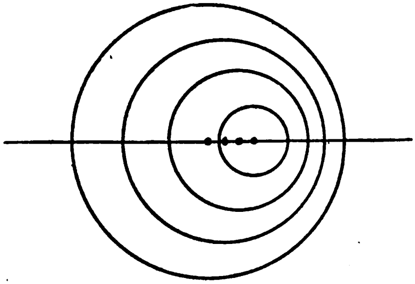


FIG. 112.

It can be explained very simply from the accompanying diagram (Fig. 112). Let V be the velocity of the sound, v the velocity of the engine, n the frequency of the whistle, and let the circles give positions of successive sound waves all drawn for the same time. The circles are not concentric; their centres are displaced in the direction of motion of the engine. In one second n waves are emitted, but these are crowded together into a distance $V - v$ in front of the engine instead of being spread over a distance V. Consequently the wave-length is diminished, and to any one in front of the engine the pitch appears to be

$$\frac{nV}{V - v}$$

Similarly in the space behind the engine we have n waves spread over a distance $V + v$ instead of over a distance V. The wave-length is consequently increased, and the pitch appears to be

$$\frac{nV}{V + v}$$

Hence the change of pitch, as the whistle passes, corresponds to an interval

$$\frac{V + v}{V - v}$$

If the whistle is stationary, and the observer approaches it with velocity v, in one second he meets the waves which occupy a distance $V + v$. Consequently the apparent pitch is

$$\frac{n(V + v)}{V}$$

After he passes the whistle, he is overtaken in one second by the waves which occupy a distance $V - v$. Consequently the apparent pitch is

$$\frac{n(V - v)}{V}$$

EXAMPLES 25

1. A siren is tuned to unison with a fork of frequency 256, and kept in unison for 7·2 secs. If the disc has 18 holes, what number of revolutions should the speed counter record for this time?

2. A disc which is provided with 36 holes arranged at equal distances from one another on a circle concentric with its edge, is rotated rapidly about an axis which passes perpendicularly through its centre. A jet of air from a bellows is directed against the holes, and the note produced is found to be in unison with a fork of frequency 320. How many revolutions does the disc make per second?

3. There are seven octaves on an ordinary piano, the lowest note being A_{III} and the highest a^{iii}. Calculate their frequencies, given that the frequency of c is 261.

4. What are the wave-lengths of these two notes? Take the velocity of sound as 340 metres/sec.

5. If the frequency of c is 274 (concert pitch), what are the frequencies of f, $f\sharp$, b, and $e\flat$ on the equally tempered scale? $f\sharp$ reads f sharp and is a semitone higher than f, $e\flat$ reads e flat and is a semitone lower than e.

$$\begin{aligned} f &= 274 \times 2^{\frac{5}{12}} = 365{\cdot}8 \\ f\sharp &= 274 \times 2^{\frac{6}{12}} = 387{\cdot}6 \\ b &= 274 \times 2^{\frac{11}{12}} = 517{\cdot}2 \\ e\flat &= 274 \times 2^{\frac{3}{12}} = 325{\cdot}8 \end{aligned}$$

6. Given that the frequency of c is 261 (international pitch), calculate the frequencies of g, b, and c', both on the natural and equally tempered scales.

7. When three notes, the frequencies of which are in the ratio of 4, 5, and 6 are sounded together, they form the major chord. Show that the diatonic scale contains major chords starting on c, f, and g.

8. A bristle is attached to one of the prongs of a tuning fork, the fork is sounded, and a smoked-glass plate is held vertically so that the bristle rests against it, and then allowed to fall under the action of gravity, so that the bristle traces a wavy line up it. Show that if N is the number of waves marked in a distance d, the frequency of the fork is $N/\sqrt{2d/g}$.

9. An engine is standing in a siding blowing a whistle, which has a frequency of 600. A train passes at 60 miles per hour. What does the frequency of the whistle appear to be to the passengers in the train, before and after passing?

10. The frequency of the whistle of an engine is 500. It passes an observer at the side of the railway at 60 miles per hour. What is the apparent frequency of the whistle before and after passing?

11. An observer is standing by the side of a railway, when a train passes with the engine sounding its whistle. As the engine passes, the pitch of the whistle appears to drop a minor third ($\frac{6}{5}$). What is the speed of the train in miles per hour?

12. Two express trains pass, and each is going at 60 miles per hour. The engine of one blows its whistle, which has a frequency of 700. What is the apparent frequency to a passenger in the other train, (i) before, (ii) after passing?

60 mls./hr. = 88 ft./sec. The train of waves, which should extend for a distance of 1100 ft. in front of the engine, is only $1100 - 88 = 1012$ ft. long; its velocity relative to the second train is 1188 ft./sec. Hence the apparent pitch before passing is

$$\frac{1188}{1012} \times 700 = 822$$

Similarly after passing it is

$$\frac{1012}{1188} \times 700 = 596$$

26. STRETCHED STRING, ORGAN PIPE, PHONOGRAPH

Stretched String.—If a violin string or piano wire stretched between two points is plucked or struck, a wave travels along the string both ways, is reflected at the ends, and a stationary wave is produced. Now it was shown on p. 108, that in a stationary wave there were points of no displacement called nodes and points of maximum displacement called loops midway between the nodes. Since the ends of the string are fixed they must obviously be nodes; consequently the string may vibrate as indicated in the adjoining figures. The full curve and dotted curve represent the extreme positions between which the string oscillates.

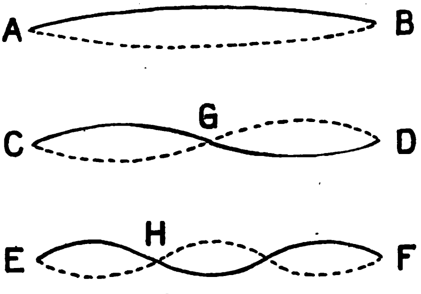


FIG. 113.

Let l be the length of the string. Then, since the curvature is small, the length measured along the curve may be taken the same as the length measured along the chord. Obviously in the case of AB the wave-length $\lambda = 2l$, in the case of CD it is $\frac{2}{2}l$, in the case of EF it is $\frac{2}{3}l$, and it is clear, that by inserting 3, 4, 5, etc., nodes between the ends of the string we obtain wave-lengths $\frac{2}{4}l$, $\frac{2}{5}l$, $\frac{2}{6}l$, etc. Now it was shown that $V = n\lambda$, where V is the velocity of the wave and n is its frequency. Consequently the frequencies of the tones emitted by the string are given by

$$n = \frac{V}{2l}, \frac{2V}{2l}, \frac{3V}{2l}, \text{etc.}$$

Monochord.—Fig. 114 represents a piece of apparatus known as the monochord or sonometer, which is used for investigating the frequencies of the notes emitted by a stretched string or wire. The wire, usually a piano wire, is fastened at D, passed over the movable bridge B and the fixed bridge A, then over a pulley to a pan P for carrying weights. AB is the part of the wire set into vibration; its length can be measured on a metre scale.

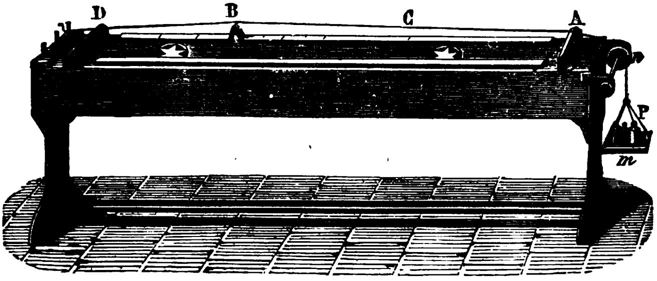


Fig. 114.

From Ganot's Physics.

It can be shown mathematically that the velocity of a wave in a wire such as this is given by $V = \sqrt{T/m}$, where T is the stretching force in the wire in dynes, and m is the mass of one centimetre of the wire in grams. If this value of V is substituted in the formulæ above, the frequencies of the tones, which may be emitted by the wire, can be calculated.

If a tuning fork of known frequency is sounded, and the weight in the pan and position of the movable bridge adjusted, so that the wire is in unison with the fork, then it is found that the frequency of the fork is given by $V/(2l)$. This note is said to be the fundamental of the wire. If, however, the position of the bridge and stretching force are kept the same, but the wire is plucked with one hand at a quarter of its length from the end, and then touched for an instant lightly and almost simultaneously at its midpoint by a finger of the other hand, a note an octave higher than the fundamental is distinctly heard. This is called its first overtone; the frequency is now given by the second formula at the top of the page. Touching the wire at its midpoint has produced the node at G in Fig. 113. In the same way, if the wire is plucked at a point $\frac{1}{6}$ of its length from one end and touched at a point H (Fig. 113) $\frac{1}{3}$ of its length from the end, it emits a note of the frequency given by the third formula above, *i.e.* an octave plus a fifth above the original note. This is called the second overtone. The overtones of a note are sometimes referred to as its upper partials.

The emission of overtones is not peculiar to strings, but is shared by organ pipes, bells, etc. But in the case of bells the frequencies of the overtones are not exact multiples of the frequency of the fundamental. When the frequencies of the overtones are exact multiples of the frequency of the fundamental, they are referred to as harmonics of the fundamental.

If the stretching force of the string is kept constant, and the position of the bridge altered to give a note a fourth, fifth, octave, etc., above the original note, the length of the string has to be diminished in the ratio $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, etc. This is probably the oldest discovery in mathematical physics, having been made by Pythagoras (572–492 B.C.), who also discovered Prop. I. 47, in Euclid; for centuries it exercised a great influence on the development of mathematics and scientific thought.

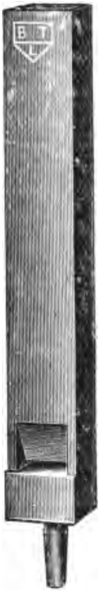


FIG. 115.

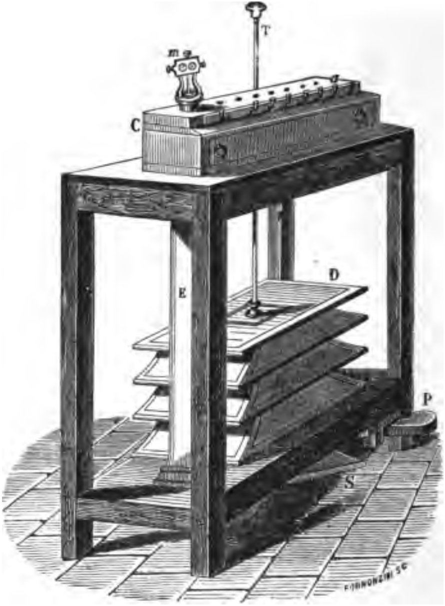


FIG. 116.

From Ganot's Physics.

Timbre.—When the monochord is tuned to unison with a tuning fork and plucked, although the pitch is the same, the quality or timbre of the two notes is not the same. This is because the note of the tuning fork is pure,[1] giving only the fundamental, while in general the overtones of the monochord are also present. If the wire on the monochord is plucked close to one end instead of near the middle, there is a distinct change in the quality of the sound. This is because the higher overtones are stronger. The characteristic of a tone which we call its timbre depends simply on the number and strength of the overtones present. If the overtones are few and weak, the note is soft and mellow; if they are strong up to the fifth, the note is richer; the presence of overtones above the sixth produces a sharp metallic quality. A trained ear can detect the presence of harmonics in a note.

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[1] Especially if fixed on a resonance box.

Organ Pipes—Fig. 115 represents an organ pipe. It can be sounded either by blowing it with the mouth or by a bellows. Fig. 116 shows a bellows of a type often used in experiments in sound. P is a pedal worked by the foot, S the bellows themselves, D a leather reservoir into which the air is pumped; the air escapes from the reservoir through the pipe E to the wind-chest C. In the top of this wind-chest there are a number of holes closed by keys, into which a siren or organ pipes can be fitted; when the key is pressed the siren or organ pipe is sounded. Increased pressure can be obtained by forcing down T.

Fig. 117 represents a section of the pipe shown in Fig. 115. The wind is supplied through *i*, and escapes through a narrow slot, opposite which there is a lip *b*. The air jet impinging against the lip sets the air column inside the pipe vibrating. The pipe may be fitted into the wind-chest by means of the foot P. The vibrations consist of stationary longitudinal waves, the air particles executing S.H.Ms. in a vertical direction as the wave passes. Now at the top the pipe is open, and at the foot it communicates with the atmosphere through the hole *ob*. The density of the atmosphere is nearly constant. Thus the density inside the tube at both ends changes very little; *i.e.* according to p. 109 the ends must be loops. Consequently the vibrations take place as shown in Fig. 118, where L stands for loop and N for node and the arrows give the direction of the motion at the loops; (*a*), (*b*) and (*c*) represent respectively the fundamental and first and second overtones. Thus, if l is the length of the air column, and V the velocity of sound, the wave-lengths of the fundamental and overtones are respectively $2l$, $\frac{2}{2}l$, $\frac{2}{3}l$, etc., and their frequencies $\frac{1}{2}V/l$, V/l, $\frac{3}{2}V/l$, etc. In general the note emitted by the pipe is a complex one with the fundamental predominating. But if the strength of the air blast is increased, the first overtone predominates.

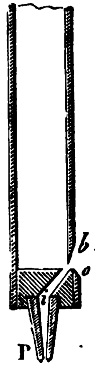

FIG. 117.

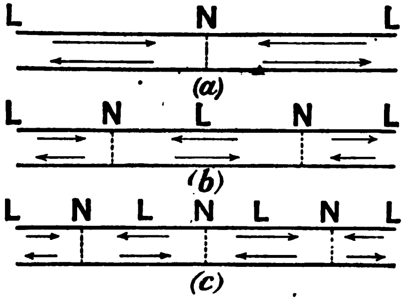

FIG. 118.

If the upper end of the organ pipe is closed, it becomes a node, for the particles cannot vibrate into the end. The wave-lengths of the fundamental and overtones are then $4l$, $\frac{4}{3}l$, $\frac{4}{5}l$, etc.

In the arrangement for determining the velocity of sound shown in Fig. 108 in which a closed pipe is used, when $V = 4nl$ the fundamental of the air column is in unison with the fork. There is also resonance, if the glass tube is long enough, when the first overtone of the air column is in unison with the fork, *i.e.* when $n = \frac{3}{4}V/l$ or $V = \frac{4}{3}nl$.

The resonance box of a middle C fork has an internal length of 31 cms. The wave-length of the note given by this fork is 34090/256 cms. at 15·9° C., according to Arago's result as given on p. 112, *i.e.* 133·1 cms. A quarter of this is 33·3, so, allowing for the disturbing effect of the end, the fundamental of the air column enclosed by the box agrees with the fundamental note of the fork.

The pipe open at both ends is approximately realized in the case of the flute. In this instrument a cylindrical air column is set into vibration by the performer blowing against the side of the mouth-hole. The air column extends from the mouth-hole to the hole that is open highest up the flute. This hole and the mouth hole both act as loops. Opening a hole is equivalent to cutting off the tube at that hole. For the lowest and middle octave the fingering is the same, but the shape of the lips and direction of the air blasts are different, the fundamental being obtained in the one case and the first overtone in the other. The principle of the penny whistle is the same as that of the flute.

Instead of the lip *b* shown in Fig. 117, organ pipes may be furnished with a thin plate of metal called a *reed*. The reed by its vibrations alternately opens and closes an aperture through which the wind passes, and the impulses derived from this intermittent supply of air set the column of air in the pipe into vibration. The pitch of the pipe is slightly modified by the action of the reed. The child's trumpet is an example of a reed pipe.

Beats.—Let us suppose we have two organ pipes of the type shown in Fig. 115, which both give a note of frequency n, and that they are placed on a wind-chest, such as is shown in Fig. 116, and sounded together. Now let the upper end of one of the pipes be partially closed, for example, by sliding a brass plate over it. Its pitch is then slightly lowered, and when the two pipes are sounded together there is no longer unison, but throbbing pulsating bursts of sound with intervals of comparative silence between them. The bursts of sound are called beats, and the two notes are said to give beats. The number of beats per second can easily be counted with a stop watch. When the two pipes are almost in unison, the beats are slow, and as the frequency of the one pipe is altered, the beats become more rapid; as the difference of frequency is further increased, the beats become too quick to count, and finally form a discord.

The phenomenon is easily explained. Let m be the frequency of the variable note. Then the fixed note emits a train of n waves per second, and the variable note a train of m waves per second. These trains superimpose at the ear of the observer. Then starting from the instant, when a condensation in the one train coincides with a condensation in the other, the one train gradually falls behind the other, until there is a difference of half a period, the waves oppose one another, and the sound is a minimum; then the difference increases to a period, two condensations coincide again, and we have a maximum of sound. Since the one train gains $n - m$ periods on the other per second, the condensations obviously coincide $n - m$ times per second. Each concidence gives a beat. There are thus $n - m$ beats per second.

Phonograph.—The phonograph is an instrument devised by Edison in 1876 for the purpose of recording and reproducing sounds. Fig. 119 represents a modern form of it. A wax cylinder is rotated by clockwork, and the words to be reproduced are spoken into the horn; the sound waves which constitute these words cause a thin glass disc at the end of the horn to vibrate, and the vibrations of the disc cause a chisel-shaped sapphire point to move up and down, cutting a groove of varying depth in the wax. To reproduce the sound a slightly different arrangement is fitted, consisting of a smooth sapphire point which passes along the indentation produced by the recording point. It sets a thin glass disc at the end of the horn into vibration, and the vibration of the glass disc sets up the sound waves in the air.

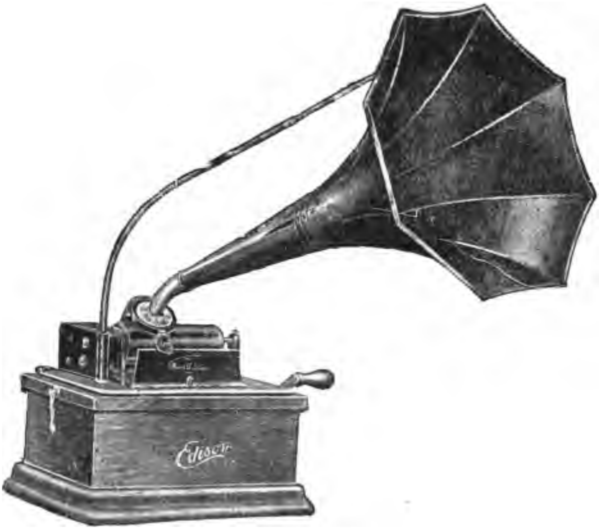

FIG. 119.

The records made by an amateur in wax do not last very well. The cylindrical records sold in the shops are made from moulds, and are in a much harder material.

Gramophone.—The gramophone, which has displaced the phonograph in popular favour, differs from the latter in not being able to make its own records, in using disc-shaped records instead of cylindrical ones, and in employing grooves of varying width instead of varying depth to record the sound. Thus in the gramophone the needle moves from side to side, not up and down like the sapphire point in the phonograph.

Fig. 120 shows the construction of the gramophone reproducer. P, the needle point, is held by a screw S in the needle carrier N, which is attached at D to the centre of the glass diaphragm. The latter is held by the cover A against a rubber ring. The tube B is connected to the horn, which presses against the rubber ring C to make an airtight joint.

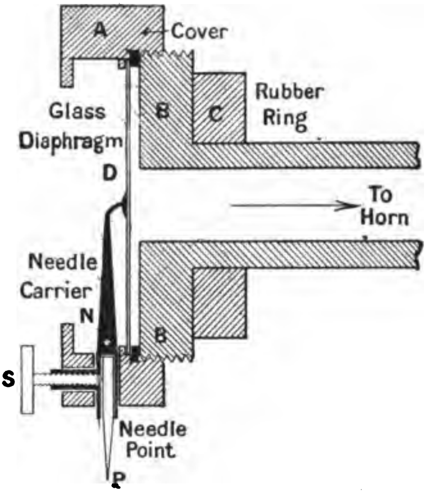

FIG. 120.

EXAMPLES 26

1. What is the frequency of the fundamental note of a wire 50 cms. in length, which is stretched by a force of 15 kilograms? A metre of the wire weighs 2·69 gms.

$$V = \sqrt{T/m} = \sqrt{15{,}000 \times 981/{\cdot}0269}$$
$$= 233{\cdot}9 \text{ metres/sec.}$$
$$\text{Frequency} = \frac{23390}{100} = 233{\cdot}9$$

2. A vibrating string gives the note *c* when stretched by a weight of 16 lbs. What weight must be used to give the note *g*? The length of the string is kept unaltered.

3. What is the stretching force in the *e* string of a violin, which vibrates 640 times per second, if the vibrating portion has a mass of $\frac{1}{10}$ gm. and a length of 33 cms.?

4. The internal length of the resonance box of a tuning fork of frequency 320 is 24·7 cms. What is the frequency of the fundamental note of the box at 0° C.?

5. What length should a glass tube open at both ends have, in order that it may produce resonance when a tuning fork of frequency 320 is sounded near one end?

6. What percentage variation will take place in the frequency of a pipe organ, if the temperature changes from 40° to 70° F.?

$$V = V_0\left(1 + \frac{t}{546}\right)$$

$$40^\circ \text{ F.} = \tfrac{5}{9} \times 8 = 4{\cdot}4^\circ \text{ C.} \quad 70^\circ \text{ F.} = \tfrac{5}{9} \times 38 = 21{\cdot}1^\circ \text{ C.}$$

The pitch varies as the velocity, hence increases in the ratio

$$\frac{546 + 21{\cdot}1}{546 + 4{\cdot}4} = \frac{5671}{5504}$$

and the increase is 3·0 per cent.

7. Two tuning forks make 4 beats per second. The frequency of one is 320. When a prong of the other one is weighted with a small piece of wax which reduces its frequency, the number of beats per second is increased. What was its original frequency?

8. An organ pipe gives a certain note at 0° C. How much must the temperature rise for the pitch to go up one semitone?

CHAPTER V

LIGHT

27. PROPAGATION OF LIGHT AND PHOTOMETRY

LIGHT travels in straight lines. This is taken for granted by every one, for we always assume that a body exists in the direction of the rays of light which enter our eye from it. Also numerous illustrations of the rectilinear propagation of light occur in daily life. For example, the rays of the sun entering a darkened room through a chink in a shutter are seen to be straight; also, if a shadow of a stick be cast by a candle flame, the flame, the top of the stick, and the shadow of the top of the stick, are all in one straight line.

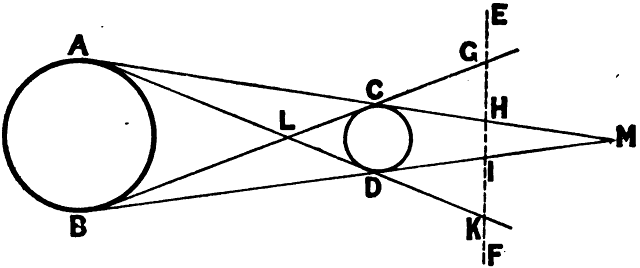

FIG. 121.

Let us suppose that AB is a spherical source of light, CD a spherical obstacle, and EF a screen. Each point of the source casts its own shadow, so that on the screen we have an infinite number of overlapping shadow discs, the point A casting, for example, the shadow HK and the point B the shadow GI. As A and B are the extreme points of the source, the overlapping portion, HI, receives no light at all. The parts GH and IK receive light from part of the sphere, and the shadow gradually becomes brighter, as we proceed from H to G and from I to K. The whole shadow consists therefore of a perfectly black disc of diameter HI, called the umbra, surrounded by a ring of gradually diminishing darkness, called the penumbra.

If the sphere AB represents the sun and the sphere CD the moon, it is only when a point on the earth's surface enters the cone CMD that the sun is totally eclipsed for that point. When it is within the penumbra, the eclipse is only partial.

Pinhole Camera.—If rays of light pass into a dark box through a small hole, they form an inverted image of external objects. A pencil of light goes out from every point in the object in the manner indicated in Fig. 122, and forms a patch of light on the wall of the box. If a photographic plate is placed there to receive the image, a picture can be taken. The hole must be small, literally a pinhole, otherwise the patches of light on the card will be too large and the image blurred. Consequently the image is very faint, and unless the object to be photographed is a very bright one, like a candle flame, a very long exposure is necessary. The pinhole camera has, however, the advantage that no focussing is necessary.

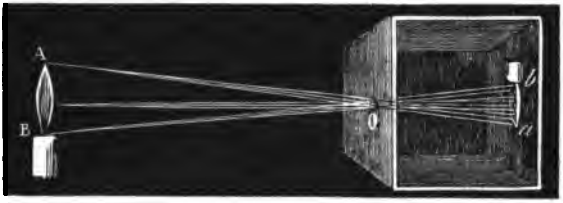

FIG. 122.
From Ganot's Physics.

Velocity of Light.—The velocity of light is so great as to appear instantaneous to all ordinary observation. It has, however, been measured by four independent methods, and consistent results been obtained. Here only one of these methods, namely, the method employed by Fizeau in 1849, will be described. The description assumes a slight knowledge of the properties of lenses, which are not dealt with in this book until p. 144, but it will be possible here to make out the principle of the method in spite of this. Fizeau's arrangement is shown diagrammatically in Fig. 123. A beam of light from a source S passes through a converging lens, is reflected by a glass plate P, and comes to a focus at F. It is then made parallel by the lens O, traverses a very great distance OL, falls on the lens L, and is brought to a focus on the surface of the concave mirror M. The radius of curvature of this mirror is equal to ML, its distance from the lens; the central ray of any pencil through the lens thus falls on the mirror

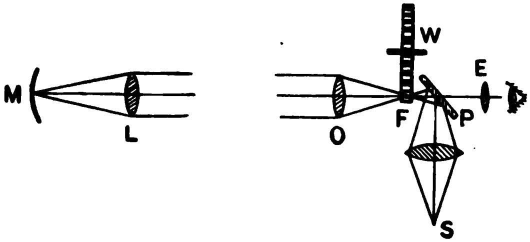

FIG. 123.

normally and is reflected back the way it comes, even though inclined to the axis of the mirror. The lens and mirror L and M thus direct the beam back through the lens O to form a real image once more at F, and the observer looks at this real image through the eyepiece E and the glass plate P.

W is a toothed wheel, and, as it rotates, its teeth pass one after another through the point F, alternately stopping and letting through the light. If the wheel is moving slowly, the eye sees a flickering image of S. If the images succeed each other faster than 15 or 20 a second, the flickering ceases owing to the persistence of vision, and the image becomes steady. It is, of course, not as bright as it would be, if the wheel were away; the teeth in passing stop some of the light.

If the speed of the wheel is still increased, so that the time taken by the light to go from F to M and back is exactly equal to the time required for a tooth to move into the position formerly occupied by an open space, the light is intercepted by a tooth on its return and the image vanishes. If the speed of the wheel is now doubled, the light passes through the next space and the image is again visible; if the speed of the wheel is trebled, the light is intercepted by the next tooth and again vanishes, and so on; as the speed increases, the image alternately appears and vanishes.

In Fizeau's experiments the wheel had 720 teeth, and the widths of the teeth and open spaces were equal. The distance between M and F was 8·6 kms. A determination was made of the angular velocity of the wheel for which the image disappeared. Let it be ω radians/sec. for the nth disappearance, and let V be the velocity of light in kms./sec. Then

$$\frac{2 \times 8{\cdot}6}{V} = \frac{(2n-1)}{2}\,\frac{2\pi}{720}\,\frac{1}{\omega}$$

i.e.

$$V = \omega\frac{720 \times 2 \times 8{\cdot}6}{(2n-1)\pi}$$

whence V can be calculated.

Determinations of the velocity of light by Fizeau's method have been made by other observers since his time, one determination being made by Young and Forbes in 1881 on the Firth of Clyde between Wemyss Bay and the hills behind Innellan, a distance of about $3\frac{1}{2}$ miles each way. The results obtained in the best determinations of the velocity of light differ very slightly from 3×10^{10} cms./sec., which, on account of its being a round number, is the value now universally adopted; as its equivalent on the ft.-lb.-sec. system we shall take 186,000 mls./sec. This is a very great velocity; light could travel round the world $7\frac{1}{2}$ times in a second.

Nature of Light.—Light is propagated by wave motion. Light waves come to us from the sun and stars. Now as we ascend in the atmosphere, the density of the air becomes less and less, finally vanishing altogether, and interstellar space is a more perfect vacuum than can be obtained by any artificial means. Hence it is inferred that light waves are propagated by the vibrations of some medium, which is different from ordinary matter. This medium is called the luminiferous ether, or simply the ether. Sound waves, as has been mentioned, are not propagated through a vacuum.

Light waves are transverse, not longitudinal like sound waves.

Inverse Square Law.—The unit of intensity of a light source in this country is the standard candle, a spermaceti candle $\frac{7}{8}$ inch in diameter, weighing six to the lb. and burning at the rate of 120 grains to the hour. A source of light, if unobstructed, radiates equally in all directions. Consequently, if a sphere is drawn with the source as centre, the quantity of light received per unit area is the same all over the surface of this sphere. The quantity of light radiated through the surface of the sphere is always the same, no matter what the area of the surface is. The quantity received per unit area must therefore vary inversely as the area of the surface, $4\pi r^2$, or, in other words, the illumination of a surface varies inversely as the square of its distance from the source. This relation, which is known as the inverse square law, enables us to compare the intensities of different sources; the instruments used for this purpose are known as photometers, and the part of the subject dealing with it as photometry.

The Rumford or Shadow Photometer.—This consists of a rod of the shape of a lead pencil, which is mounted vertically before a screen. Fig. 124 represents a plan of the arrangement; R is the rod, AC the screen, and L_1 and L_2 the two sources, the intensities of which are to be compared. BC is the shadow of the rod formed by L_1 and AB the shadow of the rod formed by L_2 on the screen. The distances of L_1 and L_2 from the screen are adjusted so as to make these shadows equally dark, and at the same time R is placed so that the shadows just touch at B. If they overlap, or if there is a bright space between them, the comparison of their intensities cannot be made so accurately.

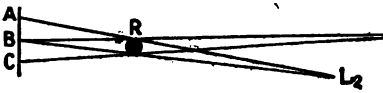

Fig. 124.

All parts of the screen outside the regions AB and BC are illuminated by both sources. The region AB being in the shadow of L_2 is illuminated solely by L_1, and the region BC being in the shadow of L_1 is illuminated solely by L_2. Consequently, if the shadows are equally intense, the illuminations produced by the two sources at B are equally great, and the intensities of the two sources are simply as the squares of their distances from B. Hence, if the intensity of the one source is known in terms of the standard candle, and the distances BL_1 and BL_2 are measured with a metre stick, the intensity of the other source can be determined.

The screen should have a rough unglazed surface. One great advantage of the shadow photometer is that the accuracy of its results is not affected much by the presence of another source of light in the room. For this third source produces a shadow of its own on the screen, which can easily be recognized, and it is only necessary to take care that the edge of this third shadow does not fall on B.

The Bunsen or Grease Spot Photometer.—This consists of a piece of white unglazed paper, in the middle of which a small portion has been rendered translucent by a drop of grease. It is placed between the two sources and in the same straight line with them. It possesses the advantage of simplicity, but it is difficult to make a grease spot with a very sharp boundary, and, of course, it is impossible to detect slight inequalities in the illumination, when the boundary is not sharp. As a substitute for the grease spot a disc of tissue paper may be used. The latter is made by placing two sheets of paper on the top of one another and with a sharp knife cutting a star-shaped opening in both of them at the same time. The tissue paper is placed between them, and they are then stuck together with the stars accurately superimposed.

The grease spot is translucent and the surrounding paper is opaque. Consequently, if the distance of the photometer is adjusted so that the grease spot appears of the same brightness as the surrounding paper, the intensities of the two sources should be as the squares of their distances from the photometer.

In practice, however, it is found that the grease spot seldom disappears for the one position, when viewed in succession from both sides of the screen. This is due to the light reflected by the white paper being scattered more than the light transmitted by the grease spot. Thus the more obliquely the surface is viewed, the darker appears the grease spot. Consequently, in working with the grease spot we set it so as to obtain the same relative intensity of the two areas when viewed from both sides, or, in other words, we adjust, not for disappearance, but for the same contrast effect.

EXAMPLES 27

1. A stick is placed in front of a screen, and an electric glow-lamp and standard candle throw equally dark shadows of the stick on the screen. The candle and lamp are respectively distant 39 and 154 cms. from the screen. Find the candle-power of the lamp.

2. How many candles are required to produce at a distance of 320 cms. the same illumination as a single candle at a distance of 50 cms.?

3. A 30 candle-power and a 16 candle-power electric glow-lamp are at the same height, 100 cms. apart. At what points on their line of centres do they give the same illumination?

4. The intensities of two glow-lamps are compared by the grease-spot photometer, and balance is obtained when the photometer is 87 cms. from the one lamp and 53 cms. from the other. The lamps are then interchanged in their sockets, and balance is obtained when the distances are 49 cms. and 81 cms. What is the ratio of the intensities of the lamps?

5. It is found that the light of the full moon falling perpendicularly on a screen gives the same illumination as a standard candle at a distance of 4 ft. Find the candle-power of the moon, given that its distance is 60 times the earth's radius.

6. Two lamps are balanced at distances of 51 and 45 cms. from a grease-spot photometer. A large plate of clear glass, which transmits 90 per cent. of the light incident on it, is then placed midway between the photometer and the stronger lamp. How much nearer must the latter be moved to restore balance?

Let I and L be the intensities of the two lamps. Let x be the new distance of the stronger lamp. Then

$$I/51^2 = L/45^2 \text{ and } \cdot 9 \times I/x^2 = L/45^2$$

Hence . $I/51^2 = \cdot 9 \times I/x^2, \quad x^2 = \cdot 9 \times 51^2$

$$x = 48\cdot 4 \text{ cms.}$$

and the stronger lamp must be moved 2·6 cms. nearer the photometer.

7. A candle is just visible to the unaided eye on a clear dark night at a distance of 3·1 kms. in the open. The diameter of the pupil is then ⅕ inch. At what distance should the candle be visible, when field glasses are used, the object glasses of which are 1¾ inches in diameter? Assume that ¼ of the light incident on the object glasses enters the eye, the remainder being lost inside the instrument.

8. In astronomy stellar distances are measured in "light years," *i.e.* the distance which light travels in a year, no star being as near the earth as three light years. How many miles are there in a light year?

9. The distance of the sun is 93,000,000 miles. Find the time taken by the light in coming from it.

10. If the light of the sun is admitted through a small hole, an image of the sun is formed on a screen placed to receive it, but if the aperture is a large one we obtain an image of the aperture. Explain this.

11. In determining the velocity of light by Fizeau's method the distance is 10 kms., the wheel has 720 teeth, and the angular velocities in radians per second for four successive disappearances of the image are 326, 457, 588, and 719. Find what disappearances these are, and calculate the velocity of light from the data supplied.

12. A very small hole is made in the shutter of a dark room, which is otherwise light-tight, and on a screen distant 200 cms. from the hole an image of a tree is produced, 30 cms. high. What is the height of the tree, given that its distance from the shutter is 50 metres?

13. The shadow which causes an eclipse is, geometrically speaking, a solid cone with its axis in the line joining the centre of the sun and the shadow-casting body. If the sun is 93,000,000 miles from the earth, and subtends an angle of 32′ at the earth, find how far the earth's shadow extends from the centre of the earth. Find also the diameter of the earth's shadow at the point where the moon crosses it. The distance of the moon is 60 times the earth's radius.

14. The angle subtended by the moon's diameter at the earth is 31′. The time taken by the moon to go round the earth is 29 days 12 hours. Hence calculate its hourly motion in minutes of arc, and find, if an eclipse of the moon is central, how long it is total, and also what time elapses between the first and last contacts of the shadow on the moon's disc. The penumbra is to be neglected altogether; it only makes the surface of the moon very gradually dimmer.

15. The mean values of the apparent angular diameters of the sun and moon are 32′ and 31′ respectively. The distance of the moon from the centre of the earth is 60 times the earth's radius or 240,000 miles. Hence show, that in general when the moon passes between the sun and earth, the umbra does not reach the earth, *i.e.* the eclipse is not total. Find how far the end of the umbra is from the earth, assuming that the moon passes across the centre of the sun.

The diameter of the moon is

$$2 \times 240{,}000 \times \tan 15\cdot 5' = 2163 \text{ mls.}$$

The distance from the end of the umbra to the moon is

$$\tfrac{1}{2} \times 2163 \cot 16' = 232{,}400 \text{ mls.}$$

Hence the umbra is 7600 miles from the earth.

16. Calculate the diameter of the moon's penumbra at the point where the earth crosses it. Hence show that no eclipse of the sun, partial or

otherwise, can be seen over a whole hemisphere of the earth. (The sun is so far away, that the directions of the rays from any point on its surface to both edges of the moon may be regarded as parallel.)

28. REFLECTION AND REFRACTION

The fact that a body not itself luminous is yet visible in all directions, when illuminated by light from a light source, shows that it must be capable of reflecting light in all directions. Such reflection is called diffuse reflection, and is subject to no simple law. It takes place at all rough surfaces. Rays that undergo diffuse reflection usually change their colour.

When a narrow pencil of light is reflected from the polished surface of a transparent medium such as glass, diffuse reflection takes place to only a very small extent, and the incident pencil of light gives rise to two pencils, a reflected one and a refracted one. The light is then said to undergo regular reflection. The point where the incident ray of light strikes the surface is called the point of incidence, and if a normal be drawn to the surface at this point, the angle which it makes with the incident ray is called the angle of incidence, and the angle which it makes with the reflected ray is called the angle of reflection. If the normal be produced downwards into the transparent medium, the angle which it makes with the refracted ray is called the angle of refraction.

The laws of reflection and refraction are as follows :—

The incident ray, the normal to the reflecting surface at the point of incidence, and the reflected ray, are all in the same plane.

The angle of reflection is equal to the angle of incidence.

The refracted ray lies in the same plane as the normal and the incident ray, and is on the opposite side of the normal from the incident ray.

The sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction for all angles of incidence, the value of the ratio depending on the nature of the media in contact at the surface at which refraction takes place.

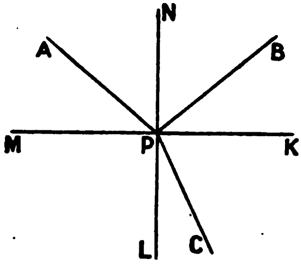

FIG. 125.

Thus, in Fig. 125, if P is the point of incidence, MK the trace of the reflecting surface, NP the normal, and AP the incident ray, the reflected ray PB and the refracted ray PC are both in the same plane as AP and PN, $\angle$ NPB = $\angle$ NPA, and sin NPA/sin LPC is constant, no matter what the value of $\angle$ NPA is.

Write
$$\frac{\sin NPA}{\sin LPC} = \mu$$

Then μ is said to be the index of refraction of the lower medium with reference to the upper, or simply the index of refraction of the lower medium. For all transparent solids and liquids μ is greater than 1, and unless the angles are measured with very great accuracy indeed, it is immaterial whether the upper medium is vacuum or air. For air $\mu = 1{\cdot}0003$. It may be stated here, simply as a fact, that the velocity of light in a medium is inversely proportional to the index of refraction of that medium.

When the angle of incidence is zero, AP and PB both coincide with NP, and PC coincides with PL. The ray is then undeviated by refraction.

If the surface on which the light falls is curved, we may divide it up into small elements of area, regard these elements as plane, draw a normal to each, and consider each separately. The light falling on each will then be reflected and refracted according to the above laws.

The laws of reflection and refraction can best be proved with the spectrometer (p. 154), but a very simple means of verifying them in the case of a glass slab will be described here.

Fix a piece of paper on a drawing board, and draw any straight line AP on it. Place the slab in position so that one of its faces rests along MK, and draw the line MK by running a pencil along its edge. Look into the surface of the slab, and the line AP will be seen by reflection in the direction A′P; then place a ruler on the paper, so that its edge seems a continuation of A′P, and draw part of the line PB. Remove the slab, and produce BP to cut MK. It should cut it at P, and it can be shown with a pair of compasses in the manner indicated in the diagram, that ∠ APN = ∠ NPB.

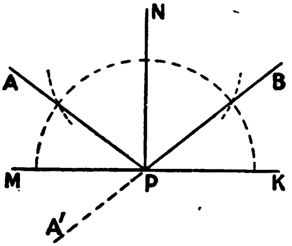

FIG. 126.

Draw a straight line AP as before, and place the slab in position so that MK and HJ are the traces of two of its parallel faces. Draw MK and HJ. Then, looking into the face HJ of the slab, draw with a ruler QR, which appears to be the continuation of the line AP. Remove the slab, draw the normal NPS, produce RQ to meet the normal at S, and join PQ. Then ∠SPQ is the angle of refraction.

The ratio of the sines can now be determined in different ways, but perhaps the neatest is as follows: It is found from the drawing that SR is parallel to AP. Consequently ∠PSQ is the supplement of ∠APN and sin APN = sin PSQ. Hence, since the sines of the angles of a triangle are proportional to the lengths of the opposite sides,

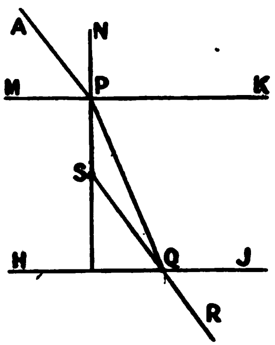

FIG. 127.

$$\mu = \frac{\sin \text{APN}}{\sin \text{SPQ}} = \frac{\sin \text{PSQ}}{\sin \text{SPQ}} = \frac{\text{PQ}}{\text{SQ}}$$

No matter how ∠APN varies, the ratio PQ/SQ is always constant.

Formation of an Image.—Let MN be the section of a plane mirror perpendicular to the plane of the paper, and let P be a point source of light. Let PA be any ray from P to the mirror; then, after reflection, it has the direction AC. Draw EA, the normal at A, draw PN perpendicular to MN, and produce CA and PN to meet at Q.

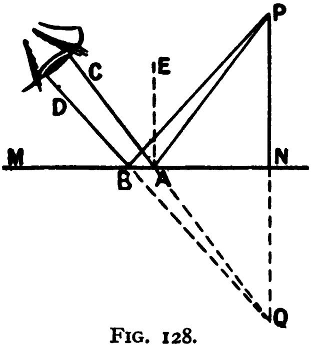

FIG. 128.

In △s APN and AQN we have AN common, ∠ANP = ∠ANQ both being right, and ∠PAN = ∠QAN, since ∠ EAN is right, and EA bisects ∠CAP. Hence the triangles are equal and PN = NQ.

The point Q is fixed and quite independent of the direction of PA. If we draw another ray PB, DB its direction after reflection must also pass through Q. All rays diverging from P and striking the mirror appear therefore to an eye at CD to diverge from Q. The point Q is thus said to be the image of P, and since the rays AC and BD themselves do not pass through Q, but only their directions produced backwards do, the image is said to be virtual. If the rays themselves had passed through Q, the image would have been real. The essential difference between a real image and a virtual image is, that a real image can be received upon a screen and made visible; a virtual image cannot.

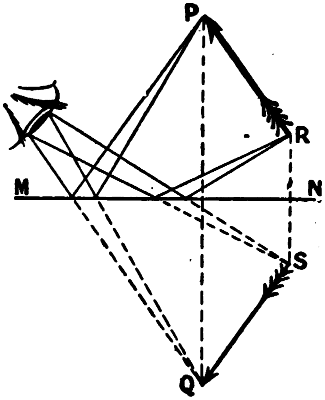

FIG. 129.

If instead of a luminous point we have a line object PR, an image is formed of every point of it as above, and these point images combine to form the line image QS as shown in Fig. 129. It will be noticed that the object is reversed by reflection, that right and left are interchanged. Consequently, if a piece of writing be held up to a mirror, its image is illegible; if, however, it be blotted, and the blotting-paper held up to the mirror, the writing, being reversed twice, becomes legible again.

The fact, that the image is as far behind the mirror as the object is in front of it, can be shown very easily with two pins, a short one and a long one, and a glass plate, the back of which has been blackened so that only the front surface can reflect light. The glass is mounted vertically and the short pin stuck up in front of it. The long pin is placed behind the glass in such a position, that it and the image of the short pin appear to occupy the same position. Then there is no parallax between the long pin and image of the short one, *i.e.* moving the eye from side to side does not cause the one to move past the other.

If the adjustment is made accurately, the long pin and the short pin are equidistant from the surface of the glass.

Rotation of a Plane Mirror.—Let MN be the intersection by the paper of a plane mirror perpendicular to the plane of the paper. Let PO be the incident ray, OQ the reflected ray, and OH the normal at the point of incidence. Let the mirror be rotated through an angle α about an axis through O perpendicular to the plane of the paper, and let OR and OG be the positions of the reflected ray and normal after the rotation. Then

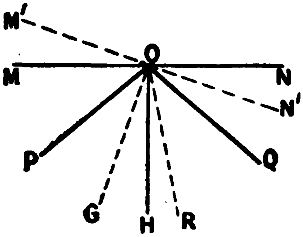

FIG. 130.

$$\alpha = \angle M'OM = \angle GOH = \angle POH - \angle POG = \tfrac{1}{2}\angle POQ - \tfrac{1}{2}\angle POR$$
$$= \tfrac{1}{2}(\angle POQ - \angle POR) = \tfrac{1}{2}\angle QOR$$

But QOR is the angle turned through by the reflected ray. Therefore, the angle turned through by the reflected ray is twice the angle turned through by the mirror.

Refraction through a Slab.—Let a ray of light, ABDE, be refracted through a plate of a transparent material with parallel sides such as is shown in Fig. 131. Let μ_{12} be the index of refraction of the plate with reference to the medium in which it is placed, and let μ_{21} be the index of refraction of the medium with reference to the plate. It is found by experiment that the direction of a ray is never altered by passing through a medium with parallel sides; hence the emergent ray DE makes the same angle α with the normal as the incident ray does. If β denotes the angle of refraction

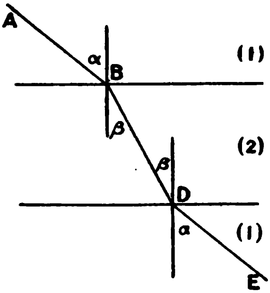

FIG. 131.

$$\mu_{12} = \frac{\sin \alpha}{\sin \beta}, \quad \mu_{21} = \frac{\sin \beta}{\sin \alpha}.$$

Therefore $\mu_{21} = 1/\mu_{12}$. For crown glass $\mu_{12} = 1{\cdot}52$; hence $\mu_{21} = \dfrac{1}{1{\cdot}52}$

Image of a Point formed by Refraction at a Plane Surface.—Let us suppose that a pencil of rays is diverging from a point P on the under surface of a parallel-sided slab of some refracting material, for example, glass. The ray PN, which meets the upper surface of the slab at right angles, does not change its direction in passing into the air, but any other ray, PS, is refracted at the surface of separation. Let SR be its direction after refraction, and let μ be the index of refraction of the glass. Then

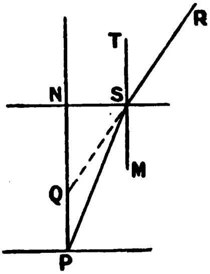

FIG. 132.

$$\mu = \frac{\sin TSR}{\sin PSM} = \frac{\sin NQS}{\sin NPS} = \frac{NS/SQ}{NS/SP} = \frac{SP}{SQ}$$

If the angle QPS is small, SP/SQ = NP/NQ; consequently, μNQ = NP, and the point Q is fixed. Thus, if a thin pencil from P be incident at right angles on the upper surface of the glass, it appears after refraction to come from Q, *i.e.* a virtual image of P is formed at Q.

An experimental method of finding the index of refraction of a glass slab is founded on this. A microscope is used, the tube of which can be raised vertically by a rack and pinion motion. This microscope is first focussed on a mark on the stage, then the slab of glass is placed on the mark and the microscope focussed on the virtual image of the mark formed by the slab. The distance through which the microscope has to be raised gives PQ. This is read on a scale. The microscope is then focussed on a scratch on the upper surface of the slab, and the height through which it has to be raised this time gives QN. Then, when PQ and QN are known, the index of refraction can be calculated.

A pool of water always appears shallower than it really is, its apparent depth being only $\frac{3}{4}$ its real depth. If in Fig. 132 NS denotes the surface of the water and P a stone at the bottom of the pool, P appears to be at Q, and NQ = $\frac{3}{4}$NP. Consequently the index of refraction of water is $\frac{4}{3}$.

Total Reflection.—The angles of incidence and refraction are connected by the relation $\sin i = \mu \sin r$. If the ray is passing from a medium with a large index of refraction to one with a small index of refraction, say from glass to air, μ is less than 1. As i, the angle of incidence inside the glass increases, r increases, and when $\sin i$ becomes equal to μ, $\sin r = 1$, and $r = 90°$. The value of i given by $\sin i = \mu$ is known as the critical angle. When the angle of incidence is greater than the critical angle, $\sin r$ is greater than 1. Consequently, r can have no real value; there is no refracted ray, and the light is said to be totally reflected. In Fig. 133 are shown corresponding angles of incidence and refraction for rays incident internally on the surface of glass of index of refraction 1·5. The ray PA makes the limiting angle with the normal. The ray PB is totally reflected.

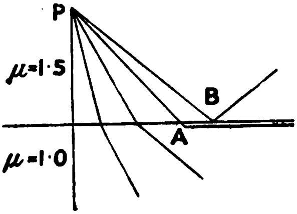

FIG. 133.

EXAMPLES 28

1. Two plane mirrors M and N are placed vertically upon a table. A ray of light is reflected by both mirrors in succession. Show that the angle through which it is deviated by the double reflection is equal to twice the angle between the mirrors.

2. Calculate the critical angle of total reflection for water (μ = 1·333) and crown glass (μ = 1·52).

$$\frac{1}{1{\cdot}333} = {\cdot}7500 = \sin 48° \, 36'$$

$$\frac{1}{1{\cdot}52} = {\cdot}6579 = \sin 41° \, 8'$$

The critical angle for water is 48° 36′ and for glass 41° 8′.

3. Water is poured into a beaker to a depth of 4·9 cms. How great is the apparent upward displacement of the scratches on the inside bottom of the beaker?

4. Find the velocity of light in water ($\mu = 1·33$) and in crown glass ($\mu = 1·52$).

5. On a moonlight night, when the surface of the sea is covered with small ripples, instead of a clear image of the moon a band of light is seen on the surface of the water extending in the direction of the moon. Explain in a diagram why this occurs.

6. A small air bubble in a glass slab appears to an eye looking normally at the surface to be 2 cms. from the latter. If 1·52 is the index of refraction of the glass, what is the real distance of the bubble from the surface?

7. A slab of glass 10 cms. thick with a refractive index of 1·52 is held with its lower surface horizontal 6 cms. above a piece of paper on which a mark is made. Where does the mark appear to be to an eye looking at it vertically through the slab? Illustrate your answer with a diagram.

8. Will a ray incident internally on the surface of water at an angle of 45° emerge or be totally reflected?

9. What is the greatest zenith distance that a star can have as seen by an eye under water? The zenith distance of a star is the angle which its direction makes with the vertical.

10. An observer stands 18 ft. from the shore of a lake, and sees the top of a mountain 2000 ft. high mirrored in the water at the lake's edge. If his eyes are 6 ft. above the level of the water, what is the distance of the mountain?

11. Show by a diagram in what direction a fish sees the setting sun.

12. The index of refraction of carbon bisulphide is 1·63. Carbon bisulphide is poured into a beaker to a depth of 8·0 cms. What is its apparent depth?

13. If a man walks towards a mirror at 3 miles an hour, with what velocity does he approach his image?

14. The earliest quantitative experiment in Light, of which we have a record, is a determination by Ptolemy (A.D. 140), 15 centuries before the discovery of the law of refraction, of the angles of refraction corresponding to angles of incidence, 10°, 20°, 30°, 40°, 50°, 60°, 70°, and 80° for water and glass. The results he obtained were 8°, 15½°, 22½°, 28°, 35°, 40½°, 45½°, and 50° for water, and 7°, 13½°, 20½°, 25½°, 30°, 34½°, 38½°, and 42° for glass. Find the mean results given by these figures for the indexes of refraction of water, and of the glass he used.

29. SPHERICAL MIRRORS

If the surface bounding two media is spherical in shape and highly polished, it is said to form a spherical mirror. It is not necessary for it to be silvered; an unsilvered glass surface gives quite as sharp images, but if the surface is silvered the images are much brighter. The glass must be silvered on the front, if the theory given here is to apply, for if the mirror consists of a thin piece of glass silvered on the back, the light suffers refraction at the front, both before and after reflection at the back.

Spherical mirrors are divided into two classes, concave and convex. In the case of the concave spherical mirror the light falls on the surface from the same side as the centre of curvature or centre of the sphere of which the surface forms part; in the case of the convex mirror the light falls on the surface from the opposite side to the centre of curvature.

Proof of Formula.—Let AB represent a section of a concave spherical mirror, let C be its centre of curvature, and let P be a point source of light. Join PC and produce it to cut the mirror in A.

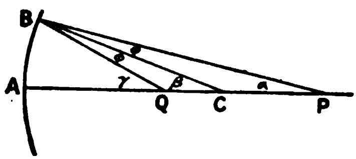

FIG. 134.

Draw any ray making a small angle α with AP to meet the mirror at B. Then CB is the normal to the element of the mirror at B. Let $\angle$ PBC = ϕ. The ray after reflection at B makes an angle ϕ with the normal and meets AP at Q. Denote $\angle$ BCQ by β and $\angle$ BQA by γ, and let AP, AC and AQ have respectively the values u, r, and v.

Then, from the figure

$$\beta = \phi + \alpha, \gamma = \phi + \beta$$

and consequently $\alpha + \gamma = 2\beta$. But since α, β and γ are small we can write

$$\alpha = \frac{AB}{u}, \beta = \frac{AB}{r} \text{ and } \gamma = \frac{AB}{v}$$

On substituting these values in the equation above and dividing out by AB we obtain

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

Rays diverging from a point P therefore, and making a small angle with the straight line joining P with the centre of curvature of the mirror, after reflection at the mirror, converge towards another point Q. Or, in other words, the mirror forms an image of the point P at Q.

In Fig. 134 keep the mirror and, consequently, the point C fixed, and rotate the line PC through a small angle about an axis perpendicular to the plane of the figure through C. P and Q then describe arcs which may be taken as straight lines, and the one line will be the image of the other.

Spherical mirrors have usually a circular rim, and the straight line through the centre of curvature perpendicular to the plane of this rim is called the axis of the mirror. We have proved, therefore, for one particular case, that a spherical mirror forms an image of a short line situated on the axis and perpendicular to the axis.

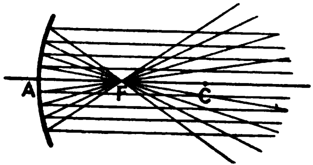

FIG. 135.

If the object is at infinity, $1/u$ is zero, and v becomes equal to $r/2$. All the incident rays are then parallel, and after reflection they converge to a point F midway between A and C. This point F is called the principal focus, or simply the focus of the mirror, and the length AF is called its focal length, and is usually written f.

Graphical Construction.—If the property of the focus, and the fact that the mirror forms images be assumed, the position and size of the image can be found very easily by a graphical construction. Let there be an object at P (Fig. 136) only one half of which, P*p*, is drawn. Draw a ray from *p* through C; it falls on the mirror perpendicularly, and consequently after reflection comes back along the same path. Draw another ray, *p*B, parallel to the axis; after reflection it passes through the focus. The point *q* where BF cuts *p*C is therefore the image of *p*, and the perpendicular *q*Q to the axis consequently the image of *p*P.

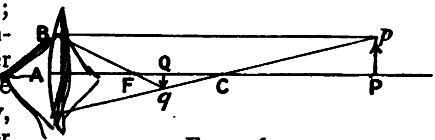

FIG. 136.

Since AB is small in comparison with the radius of curvature—it is greatly exaggerated in the figures for the sake of clearness—it may be regarded as straight, and at right angles to AP. Then AB = P*p*, and consequently AB/Q*q* = P*p*/Q*q*. Triangles ABF and Q*q*F are similar, as are also triangles Q*q*C and P*p*C. Hence

$$\frac{AB}{Qq} = \frac{AF}{QF} \quad \text{and} \quad \frac{Pp}{Qq} = \frac{PC}{QC}$$

But the left-hand sides of these two equations are already equal; hence

$$\frac{AF}{QF} = \frac{PC}{QC}$$

or

$$\frac{f}{v - f} = \frac{u - 2f}{2f - v}$$

which gives

$$f(2f - v) = (u - 2f)(v - f)$$

or

$$fu + fv = uv$$

and on dividing out by *uvf*

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{r}$$

the equation which has already been found.

The relative sizes of the image and object can also be obtained from Fig. 136. For

$$\frac{Qq}{Pp} = \frac{QC}{CP} = \frac{r - v}{u - r} = \frac{v\left(\frac{1}{v} - \frac{1}{r}\right)}{u\left(\frac{1}{r} - \frac{1}{u}\right)}$$

But since

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}, \quad \frac{1}{v} - \frac{1}{r} = \frac{1}{r} - \frac{1}{u}$$

and

$$\frac{Qq}{Pp} = \frac{v}{u}$$

The ratio Q*q*/P*p* is termed the linear magnification. The above equation can be put into the following useful rule: the linear dimensions of the image and object are in the ratio of their distances from the mirror.

Virtual Image.—So far we have considered only the case where the image is on the same side of the mirror as the object. It is then real, and can be received on a screen. But suppose that the object lies between the surface of the mirror and the focus; for example, let $r = 10$ cms. and $u = 3$ cms. Then, by the equation,

$$\frac{1}{3} + \frac{1}{v} = \frac{1}{5}$$

which gives $v = -7{\cdot}5$ cms.

If we use the graphical method we obtain the same result, namely that the image is 7·5 cms. behind the mirror. Now the light rays themselves cannot get behind the mirror; it is only their directions produced backwards that meet there. Consequently the image is virtual, and will not show up on a screen.

The same graphical construction holds for the convex mirror as for the concave. In the case of the convex mirror the focus is behind the surface halfway between it and the centre of curvature, and the image is always virtual when the object is real. The formula holds for all cases for both concave and convex mirrors, if u, v and f are considered negative when measured to the left of the mirror. It is much better, however, for beginners to use the formula only for the case for which we have proved it, and treat all other cases by the graphical method.

Phantom Bouquet and Pepper's Ghost.—An interesting optical illusion called the "Phantom Bouquet" can be produced with a large concave mirror of about a metre radius of curvature. A bouquet is placed in front of the mirror in a darkened room in such a position as to produce a real and magnified image. The bouquet itself is suspended upside down, and the image is erect. The mirror and bouquet are placed in a large box and the bouquet illuminated strongly by lamps. These lamps are placed in the box in such a position that they cannot be seen from outside. After reflection by the mirror, the rays from the bouquet emerge from an opening in the box, and form the image in front of it. An empty vase is placed below the image. If an observer looks at the vase, so that his direction of vision meets the mirror, he sees the bouquet in the vase, but if he looks at it from the side, the vase appears empty.

The illusion known as "Pepper's Ghost" requires a large plane sheet of plate glass, which is placed vertical and at an angle of 45° with the line of vision of the spectators. The latter consequently see two scenes superimposed, the background, which is visible by the direct rays that come through the glass, and objects at the side of the stage which are visible by the rays reflected through 90° by the glass. The background is usually somewhat dim, and at first the actor who is to fill the rôle of the ghost is kept in the dark at the side. He is too far to the side to be seen by the spectators directly. When it is time for him to "appear," the limelight is turned on him, and his image at once appears superimposed on the background by the reflected light. The image is, of course, transparent; bright parts of the background can be seen through it.

Examples 29

1. A concave spherical mirror has a focal length of 12 cms. Find the position, size, and nature of the image, when the object is 2 cms. high and distant (i) 30 cms., (ii) 8 cms. from the mirror. Verify by the graphical construction.

2. A concave spherical mirror has a radius of curvature of 20 cms. Find the position, size, and nature of the image when the object is 1 cm. high and distant (i) 15 cms., (ii) 5 cms. from the surface of the mirror. Verify by the graphical construction.

3. A convex spherical mirror has a focal length of 12 cms. Find by the graphical construction the position, nature, and size of the image, when the object is 2 cms. high and distant (i) 30 cms., (ii) 8 cms. from the mirror.

4. A convex spherical mirror has a radius of curvature of 10 cms. Find the position and size of the image when the object is 1 cm. high and distant (i) 10 cms., (ii) 4 cms. from the surface of the mirror.

5. A real image produced by a concave mirror is found to be three times the height of the object. If the mirror is 16 cms. from the object, what is its focal length?

6. An unlit gas burner is placed 3 feet in front of a concave mirror of radius of curvature 2 feet. Where must a flame be placed so that its image may appear to come out of the unlit burner? Should the flame be erect or inverted?

7. An object is placed at a distance of $\frac{1}{3}r$ in front of a concave mirror of radius r. Show that the image is erect, virtual, and magnified three times.

8. An image produced by a concave mirror is n times the size of the object. What is the distance of the latter from the mirror in terms of the focal length?

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}, \quad v=nu$$

$$\frac{1}{u}+\frac{1}{nu}=\frac{1}{f}, \quad \frac{1}{u}\left(1+\frac{1}{n}\right)=\frac{1}{f}$$

and
$$u=\frac{(n+1)}{n}f$$

9. If the image formed by a concave mirror is 10 cms. distant from the object, and the mirror itself is 8 cms. distant from the object in the other direction, find the radius of curvature of the mirror.

10. Prove the formula giving the position of the image formed by a concave mirror for the case when the object is between the focus and the surface of the mirror.

11. Derive the magnification law for the same case.

12. The focal length of a concave mirror is obtained by forming the image of a lamp several metres distant. The result is 32·1 cms. How far away must the lamp be for the result to be accurate to 2 mm.?

Let u be the distance of the lamp when the result is accurate to 2 mm. Then $v = 32{\cdot}1$ and $f = 32{\cdot}1 - {\cdot}2 = 31{\cdot}9$. Consequently

$$\frac{1}{32{\cdot}1}+\frac{1}{u}=\frac{1}{31{\cdot}9}$$

which gives $u = 5120$ cms.

30. LENSES AND THE OPTICAL BENCH

A lens is a portion of a refracting medium bounded by two spherical surfaces or by one spherical surface and a plane surface. The straight line joining the centres of curvature of the surfaces is called the axis of the lens, or, if one of the surfaces is plane, the axis is the straight line normal to it drawn through the centre of curvature of the other. A plane through the axis is said to be a principal section of the lens. A lens forms an image of an object situated on its axis.

Lenses are divided into two classes. The first class, convex or converging lenses, cause a beam of parallel rays to converge; the second class, concave or diverging lenses, cause a beam of parallel rays to diverge. Fig. 137 gives the principal sections of some typical lenses. Of these, A, B, and C are convex; D, E, and F are concave. A is termed a double-convex or bi-convex lens, B a plano-convex lens, and C a convex meniscus, while D is termed a double-concave or bi-concave lens, E a plano-concave lens, and F a concave meniscus. C and F are termed also, somewhat indiscriminately, convexo-concave and concavo-convex lenses.

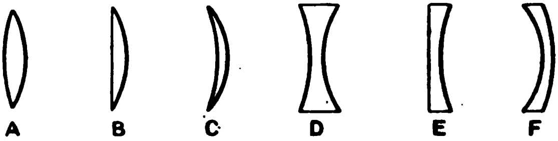

FIG. 137.

Figs. 138 and 139 show beams of parallel rays incident on a convex and a concave lens respectively. After passing through the convex lens the rays meet in a point, the focus of the lens; after passing through the concave lens the directions of the rays produced backwards meet in a focus. The distance from the focus to the lens is in each case called the focal length of the lens, and is usually denoted by f.

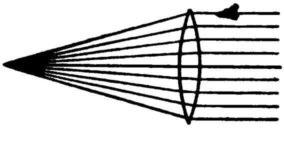

FIG. 138.

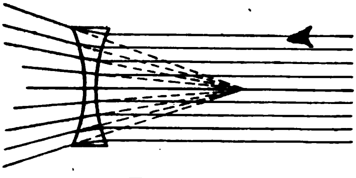

FIG. 139.

The direction of a ray of light which passes through the centre of a lens is unaltered, because to such a ray it acts simply as a parallel plate.

Graphical Determination of Position of Image.—If the properties of the focus and the centre of the lens are assumed, the position of the image can be determined very easily graphically. For, let Pp represent one half of the object. From p draw a ray parallel to the axis to meet the lens in B. After refraction by the lens it either passes through F (Fig. 140), or appears to pass through F (Fig. 141). From p draw

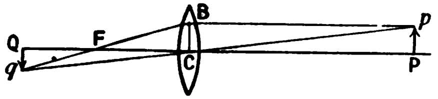

FIG. 140.

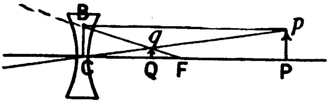

FIG. 141.

another ray through C, the centre of the lens. It passes through undeviated. The point where these two rays intersect, that is, q, is the image of p, and the perpendicular to the axis, qQ, is the image of pP.

Formula for Convex Lens.—The formula giving the position of the image formed by a convex lens can be derived very easily from Fig. 140. For $\mathrm{CB} = \mathrm{P}p$, and consequently $\mathrm{CB}/\mathrm{Q}q = \mathrm{P}p/\mathrm{Q}q$. Triangles FQ$q$ and FCB are similar, as are also triangles CQq and CPq. Hence

$$\frac{\mathrm{CB}}{\mathrm{Q}q} = \frac{\mathrm{CF}}{\mathrm{QF}} \quad \text{and} \quad \frac{\mathrm{P}p}{\mathrm{Q}q} = \frac{\mathrm{PC}}{\mathrm{QC}}$$

But the left-hand sides of these two equations are already equal; hence

$$\frac{\mathrm{CF}}{\mathrm{QF}} = \frac{\mathrm{PC}}{\mathrm{QC}} \quad \text{or} \quad \frac{f}{v-f} = \frac{u}{v}$$

if u denotes the distance of the object and v the distance of the image from the lens. This gives $vf = uv - uf$, and on dividing out by uvf

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

A similar formula can be derived for the concave lens, but lies outside the scope of this book; questions given in this book on concave lenses should be solved by the graphical construction.

The relative sizes of image and object are easily obtained from Figs. 140 and 141.

For
$$\frac{\mathrm{Q}q}{\mathrm{P}p} = \frac{\mathrm{CQ}}{\mathrm{CP}} = \frac{v}{u}$$

that is, the magnification is equal to image distance divided by object distance.

Optical Bench.—The apparatus used most in physical laboratories for determining the focal lengths of lenses is the optical bench. Fig. 142 shows a very useful and simple wooden bench together with its

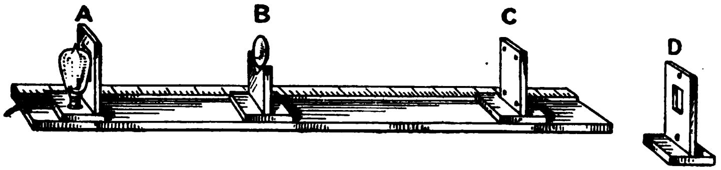

FIG. 142.

fittings. The base is mahogany. At the side is a scale two metres long for reading the positions of the pieces. The stand A carries an incandescent electric lamp, in front of which is a wooden upright with a rectangular hole in it, across which are stretched cross-wires. These cross-wires are the object, and it is usually advisable to fasten a piece of tissue paper with drawing pins between the hole and the lamp to give a more uniform background to the object. Stand B is for carrying the lens or mirror; it has a V-shaped top with a groove in it. C is a screen for receiving the image, consisting of an upright with a piece of paper fastened on with drawing pins, and D a similar screen with a hole in it, also used for receiving images. The positions of the various stands can be read by marks on their bases.

To find the focal length of a convex lens it is simply put on the stand B, and an image of the cross-wires focussed on the screen. Then u and v, the distances of the cross-wires and the image from the lens, are measured, and f is calculated by the formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

The length of one of the cross-wires and the length of its image can at the same time be measured, and thus the linear magnification found. It should, of course, be equal to v/u.

Another way of finding the focal length is to place the lens at the middle of the bench and arrange the cross-wires and image screen at equal distances from it. Then, in general, there will be no image formed. If now the distances of the cross wires and image screen from the lens be gradually increased or decreased, their values being always kept equal to one another, positions will eventually be reached in which a sharp image is formed on the screen. Then u is equal to v, and each is equal to $2f$. If d be the distance between the cross-wires and image screen, f is equal to $d/4$.

A third method is simply to measure the distance from the lens of the image of a lamp situated at a distance, which is great in comparison with the focal length of the lens.

To determine the focal length of a concave mirror on the optical bench the mirror is mounted on the stand B, and the screen D with the hole in it placed between it and the cross-wires. The rays from the cross-wires fall on the mirror through the hole in the screen, and are reflected to form a sharp image on the screen. The distances u and v of the cross-wires and image from the mirror are then measured, and f calculated by the formula.

EXAMPLES 30

1. A convex lens has a focal length of 20 cms. Find by the graphical method the position, size, and nature of the image formed of an object 1 cm. high, when the latter is situated (i) 50 cms., (ii) 10 cms. from the lens. Verify your results by the formula.

2. A convex lens has a focal length of 14 cms. Find by the graphical method the position, size, and nature of the image formed of an object 1 cm. high, when the latter is situated (i) 50 cms., (ii) 20 cms. from the lens. Verify the results by the formula.

3. A concave lens has a focal length of 20 cms. Find by the graphical

method the position, size, and nature of the image formed of an object 1 cm. high, when the latter is situated (i) 50 cms., (ii) 10 cms. from the lens.

4. A concave lens has a focal length of 14 cms. Find by the graphical method the position, size, and nature of the image formed of an object 1 cm. high, when the latter is situated (i) 50 cms., (ii) 20 cms. from the lens.

5. A convex lens is mounted on an optical bench, and it is found, that when the cross-wires are distant 32·0 cms. from the lens, a real image is formed at a distance of 18·4 cms. on the other side of the lens. Find the focal length.

6. A convex lens is mounted on an optical bench, and it is found, that when the object distance is 41·0 cms. the image distance is 64·1 cms., and that when the object distance is 84·0 cms. the image distance is 35·4 cms. Find the mean value of the focal length of the lens.

7. It is found, when the cross-wires and image screen on an optical bench are 84·0 cms. apart with a convex lens mounted exactly midway between them, that a clear image of the cross-wires is formed by the lens on the image screen. What is the focal length of the lens?

8. The cross-wires and image screen are a distance d apart, and a lens is mounted between them, nearer the cross-wires, so as to form a sharp image on the screen. It is then moved a distance a towards the image screen, and it is found that in this new position it also forms a sharp image on the screen. Show that the focal length of the lens is given by

$$f = \frac{d^2 - a^2}{4d}$$

Let u be the first object distance and v the first image distance. Then $u + v = d$. Now if in the equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we interchange v and u, *i.e.* make u the image distance and v the object distance, the equation still holds, consequently a sharp image is still formed, and the distance between object and image is still d; thus this gives the second position in which the image is formed.

Hence
$$a = v - u$$
$$d = v + u$$
$$v = \tfrac{1}{2}(d + a), \quad u = \tfrac{1}{2}(d - a)$$

and by substituting these values in the formula,

$$\frac{2}{d + a} + \frac{2}{d - a} = \frac{1}{f}$$

which gives the required result.

9. Show that in the preceding question, if the magnification is m in the first position, it is $1/m$ in the second.

10. Show graphically that a convex lens cannot form a real image on a screen which is distant less than $4f$ from the object.

11. In order to determine the focal length of a convex lens it is used to form an image of a lamp at a distance of about 5 metres. The lamp is supposed to be infinitely far off, and the distance of the image from the lens, which is found to be 28·3 cms., is then taken to be the focal length of the lens. Calculate the magnitude of the error involved.

12. A candle stands at a distance of 6 ft. from a wall; where must a convex lens of 18 inches focal length be placed, in the perpendicular from the candle to the wall, so as to produce upon the wall a distinct image of the candle?

13. If an object at a distance of 12·0 cms. from a convex lens has its image magnified three times, what is the focal length of the lens?

We have $u = 12$, $v = 36$, hence

$$\frac{1}{f} = \frac{1}{36} + \frac{1}{12} = \frac{1}{9}, \text{ and } f = 9 \text{ cms.}$$

In this case the image is real. The image may, however, be virtual (cf. Fig. 143); then it may be shown graphically that $f = 18$ cms.

14. At what distance from a convex lens must an object be placed, in order that the image may be real and magnified three times? Express your result in terms of the focal length.

15. A camera is provided with a little celluloid scale, on which are marked the positions at which the lens must be placed in order to photograph objects at infinity, 15, 10, 5, 3, and 2 ft. respectively. Construct a plan of this scale, given that the focal length of the lens is 5·5 inches.

16. An image of the moon is formed on a piece of white cardboard by means of a lens, and the diameter of the image is measured and found to be 8 mm. Find the focal length of the lens, given that the moon subtends an angle of 31′ at the surface of the earth.

31. OPTICAL INSTRUMENTS

Magnifying Glass.—The normal eye focusses best on an object distant 10 or 12 inches from it. This distance is called the distance of distinct vision. If we attempt to increase the detail visible by bringing the object nearer, an exertion is required to see it distinctly. Hence the distance of distinct vision is the most favourable position for examining the detail of an object.

Let AB be a convex lens of small focal length placed before the eye at E, let F be its focus, and let Pp be an object within its focal length.

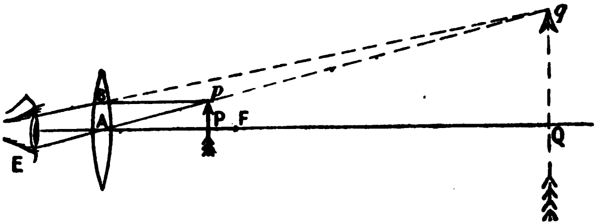

FIG. 143.

Then on applying the usual graphical construction, the image is found to be at Qq. All rays proceeding from Pp to the eye, after refraction by the lens, appear to come from Qq.

In using a magnifying glass we hold it so that the image is at the distance of distinct vision. Thus the distance of Q from the eye is equal to D, the distance of distinct vision, or, since the eye is near the lens, we may put AQ = D. If the object Pp were to be examined by the eye direct, it would have to be placed at Q. The magnification, that is, the ratio of the apparent lengths with and without the lens, is thus Qq/Pp. But by similar triangles

$$\frac{Qq}{Pp} = \frac{AQ}{AP} = \frac{D}{f}$$

since AP is almost equal to f, the focal length of the lens. The magnification is consequently equal to D/f.

A single convex lens used as above is sometimes called a simple microscope.

Astronomical Telescope.—The optical system of an astronomical telescope consists of two parts, an object glass and an ocular or eyepiece. The eyepiece is usually a compound one, consisting of two lenses. It fulfils the same function as the magnifying glass just described, and could be replaced by a single convex lens. In order to make the explanation of the instrument simpler, we shall therefore suppose the compound eyepiece replaced by a single magnifying lens. The action of the telescope is then represented in Fig. 144.

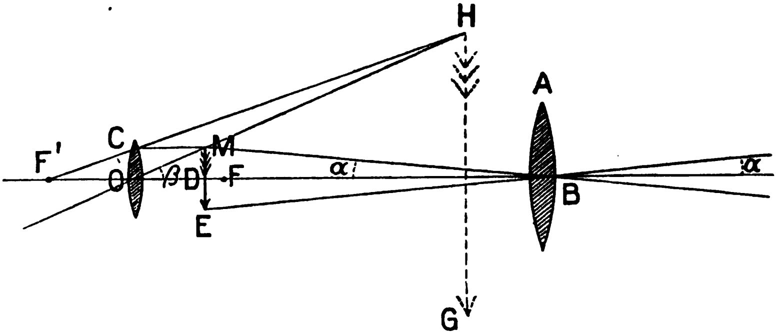

FIG. 144.

AB is the object glass and CO the eyepiece. The object glass forms a real and inverted image ME of the distant object very near its focus. BM and BE are the two undeviated rays from the extremities of the distant object through the centre of the object glass to the extremities of this image. Let F and F′ be the two foci of the eyepiece. Draw from M a ray MC parallel to the axis and a ray MO through the centre of the eyepiece; then by the graphical construction we find that a virtual image of ME is formed at HG.

The magnifying power of the telescope is measured by the ratio of the angle subtended at the eye by the image in the telescope to the angle subtended by the distant object itself. Now the length of the telescope is short compared with the distance of the distant object. Hence the angle subtended by the distant object at the eye is 2α (see fig.). The eye is held close up to the eyepiece; hence the angle subtended by the final image HG at the eye is 2β. Consequently the magnifying power is

$$\frac{2\beta}{2\alpha}=\frac{\beta}{\alpha}=\frac{\mathrm{MD/OD}}{\mathrm{MD/BD}}=\frac{\mathrm{BD}}{\mathrm{OD}}$$

since the angles are small. But BD and OD are almost equal to the focal lengths of the object glass and eyepiece respectively. Hence the magnifying power of the telescope is equal to

$$\frac{\text{Focal length of object glass}}{\text{Focal length of eyepiece}}$$

Compound Microscope.—Suppose that in the case of the telescope the object is brought nearer and nearer the object glass, but that the focal length of the latter is made shorter and shorter, so that the image ME remains in the same place. Then finally the object is close up in front of the object glass, the focal length of the object glass is smaller than the focal length of the eyepiece, and the instrument will have been converted into a compound microscope.

Fig. 145 represents a compound microscope of a type used in physical laboratories. The tube, which is shown vertical, can be turned horizontal, and its position can be adjusted in both a vertical and horizontal direction, and read on either of the two scales. The instrument is thus very convenient for making accurate measurements of changes in length or height.

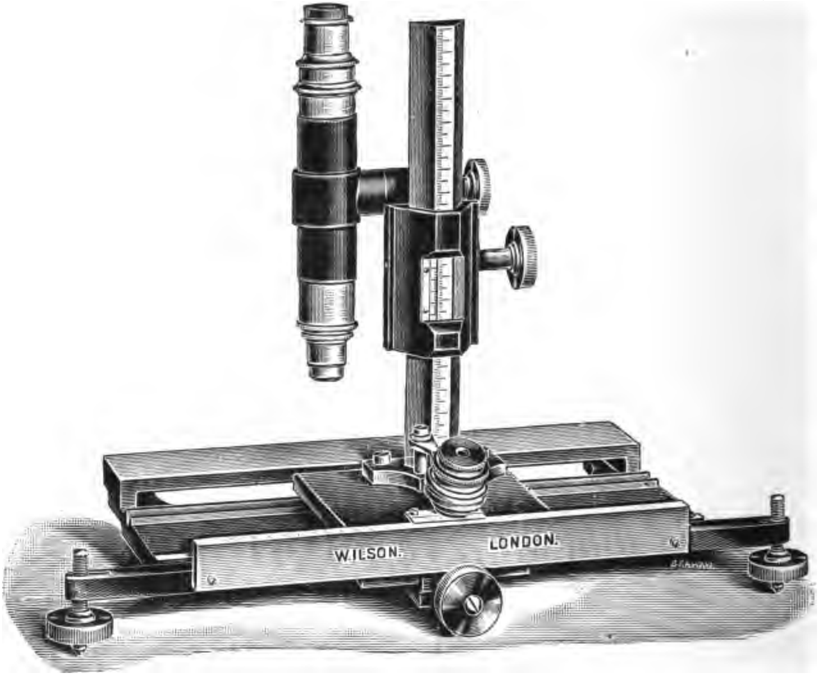

FIG. 145.

Galilean Telescope.—In the Galilean telescope the convex eyepiece of the astronomical telescope is replaced by a concave lens; the real image formed by the object glass is inverted and magnified by this lens, and thus objects are seen in the Galilean telescope the right way up. It is consequently better suited for terrestrial use than the other telescope, although it gives neither such a large field nor such a bright image; when a telescope with a convex eyepiece is used for terrestrial work, it has to be fitted with two erecting lenses in front of the eyepiece, and this has the disadvantage of adding to the length of the tube. It is immaterial in astronomical work whether the image is erect or inverted. Ordinary field glasses consist of two Galilean telescopes mounted side by side and focussed by the same screw.

Photographic Camera.—A photographic camera consists essentially of a rectangular box at one end of which is placed the sensitive plate or film, and at the middle of the opposite end of which is placed the lens. The distance of the lens from the plate can be altered, so as to focus the picture sharply on the latter, and no light must enter the camera except through the lens. By means of a series of stops placed at the lens the aperture of the pencils of light forming the image can be regulated. The diameter of the aperture is always expressed as a fraction of the focal length of the lens; thus, if a lens is working at $f/16$, we mean that the diameter of the stop is $\frac{1}{16}$ of its focal length. A lens working at $f/8$ thus admits about 8 times as much light as one working at $f/22$, but, of course, does not give such a sharp picture.

The modern photographic plate consists of an emulsion of silver bromide in gelatine, which has been poured while warm on to a large plate and allowed to set. The glass plate is then cut into smaller sizes. The light produces no visible action on the plate until the developer is poured over it. Then the particles of silver bromide on which the light was incident are reduced to black metallic silver. After development the plate is "fixed" by immersion in a solution of hyposulphite of soda, which dissolves away all the sensitive particles on which the light has not acted. It is then washed and dried. As the bright parts of the image, those on which most light falls, come out black on the plate, and the dark parts come out light, the picture is said to be a negative. By placing it in contact with sensitive paper, and allowing light to act through the negative on the paper, an image called a positive is produced on the paper. In this image the light and shade are correctly rendered; it is the picture wanted, the negative being only an intermediate step.

Optical Lantern.—Fig. 146 depicts the optical system of a lantern used for projecting slides. AB represents the slide; it is placed upside down. C is the objective or projecting lens. It forms an erect image of the picture on the screen, and by focussing it the image can be made sharp for different positions of the screen. The source of light, E, is an electric arc, the positive carbon of which is horizontal and the

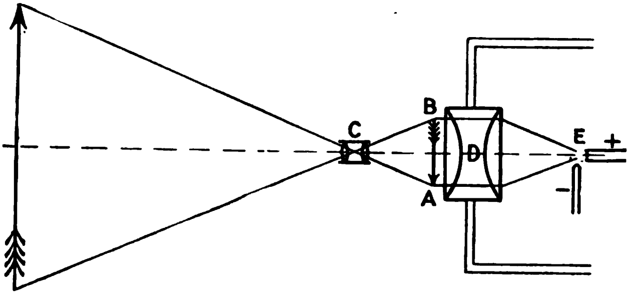

FIG. 146.

negative vertical. By this arrangement the full light from the crater falls upon the condenser D. The latter consists usually of two plano-convex lenses with their plane faces outwards, and its function is to make as much light as possible pass through the slide in such a direction that it also passes through the objective. The electric arc is enclosed in a box, so as to keep in all light except that which forms the image on the screen. Limelight may be used instead of the arc, but the arc is brighter and more convenient, and is always to be preferred if available.

Cinematograph.—Visual impressions on the retina persist after the removal of the stimulus for about $\frac{1}{16}$ of a second. Thus, if photographs are taken of a moving object at a rate of not less than 16 per second, and if these photographs are projected on a screen at the same rate, the discontinuous pictures fuse together, and produce an illusion of continuous motion. This is the principle of the cinematograph. The pictures are smaller than lantern slides, the regulation size being about 1 inch by $\frac{3}{4}$ inch, and they are projected by a lens of about $2\frac{3}{4}$ inches focal length. They are printed on a film of celluloid or a similar preparation, and the film is uncoiled from one spool, is jerked by a mechanism through the "gate" in the focal plane of the lens, and coiled up on another spool. Each jerk moves the film a distance equal to the height of a picture. Each picture rests for an instant while in the gate and is

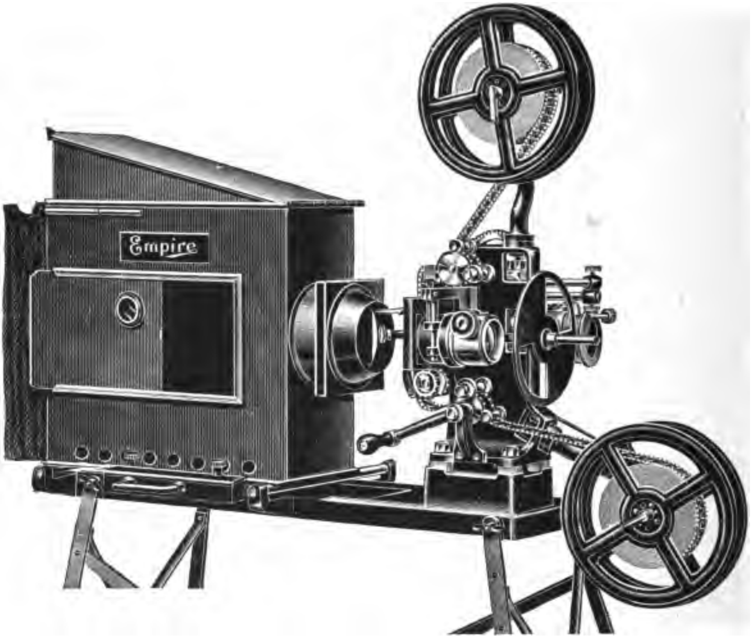

FIG. 147.

W. Butcher & Sons.

then projected, and while it is being jerked out and the next one jerked in, a sector passes up in front of the projecting lens and cuts off the light. The films are tough enough to stand the strain of being pulled through, but the celluloid ones are inflammable, and the image of the crater of the arc on one for a short time is sufficient to set it on fire.

Fig. 147 represents a simple cinematograph together with its projecting lantern.

EXAMPLES 31

1. Rule two lines together on a card, and look at them through a magnifying glass held close up to one eye. At the same time look with the other eye at a scale placed at the distance of distinct vision. By getting the image of the lines to superimpose on the scale, and measuring the distance between the lines, the magnification can be determined. Compare it with the theoretical value.

2. The focal length of the object glass of an astronomical telescope is 14 inches and the focal length of its eyepiece is $\frac{5}{8}$ inch. Calculate its magnifying power.

3. If half the object glass of a telescope which is pointed at the moon is covered, how will the appearance of the moon as seen through the telescope be affected?

4. A snapshot camera is provided with stops marked *f*/5·6, *f*/8, *f*/11, *f*/16, *f*/22, and *f*/32. If the correct exposure for *f*/8 is $\frac{1}{100}$ sec., calculate what it must be for the other stops.

5. A stick 6 inches long is spun about an axis through its midpoint and perpendicular to its length. How many revolutions must it make per second in order to present the appearance of a disc? Visual impressions persist on the retina for $\frac{1}{16}$ second.

The stick must make half a revolution in $\frac{1}{16}$ sec., *i.e.* 8 revolutions per second.

32. THE SPECTROMETER AND DISPERSION

Fig. 148 represents a section of a glass prism ABC. PQ is a ray of light from a sodium flame incident on the prism at Q. A sodium flame is produced by heating a sodium salt in a bunsen burner. The best salt to use is the bicarbonate; a bead of this salt formed in a small loop at the end of a thin platinum wire and held in the edge of the flame gives an intense colour to the flame for a long time. After refraction at Q the ray travels in the direction QR, and after refraction at the second face at R it emerges in the direction RS. The edge A, *i.e.* the

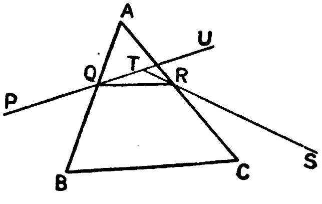

FIG. 148.

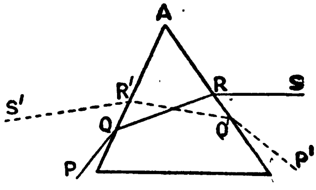

FIG. 149.

edge in which the faces that the light passes through meet, is termed the refracting edge of the prism. Produce PQ to U, and produce SR to meet it in T; then the angle UTS between the directions of the incident and emergent rays is the deviation produced by the prism.

If the angle of incidence of the ray PQ on the face AB alters, the deviation alters. If the incident and emergent rays are equally inclined to the prism, *i.e.* if AQ = AR, the deviation is a minimum, and the prism is said to be in the position of minimum deviation. This is easily seen. For, let the deviation of any other ray PQRS (Fig. 149) be a minimum; then the deviation of the ray P′Q′R′S′ which passes through the prism in the symmetrically opposite direction, *i.e.* so that AQ = AQ′ and AR = AR′, is the same as the deviation of PQRS. Consequently the latter cannot be a minimum.

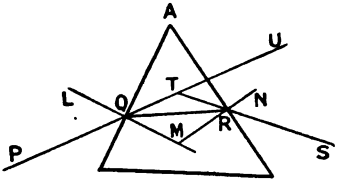

FIG. 150.

Formula for Minimum Deviation.—Suppose now that Fig. 150 represents the position of minimum deviation, that LM and MN are the normals at the points of incidence and emergence of a ray PQRS. Write

$$\angle PQL = \angle SRN = i \quad \text{and} \quad \angle RQM = \angle QRM = r,$$

and let the angle of minimum deviation be δ. Then

$$\begin{aligned} \delta &= \angle UTS = \angle TQR + \angle TRQ \\ &= \angle TQM - \angle RQM + \angle TRM - \angle QRM \\ &= 2(i - r) \end{aligned}$$

Also

$$\angle AQR + \angle ARQ = \pi - A$$

and

$$\begin{aligned} \angle AQR + \angle ARQ &= \angle AQM - \angle RQM + \angle ARM - \angle QRM \\ &= \pi - 2r \end{aligned}$$

Hence $A = 2r$, *i.e.* $r = \frac{1}{2}A$, and on substitution in the expression above for δ,

$$\delta = 2(i - \tfrac{1}{2}A) \quad \text{or} \quad i = \tfrac{1}{2}(A + \delta)$$

But the index of refraction, μ, is defined by

$$\mu = \frac{\sin i}{\sin r}$$

On substitution for i and r this gives

$$\mu = \frac{\sin \frac{1}{2}(A + \delta)}{\sin \frac{1}{2}A}$$

Hence, if A and δ are known, μ can be calculated. A useful way of remembering this formula is by considering what happens if the prism is made of air instead of glass,—δ becomes 0 and μ is equal to 1.

Spectrometer.—The spectrometer or goniometer is an instrument for determining A and δ. It is represented in Fig. 151, arranged for determining δ. The tube on the left is called the collimator; at its left end it carries a slit. The sodium flame is placed in front of this slit. At its other end it carries a convex lens which makes the rays from the slit parallel. They then pass through the prism, and are received by the telescope. An eye looking into the telescope sees the slit as a thin yellow vertical rectangle. In determining δ the prism is rotated, so that (see fig.) this image moves as far to the right as possible. The cross-wires in the field of the telescope are next set upon it. The prism is then removed, and the telescope rotated to bring the cross-wires again on to the image. δ is equal to the angle rotated through by the telescope, as read on the circle with which the instrument is provided.

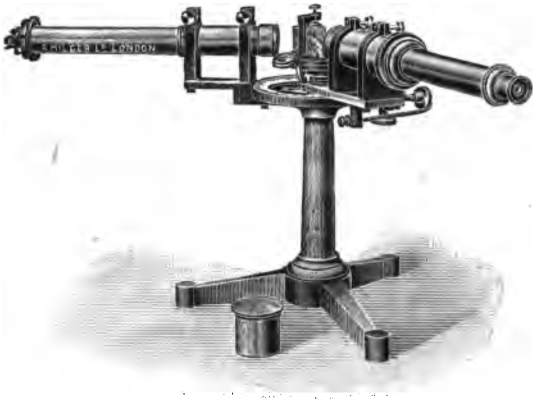

FIG. 151.

If we wish to determine the refracting angle A of the prism, the latter is placed upon the spectrometer table, so that the light from the collimator falls on it in the direction PA. Then some is reflected in the direction AR and some in the direction AQ, and images are formed in these directions. The angle RAQ between these images is read by the telescope. Then

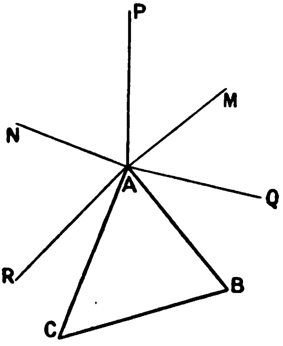

FIG. 152.

$$\angle RAQ = 2\pi - \angle RAP - \angle QAP = 2\pi - 2\angle NAP - 2\angle PAM$$
$$= 2(\pi - \angle NAM) = 2A$$

Dispersion of Light.—Let PQ (Fig. 153) be a beam of sunlight admitted through a hole in a shutter, and incident at Q on a glass prism in a darkened room. Then it is found that the beam of light, which was originally parallel owing to the great distance of the sun, is broken up by the prism into rays of different colours, and if a screen SS is placed to receive these rays, a coloured band called a spectrum is produced on this screen. The colours of the spectrum are, in order, red, orange, yellow, green, blue, indigo, violet; of these red is deviated least and violet most, the positions of red, green, and violet being denoted by the letters R, G, and V in the fig. Indigo or dark blue is seen as a separate colour only by people of exceptionally good colour vision. The seven colours of the spectrum are not sharply defined, but merge into one another gradually.

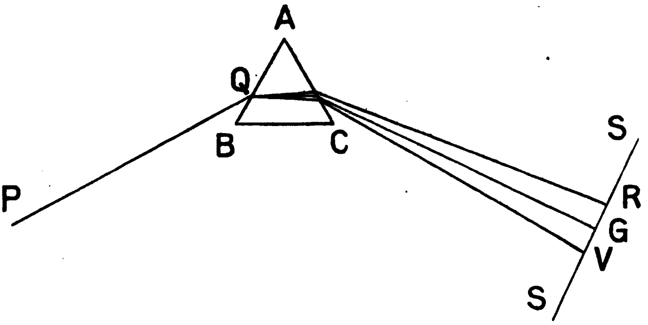

FIG. 153.

The spectrum was discovered by Newton in 1666, and he showed that the colours are deviated to a different extent because their indices of refraction are slightly different, also that they could be combined together again to form white light.

Spectroscopy.—The best way of studying spectra is to employ a glass prism mounted on a spectrometer; when used for this purpose the instrument is called a spectroscope. If the light of a white gas flame or of an electric glow lamp is allowed to fall upon the slit, and we look into the telescope, we see a continuous spectrum like the first spectrum shown in the frontispiece. If sunlight falls on the slit, we see a continuous spectrum crossed with dark lines, known after their discoverer as the Fraunhofer lines, and denoted by letters of the alphabet. If sodium light falls upon the slit, we see only two narrow yellow lines; these are known as the D lines, and are so close together that, unless the slit is very narrow, they appear as one. Sodium light is thus practically homogeneous; that is why it is used in determining the index of refraction of a glass prism. If a mercury vapour lamp is placed before the slit, we obtain the fourth spectrum in the frontispiece; this is a good example of a line spectrum. If the light of a glow lamp is allowed to fall upon the slit through a piece of blue glass coloured by cobalt oxide, we obtain the fifth spectrum, the three black bands being due to the cobalt oxide; this is of a type known as an absorption spectrum. The blue cone of the bunsen flame gives the sixth spectrum, called the Swan spectrum after its discoverer, which is of a type known as a band spectrum.

Each chemical element has its own characteristic spectrum, and we can investigate the nature of a substance by heating it or causing it in any other way to emit light, and examining the spectrum of that light. This procedure is known as *spectrum analysis*.

It will be noticed that the yellow line of the sodium spectrum coincides in position with the dark D line of the solar spectrum. It is found that if light, which would otherwise give a very bright continuous spectrum, passes through sodium vapour, the dark D line appears in its spectrum. The sodium vapour has absorbed rays of the same wave-length as it emits. It is natural to assume that the body of the sun gives a continuous spectrum, but that it is surrounded by a gas which absorbs in this way some of the rays of this continuous spectrum. The Fraunhofer D line thus becomes a proof of the existence of sodium vapour in the sun; in a similar way the existence of other terrestrial elements has been proved in the sun.

Ultra-violet and Infra-red.—Each colour of the spectrum has a different wave-length, the wave-lengths of the red and violet ends and the mean value for the sodium lines being $7{\cdot}6 \times 10^{-5}$, $4{\cdot}0 \times 10^{-5}$, and $5{\cdot}893 \times 10^{-5}$ cms. respectively. The wave-lengths of the different colours are given by the scale in the frontispiece in 10^{-8} cms.

If in Fig. 153 the spectrum is received on a photographic plate instead of on a screen, it will be found that the red light produces no blackening on the plate, that the blue, indigo, violet, and the ultra-violet, or region lying beyond the violet, have a most pronounced blackening effect, and that green and yellow have an effect intermediate between blue and red. It is because red does not affect the plate, that photographs are developed in a red light. The fact that the region beyond the violet produces an effect on the plate, shows that there are invisible rays falling on the plate there; these rays are called the ultra-violet rays.

If a thermopile, which is an instrument for measuring very small increases of temperature, is moved along a spectrum, it is found that the violet end has very little heating effect, and that the heating effect increases as we move along the spectrum, and becomes a maximum beyond the red end of the spectrum. There are here consequently invisible heat rays, usually referred to as infra-red rays. Heat rays are given out by all hot bodies; the lower the temperature of the body, the longer is the wave-length of the heat radiation given out by it.

The electric waves used in wireless telegraphy are of the same character as heat waves, only very much longer still, hundreds of metres in length.

Thus the spectrum that we see is only part of a much larger invisible spectrum; light in the widest sense of the word is a wave-motion, the properties of which vary with the wave-length, which produces heating and chemical effects, and is visible only for a comparatively short range of wave-lengths.

EXAMPLES 32

1. A prism is to be made of crown glass, the refractive index of which is 1·517, and is required to produce a minimum deviation of 20°. To what angle must it be ground?

$$\mu \sin \tfrac{1}{2}A = \sin \tfrac{1}{2}(A + \delta) = \sin \tfrac{1}{2}A \cos \tfrac{1}{2}\delta + \cos \tfrac{1}{2}A \sin \tfrac{1}{2}\delta$$
$$\mu \tan \tfrac{1}{2}A = \tan \tfrac{1}{2}A \cos \tfrac{1}{2}\delta + \sin \tfrac{1}{2}\delta$$

hence $$\tan \tfrac{1}{2}A = \frac{\sin \tfrac{1}{2}\delta}{\mu - \cos \tfrac{1}{2}\delta} = \tan 18^\circ\ 5'$$

and $$A = 36^\circ\ 10'$$

2. In determining the index of refraction of a glass prism by the spectrometer for sodium light A is found to be 44° 46′ and δ to be 29° 32′. What is the value of the index of refraction?

3. If the index of refraction of a glass prism for sodium light is 1·517 and the angle of the prism 59° 57′, what is the minimum value of the deviation it produces?

4. A glass prism has a refracting angle of 30°. Its index of refraction is 1·517. A ray of light falls perpendicularly on one face of the prism, enters the glass, and is refracted at the other face. What is the value of the deviation produced?

5. If the wave-lengths of the extreme red and violet rays of the spectrum are $7{\cdot}6 \times 10^{-5}$ cm. and $4{\cdot}0 \times 10^{-5}$ cm. respectively, and the velocity of light is 3×10^{10} cms./sec., calculate the values of their periods.

33. THE EYE AND COLOUR VISION

The Eye.—The eye consists practically of a spherical chamber with a circular opening in the front; by means of a system of lenses the light entering this opening forms an image on the back of the chamber, just as in the case of a photographic camera.

Fig. 154 represents a section of the human eye. It is surrounded by a coating S called the sclerotic. The front portion of this coating, C, is transparent and is called the cornea. L is the crystalline lens, which is attached to the walls of the eye by the ciliary muscle. In front of the lens is a diaphragm, I, called the iris, which is coloured; it is the colour of the iris that is meant when the colour of an eye is referred to. In the centre of this diaphragm there is a circular aperture called the pupil. The space A between the lens and cornea, *i.e.* the anterior chamber, is filled with a watery liquid containing a little salt in solution, which is called the aqueous humour. The space V behind the lens, *i.e.* the posterior chamber, is filled with a transparent gelatinous substance termed the vitreous humour.

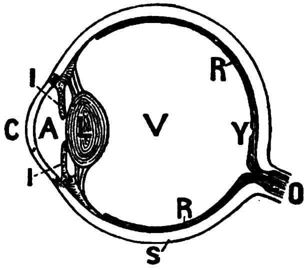

FIG. 154.
From Watson's Physics.

After passing through the cornea, aqueous humour, lens and vitreous humour, the rays fall on the retina R. The retina consists principally of a network of nerve fibres connected with the brain by the optic nerve O. In the retina there are two kinds of vision cells, termed rods and cones. In the retina directly behind the pupil is the yellow spot Y, or macula lutea, which has a depression in the centre called the fovea centralis; here vision is most distinct. There are no rods in the fovea centralis. The point in the retina at which the optic nerve enters is not sensitive to light, and is called the blind spot.

Usually, when the eye is at rest, it is adjusted so that the image of a distant object is in focus on the retina. If then it is turned to a nearer object, its pose must be altered, it must be accommodated to the nearer object in order that the latter may be clearly seen. This is done by a forward motion of the lens and an increase in curvature of both its surfaces.

Defects of Vision.—There are four defects of the eye of frequent occurrence, which are remedied by the use of spectacles. These are (1) Myopia, or Short Sight; (2) Hypermetropia, or Long Sight; (3) Presbyopia; (4) Astigmatism.

In myopia, or short sight, the focal length of the eye is too small, and parallel rays are brought to a focus in front of the retina. Consequently distant objects cannot be seen distinctly. Near objects can be seen quite well, and the distance of distinct vision is less than in the case of the normal or emmetropic eye. The defect is corrected by the use of a concave spectacle lens.

In hypermetropia, or long sight, the focal length of the eye is too great, and when the eye is relaxed, parallel rays are brought to a focus behind the retina. By an act of accommodation the focus can be brought on to the retina in the case of distant objects, and consequently they can be seen clearly. But, even when the lens is curved as much as possible, the images of near objects still fall behind the retina and appear blurred. In the hypermetropic eye the distance of distinct vision is greater than for the normal eye. Hypermetropia is corrected by the use of a convex spectacle lens.

Presbyopia is a loss of accommodating power occurring in the case of elderly people owing to the stiffening of the muscle that alters the curvature of the crystalline lens. People afflicted with it can see distant objects clearly, but not near objects. Sometimes the accommodation almost entirely disappears. Convex spectacle lenses, usually powerful ones, must be used for reading and writing, and less powerful ones for getting about with.

In astigmatism the focal length of the eye is different in two planes at right angles to one another, that is, if on a card two sets of parallel lines are ruled at right angles to one another, and the card is rotated so that the directions of the lines correspond with the above two planes, only the one set of lines can be seen distinctly at once. If the other set is to be seen distinctly, the accommodation of the eye must be altered and the first set put out of focus. Astigmatism is due to the surfaces of the lens and the cornea, principally the surface of the cornea, not being symmetrical about their axis. It is corrected by the use of cylindrical, sphero-cylindrical, and toric lenses. A sphero-cylindrical lens is one, one surface of which is spherical and the other surface of which is cylindrical. A toric or toroidal, or more strictly a plano-toric lens, has one surface plane and the other with different curvatures in its two principal meridians.

The power of spectacle lenses is always measured in diopters. The power of a lens in diopters is obtained by dividing 40 by its focal length in inches or, more accurately, by dividing 100 by its focal length in cms. A + is prefixed to the result if the lens is convex, and a − if it is concave.

Colour Mixing.—If we mix the red and green paints in a water-colour paint box, we get a dirty brown. But if we take a circular disc, divide its surface into two sectors as in Fig. 155, paint one sector red and the other green, and rotate the disc at a high speed, the two colours merge owing to the rotation, and, if the angles of the sectors have been properly chosen, the disc appears yellow. This is because, when the disc is rotated, the two colours are added, but, when pigments are mixed, we obtain the colour absorbed by neither of them, which is quite a different thing. In experimenting with the colour disc it is not necessary to restrict ourselves to two colours; we may use three or more sectors, each coloured differently. It is difficult to get colours which are spectrally pure, *e.g.* to obtain a red paint which does not contain any orange or blue in it; the colour of paints is always somewhat paler than the corresponding colour in the spectrum. But, if we make allowance for this, we find that any colour whatever can be reproduced on the colour disc by mixing three colours, namely, red, green, and violet. The method of proceeding is to get discs of these three colours, cut a slit along a radius of each, and fit each two through the slit of the other. A trial with three paper discs will explain what is meant. We thus obtain a disc with a red, a green, and a violet sector, each of variable angle, and we find that by rotating this disc and taking sectors of different angle, we can reproduce any colour we please. Red, green, and violet, or red, green, and blue, for the experiment succeeds almost as well with blue instead of violet, are therefore termed primary colours.

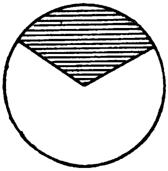

FIG. 155.

The application of the theory of primary colours to printing is shown by the last diagram in the frontispiece. The first patch, the yellow one, reflects red and green and absorbs the other primary, blue; the next patch, the carmine one, reflects red and blue and absorbs green; and the third patch reflects blue and green and absorbs red. The fourth patch is obtained by printing the third on the top of the first, and reflects only green; the fifth and sixth, which are obtained by printing the second on the first, and the third on the second, reflect respectively red and blue. In printing the spectra four colours were used; the reproduction of a spectrum is too severe a test of three-colour printing.

Two colours are said to be complementary if, when added together by the colour disc, they give white.

Colour Vision.—It is well known that many people are colour blind. The colour blindness may be slight, and reveal itself only as a difficulty in distinguishing blues from greens, or it may be of such a nature that blood red cannot be distinguished from grass green. Engine drivers and navigating officers have their colour vision tested to ensure that they can distinguish the signal lights. The two principal methods of testing are by means of Holmgreen's wool test and Edridge-Green's lantern test. The wool test simply consists in giving the man under examination a great number of different-coloured skeins of wool, and asking him to match given colours. The colour-blind man makes wrong matches. In the lantern test the case of the lantern has a hole of variable diameter, in front of which coloured glasses can be put in succession, so as to give the same appearance as a signal light under difficult conditions—mist, great distance, etc.—and the man under examination is asked to state rapidly, what the colour is.

Formerly the Young-Helmholtz theory of colour vision was in favour. According to this theory there were in the retina three sets of nerves corresponding to the three primary colours. If one set was wanting, the eye was colour blind. Thus cases were described as red blind, green blind, or violet blind. But there is nothing in the structure of the retina to justify the assumption that there are three different sets of nerves, and for this and other reasons the Young-Helmholtz theory is not now so popular as it was formerly.

EXAMPLES 33

1. A long-sighted person can see distinctly only objects which are at a distance of 50 cms. or more. Find graphically the power of spectacle lenses which will enable him to see distinctly objects at a distance of 30 cms.

2. A person whose distance of distinct vision is 75 cms. employs convex spectacles with a power of 3 diopters. Find graphically what his distance of distinct vision becomes.

3. A short-sighted person sees printed matter most distinctly when it is at a distance of 10 cms. from his eyes. Find graphically the power of spectacle lenses which will change his distance of distinct vision to 30 cms.

4. What is the apparent colour of a red body, when it is in a room to which only green light is admitted ?

CHAPTER VI

ELECTRICITY AND MAGNETISM

34. MAGNETISM. FUNDAMENTAL FACTS

In ancient times, in the province of Magnesia in Asia Minor, an iron ore was found, fragments of which had the property of attracting small pieces of iron such as filings or nails. They were hence called magnets or lodestones. Their chemical constitution is represented by the formula Fe_3O_4, that is, they consist of equivalent proportions of the two oxides of iron, FeO and Fe_2O_3.

Lodestones as found in nature have irregular forms. They do not exhibit the characteristic property equally all over their surface, but if they are dipped into a box containing iron filings, the filings adhere in clusters to corners and edges, leaving other parts clear.

Artificial Magnets.—If a small bar of steel, for example, an ordinary sewing-needle, is drawn across a corner of a lodestone several times, always in the same direction, it also acquires magnetic properties, which may be even more strongly marked than in the case of the original lodestone. It is then said to be an artificial magnet. All the magnets used nowadays are artificial magnets, but they are made, as will be explained on p. 204, by means of an electric current; lodestones, and the method of making magnets by stroking them on lodestones, are now only of historical interest. Consequently, when we speak of magnets, it is always artificial magnets that are referred to.

Poles.—If a magnetized needle or bar magnet is dipped into filings, they adhere to it in two tufts, one at each end. Fig. 156 shows the nature of the tufts in the case of a flat bar magnet. The ends of the magnet, where the power of attracting filings is apparently situated, are called its poles. If a magnetic needle is suspended in a horizontal position, so that it is free to turn about a vertical axis through its midpoint, and if there is no iron near it, it always comes to rest in this country with its length pointing some degrees west of north. It is always the same end which points towards the north. If the magnet is displaced from its equilibrium position, it

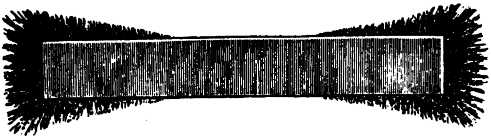

Fig. 156.

oscillates about it before coming to rest. The pole which points towards the north is called the north pole or north-seeking pole of the magnet; the other one is called the south pole or south-seeking pole.

If a magnet is suspended, as shown in Fig. 157, and the north pole A of another magnet brought near its north pole *a*, the latter is repelled. If the south pole of the other magnet is brought up to *a*, the latter is attracted. If the north pole A is brought near the south pole *b*, the latter is attracted. If the south pole of the other magnet is brought near *b*, the latter is repelled. These results may be summed up in the following rule:—

Like poles repel, unlike poles attract.

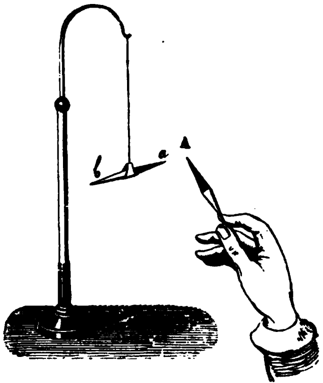

FIG. 157.

Magnetic Induction. — When one pole of a strong magnet is held near a small piece of soft iron, the latter jumps up to meet it. This is because the mere presence of the pole is sufficient to convert the piece of soft iron into a magnet with its nearer end a pole of the opposite kind; the two unlike poles then attract one another, and the piece of soft iron jumps up to adhere to the pole. The soft iron is then said to possess induced magnetism. This induced magnetism is almost entirely of a temporary nature; when the soft iron is taken to a distance from the magnet, it loses nearly all its magnetism. But when a steel needle is magnetized, it retains its magnetism permanently.

When a magnet is dipped into a box of filings, we have little chains of filings formed. The first filing of each chain sticks to the magnet, the second filing to the end of the first filing, the third filing to the end of the second filing, and so on. Each of the filings becomes a little temporary magnet owing to the inductive action of the magnet upon it.

In addition to iron and steel, the metals nickel and cobalt are also magnetic, but to a much less extent. When iron, steel, nickel, and cobalt are heated to a red heat, they lose their magnetism altogether.

Unit Quantity of Magnetism.—Magnetic poles always occur in pairs, one at each end of the magnet. If a long, thin magnetized knitting needle is broken into two, new poles appear one on each side of the break, a south pole appearing on the piece which contains the original north pole, and a north pole appearing on the piece which contains the original south pole. Therefore we cannot experiment with one pole by itself.

If, however, we take two long magnetized knitting needles, and bring the end of the one close up to the end of the other, the effects of the distant poles can be neglected, and the force between the magnets is almost wholly due to the two poles close together. By experiments made in this way it is found that the force between two poles is proportional to the product of their strengths, and inversely proportional to the square of the distance between them. This result is known as Coulomb's law.

Two like poles of equal strength are said to have unit quantity of magnetism, when they repel one another at a distance of one centimetre with a force of one dyne, both being in air.

If m is the pole strength of a magnet, and $2l$ the distance between the poles, the product $2ml$, usually denoted by M, is defined to be the magnetic moment of the magnet.

A compass needle, provided there is no iron near, always points the same way. There must consequently be forces on its poles, a force on the north pole pulling it towards the north, and a force on the south pole pulling it towards the south. These forces are said to be due to the horizontal component of the earth's magnetic field, the strength of which is usually denoted by H. If the pole strength of the magnet is m, the northward force exerted by the earth's field on the north pole is Hm, and the southward force exerted by the earth's field on the south pole is also Hm. It can be shown that the forces exerted on the two poles are equal and opposite by mounting the compass needle on a cork floating on water, instead of on an agate point. For the needle merely turns round under the influence of the field; it does not move as a whole in the northward or southward direction, as it ought to do, if the force on the one pole were stronger than the force on the other.

Mapping a Magnetic Field.—The space round a magnet in which it influences the deflections of a compass needle is called its field, and the mapping of a magnetic field is generally the first experiment performed by a student in magnetism. The method of procedure is usually as follows:—

A bar magnet about 15 cms. long, or thereabouts, is placed on a large sheet of drawing paper, which is fastened down on the table. Then a compass needle is placed on the paper near the north pole of the magnet, and, after it comes to rest, marks are placed on the paper opposite the poles of the compass needle. The needle is then shifted so that its first pole comes opposite the second mark, and a third mark drawn opposite its second pole. It is then shifted a second time, so that its first pole comes opposite the third mark, and a fourth mark drawn opposite the second pole, and so on. By proceeding in this way a series of marks, 1, 2, 3, 4, etc. (cf. Fig. 158), is obtained, and on joining these marks a closed curve is obtained extending from the north to the south pole of the magnet. Such a curve is called a line of force; it is tangential everywhere to the direction of a compass needle placed upon it, and if a small north pole were placed by itself on the curve near N, the curve gives the path it would be repelled along to S.

Then, when the one curve is completed, a fresh start is made at another point and another curve drawn, and so on, until the paper is filled with curves.

The appearance of the resulting diagram depends on the position in which the bar magnet was originally placed with reference to the earth's magnetic field. In Fig. 158 the direction of the earth's field is given by the thick arrow marked H; this is the direction in which the compass needle would set itself over the whole paper, if the bar magnet were removed. So Fig. 158 represents the lines of force for the case in which the bar magnet is placed parallel to the earth's field with its north pole pointing north. Of course, only one half of the field is shown; the lines of force have exactly the same shape on the other side of the magnet.

It will be noticed that the curves of force between P and the magnet go the one way, but beyond P they go the other, *i.e.* between P and the magnet the magnet's field is the stronger, but beyond P the earth's field is the stronger. If the compass needle is slowly moved out along the dotted line CP, which is drawn perpendicularly through the centre of the magnet, and the case is tapped gently at the same time, at P the direction of the needle reverses. P is termed the neutral point. At P the earth's field balances the field of the magnet.

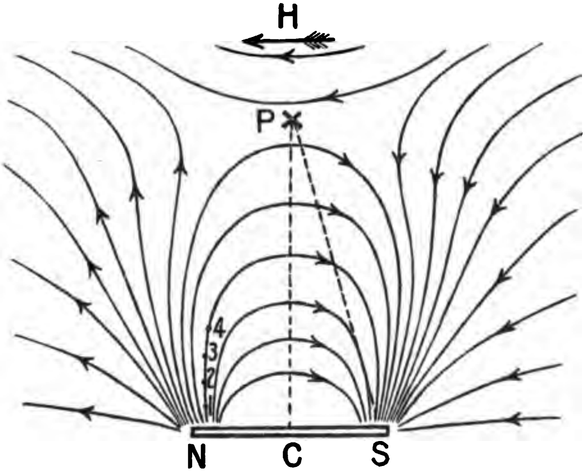

FIG. 158.

Let the distance between the poles of the magnet be $2l$, let the strength of its poles be m, and let CP = d. Then, if a unit north pole is placed at P, the earth's field will act on it with a force H, and the south pole of the magnet will act on it with a force m/PS^2 in the direction PS. If we regard △PCS as a triangle of forces, the force along PS can be regarded as equivalent to a force along PC and a force along CS. The latter is equal to

$$\frac{\mathrm{CS}}{\mathrm{PS}}\cdot\frac{m}{\mathrm{PS}^2}=\frac{ml}{(d^2+l^2)^{3/2}}$$

But the unit pole at P is also acted on by a force m/NP^2 along NP. This also resolves into two components, one of which neutralizes the force already existing along PC, and the other of which is equal to the force above.

The resultant force exerted by both poles on the unit positive pole at P is consequently

$$\frac{2ml}{(d^2 + l^2)^{3/2}}$$

and for the neutral point we have

$$H = \frac{2ml}{(d^2 + l^2)^{3/2}}$$

Hence if H is known, and d and l are measured, m can be calculated. It should be stated, that in the case of a thick magnet the poles are some distance from the ends.

Provided the magnet is strong enough, there are always two neutral points, no matter how it is placed with reference to the earth's magnetic field. But the above formula holds only for the one particular case.

Couple on a Magnet in the Earth's Field.—Let NS represent a magnet of pole strength m and length $2l$, pivoted at its middle point O, and let the arrow NA represent the direction of the earth's magnetic field. Then on the north pole of the magnet there is a force = Hm in the direction of the arrow NA, and on the south pole of the magnet there is a force = Hm in the direction of the arrow SB. The moments of these forces about the pivot are respectively HmOC and HmOD. Let θ be the angle which SN makes with the direction of H, *i.e.* $\theta = \angle$ONC. Now OD = OC = ON sin ONC = $l \sin \theta$. Hence the sum of the moments of the two forces about O is

$$2Hml \sin \theta = HM \sin \theta$$

where M is the magnetic moment of the magnet.

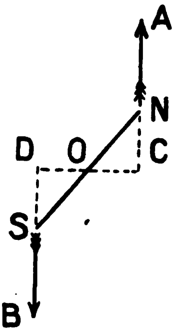

FIG. 159.

A system of two equal and parallel forces acting in opposite directions, such as is represented in Fig. 159, is called a couple, and the moment of the couple is defined as the product of either force and the perpendicular distance between the lines of action of the two forces. It is thus equal to the sum of the moments of the two forces, and measures the power of the couple to produce rotation about the axis. The moment of a couple is sometimes referred to as its magnitude.

It will be noticed that the moment of the couple is a maximum when the magnet is at right angles to the earth's field, and is zero when the magnet is parallel to the earth's field.

EXAMPLES 34

1. With what force will a north pole of strength 5 units (i) attract a south pole of strength 8 units at a distance of 4 cms., (ii) repel another north pole of strength 6 units at a distance of 5 cms.?

2. The repulsive force between two poles is 20 dynes when they are 5 cms. apart; what is it when this distance is increased by 1 cm.?

3. Two thin straight magnets each 12 cms. long are laid in one straight

line with their north poles facing one another and 4 cms. apart. Calculate in dynes the repulsion between the two magnets on the assumption that the poles are at the ends of the magnets, and that the pole strengths are 20 and 30 units respectively.

$$\text{Repulsion between N. poles} = \frac{20 \times 30}{4^2} = 37{\cdot}50 \text{ dynes}$$

$$\text{Repulsion between S. poles} = \frac{20 \times 30}{28^2} = {\cdot}76 \text{ dynes}$$

$$\text{Attraction between a N. and a S. pole} = \frac{20 \times 30}{16^2} = 2{\cdot}34 \text{ dynes}$$

$$\text{Resultant repulsion} = 37{\cdot}50 + {\cdot}76 - 2 \times 2{\cdot}34 = 33{\cdot}58 \text{ dynes}$$

4. A magnet of moment 450 c.g.s. units and magnetic length 20 cms. is placed horizontal and at right angles to the magnetic meridian. Find the resultant horizontal force on a unit pole placed on the axis of the magnet produced and at a distance of 10 cms. from the north pole. H = ·17.

5. A bar magnet of pole strength m, the distance between the poles of which is $2l$, is placed in the magnetic meridian with its south pole pointing north. Show that the neutral points are situated on the axis of the magnet at a distance d from its midpoint, where d is given by

$$H = \frac{4mdl}{(d^2 - l^2)^2}$$

6. Prove that the intensity of the magnetic field due to a bar magnet at a point on the axis produced a great distance from the magnet, is approximately twice the intensity at a point the same distance from the centre of the magnet, but on a line at right angles to its length through the centre.

7. Prove that the magnetic force due to a small magnet at any point on its axis produced is nearly inversely proportional to the cube of the distance between the point and the centre of the magnet.

The force at a point on the axis distant d from the centre is

$$\frac{4mdl}{(d^2 - l^2)^2}$$

If l^2 is small in comparison with d^2, it can be neglected, and the formula becomes

$$\frac{4mdl}{d^4} = \frac{4ml}{d^3}$$

8. A magnet 10 cms. in length lies in a field of intensity H = ·18, and the strength of each of its poles is 6 units. Find the moment of the couple required to deflect it (i) through an angle of 30° from the magnetic meridian, (ii) through an angle of 60° from the magnetic meridian.

9. A cylindrical magnet 20 cms. long and 9 mms. in diameter was placed in the magnetic meridian with its north pole pointing north, and the neutral points were determined to be 27·5 cms. from the magnet. Calculate the pole strength, given that the poles are distant 1 cm. from the ends of the magnet and that H = ·17 c.g.s. unit.

10. A similar magnet is placed in the magnetic meridian at the same place, but with its north pole pointing south. The neutral points are found to be in the line of the magnet at a distance of 30 cms. from its midpoint. Calculate the pole strength of the magnet.

11. A little cylindrical magnet 10 cms. long has a pole strength of 68 units. The poles are 1 cm. from the ends, and the diameter of the magnet is about 3 mms. Find the position of the neutral points, when the magnet is placed in the magnetic meridian with its north pole pointing north, given that H = ·17.

35. TERRESTRIAL MAGNETISM

Declination.—It has been mentioned on p. 162 that in this country when a compass needle comes to rest, it points some degrees west of the geographical north. The direction in which it points is called the magnetic north, and the angle between the magnetic north and the geographical north at any place is called the declination or variation at that place. The declination may also be defined as the angle between the geographical meridian and the magnetic meridian at a place. The geographical meridian is the plane through the vertical at the place and the earth's axis; the magnetic meridian is the plane through the vertical and the direction in which the magnetic needle points at the place.

The declination varies from place to place. For example, in 1909 it was 18° 30′ W. at Eskdalemuir, 15° 48′ W. at Greenwich, 0° 2′ E. at Hong Kong, and 9° 26′ E. at Honolulu, *i.e.* at the two latter places the needle pointed east of the geographical north. The change in declination throughout a country can be represented by maps in which places having the same declination are joined by curves; such curves are called isogonal lines.

The declination also varies from time to time. The following table gives some of its values for London during the time within which observations have been carried out there :—

1580	11° 15′ E.	1795	23° 57′ W.
1634	4° 6′ E.	1820	24° 34′ 30″ W.
1657	0° 0′ E.	1860	21° 39′ 51″ W.
1692	6° 30′ W.	1880	18° 57′ 59″ W.
1748	17° 40′ W.	1893	17° 27′ W.

This change throughout the years is called the secular change. In addition to the secular change, the declination is subject to small periodic annual and daily changes.

Dip.—The ordinary compass needle turns about a vertical axis, and is balanced so as to remain horizontal. Suppose, however, that an unmagnetized needle is mounted free to rotate about a horizontal axis through its centre of gravity, so that it balances in any position in which we care to set it, and that it is then magnetized; as soon as it is magnetized, it comes to rest only in one position making a definite angle with the horizontal. The magnitude of this angle depends on the vertical plane in which the needle happens to be. If the needle is in the meridian, the angle is a minimum, and is then called the dip, and the needle itself is called a dip needle. When placed in the magnetic meridian at any place, the dip needle gives the direction of the earth's magnetic field at that place. Like the declination, the dip is also subject to secular, annual, and diurnal changes, and varies from place to place, and its variation over the surface of the earth for any one time can be represented by a map, in which places of equal dip are joined by lines known as isoclinal lines.

If such a map of the world is examined, it will be found that the earth has a magnetic equator for which the dip is zero, of irregular form but roughly coincident with the geographical equator; as we move away from the magnetic equator the dip increases, until in the northern hemisphere it becomes 90°, *i.e.* the dip needle points vertically downwards, at a point to the north of Canada, and in the southern hemisphere it becomes 90° at a point in the Antarctic continent to the south of Australia.

In the middle ages the pointing of the compass needle towards the north was ascribed to an attraction by the pole star. The fact that the dip needle is vertical at two places on the earth's surface shows that the earth itself is a magnet, and that these places are its magnetic poles. It should be noted that the earth's north magnetic pole is really a south pole, since it attracts the north or north-seeking pole of the compass needle, and that similarly the earth's south magnetic pole is really a north pole, also that the earth's magnetic poles are not nearly at the ends of the same diameter.

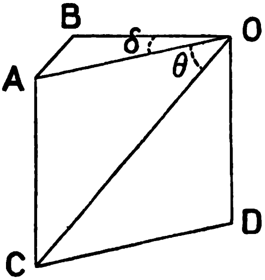

FIG. 160.

Magnetic Elements.—When H, the horizontal component of the earth's field, together with the declination and dip, is known, the magnetic intensity at a place is fully determined. These three quantities are called the magnetic elements. Their values at Eskdalemuir in Dumfriesshire, where the principal magnetic observatory in this country is situated, in order to get it away from all disturbing influences, were in 1917 respectively ·167, 17° 20′, and 69° 38′, and their values at Glasgow are at present about ·16, 19°, and 70°.

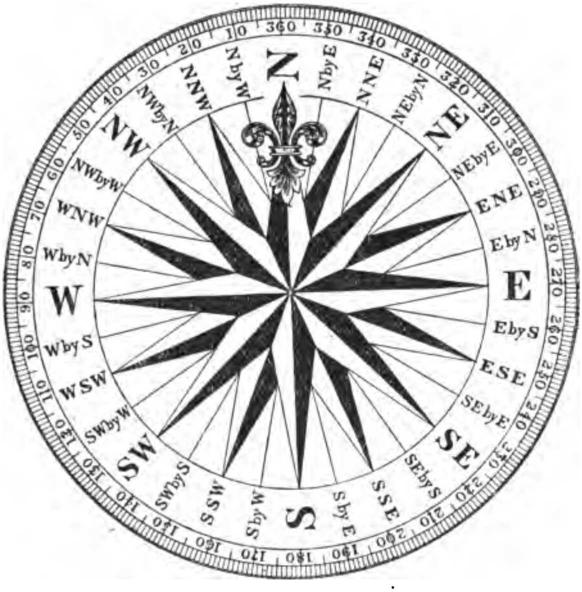

FIG. 161.

Let δ be the declination and θ the dip. Then it is clear, from Fig. 160, that the vertical component of the earth's field is H tan θ, that its resultant value is H sec θ, and that its component in a horizontal direction parallel to the geographical meridian, *i.e.* along OB, is H cos δ.

Magnetic Storms.—At a magnetic observatory the magnetic elements are recorded continuously by self-recording apparatus, and on certain days they are found to undergo sudden and irregular changes. A "magnetic storm" is then said to take place. The cause of these storms has not been determined, but they occur when there is a large number of spots on the sun, and are often accompanied by the Aurora Borealis.

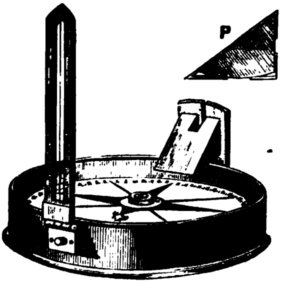

FIG. 162.

Magnetic Compass.—Fig. 161 shows the card of a ship's compass. The magnetic bars, which form the needle of the compass, are attached to the lower side of the card, and are pivoted by means of an agate cap. When they turn, the whole card turns.

Fig. 162 shows a prismatic compass, which is used for surveying and military purposes. Its case is a shallow metal box about $2\frac{1}{2}$ inches in diameter. To use it to take the bearing of a distant object the eye looks through the slit above the prism and gets the distant object in line with the vertical wire in the upright frame; the reading of the card is then seen reflected in the prism directly below the distant object. A section of the prism is shown at P; its under surface is curved so as to magnify the reading of the card.

EXAMPLES 35

1. When a needle is magnetized, does its weight suffer an apparent increase?
2. Graph the figures given on p. 168 for the declination at London, and hence find the value in 1622 and 1802.
3. Calculate the numerical value of the vertical component and resultant of the earth's magnetic field at Eskdalemuir from the data supplied on p. 169.
4. How will a dip needle set itself when its axis of rotation is in the magnetic meridian at the magnetic equator?

36. ELECTROSTATICS

Fundamental Experiments.—It was discovered by Thales (600 B.C.), that when amber was rubbed it acquired the property of attracting threads and other light objects. It was found by Gilbert about the year 1600 that this property was shared by many other substances such as glass, sealing wax, etc., and he named the effect produced on these substances by rubbing them "electrification," after elektron, the Greek word for amber.

The easiest way of exhibiting the effect is to tear off some little squares of paper about 2 mms. side, and rub the end of a vulcanite fountain pen on your sleeve. It then becomes electrified, and if it is brought near the little bits of paper, the latter jump up and adhere to it.

If a pith ball is suspended by a dry silk thread like the bob of a pendulum, and if a glass rod is rubbed with a piece of silk and held towards the pith ball, the latter is attracted towards the glass rod, touches it, and then is repelled by it. If another pith ball is suspended by a dry silk thread, and a stick of sealing wax rubbed with dry flannel and held towards this other pith ball, the latter is attracted towards the wax, touches it, and then is repelled by it. But if the glass rod is then held towards the second pith ball, it attracts it, and, if the sealing wax is held towards the first pith ball, it also attracts it. Manifestly, the pith ball electrified by the glass rod is in a different state from the pith ball electrified by the sealing wax.

These experiments are fundamental. They are explained by assuming that all matter is permeated with a subtle fluid called negative electricity, that this fluid repels itself but is attracted by matter, and consequently spreads throughout the volume of any matter with which it is associated. When a body contains its proper quantity of this fluid, uniformly distributed, it has no special properties. But if it has an excess of the fluid or, as we may put it, a charge of negative electricity, it repels a body with a similar excess, *i.e.* with a similar negative charge. If it has a deficiency of the fluid or, as it is put, a charge of positive electricity, it repels a body with a similar deficiency, *i.e.* with a similar positive charge. And a body with an excess of the fluid attracts a body with a deficiency of it, *i.e.* a negative charge attracts a positive one.

Formerly it was assumed that there were two fluids, a negative one and a positive one, and that in ordinary unelectrified matter these two fluids neutralized one another. But though we shall use the language of the two-fluid theory on account of its great convenience, this theory is not in accordance with modern thought. A body with a positive charge is merely one with a deficiency of negative charge; the negative charge is something that can be measured and investigated apart from ordinary matter, while the positive cannot. The laws of the behaviour of positive and negative charges can be summed up in the sentence—

Like repels like and attracts unlike.

Let us now consider the experiment with the pith balls. When the glass rod is rubbed with the silk, it loses negative electricity to the silk, and thus acquires a positive charge. When it is brought up to the first pith ball, the negative electricity on the latter is attracted to the side nearest the rod, the attractive force on the pith ball preponderates, and it is attracted so as to touch the rod. But as soon as it touches the rod, it acquires a positive charge from the latter, and so is repelled by it.

When the stick of sealing wax is rubbed with the flannel, it gains negative electricity from the flannel. When it is brought up to the second pith ball, the negative electricity on the latter is repelled to the side farthest from the stick, consequently the attractive force on the pith ball preponderates, and it is attracted so as to touch the stick. But as soon as it touches the stick, it acquires a negative charge from the latter, and is repelled by it. The second pith ball has finally a negative charge, and so is attracted by the glass rod, while the first pith ball has finally a positive charge, and so is attracted by the sealing wax.

When the approaching glass rod or stick of sealing wax causes a separation out of the charges on the pith ball, it is said to exert an inductive action on the latter.

The names positive and negative are quite arbitrary. There is no reason, except custom, why the charge given by the glass should be called positive rather than negative; hence Lord Kelvin always called it vitreous electricity and the other kind resinous electricity, since it can be produced by using resin instead of sealing wax.

The question arises as to why in the experiment with the fountain pen the pieces of paper stuck to the vulcanite, and were not repelled as soon as they touched. It is because the vulcanite becomes negatively charged, and at first repels all the negative electricity out of the paper into the table, and the negative electricity which flows into the paper from the vulcanite after contact is not enough to make up this deficiency. Consequently there is an excess of positive in the paper, and it remains sticking to the vulcanite.

Conductors and Insulators.—When a pith ball suspended by a dry thread of unbleached silk is given a charge of electricity, it retains this charge for a time. But if it were suspended by a thin copper wire, it would lose the charge at once. The silk thread is said to be a good insulator or dielectric or non-conductor, while the copper wire is a good conductor and conducts the electricity away at once. The best non-conductors are glass, ebonite, shellac, quartz, paraffin, sealing wax, and silk. The best conductors are the metals and solutions of inorganic salts in water. There is no sharp division between the classes; for example, dry wood and paper are semi-conductors. All the non-conductors can be electrified by rubbing; if a conductor were electrified, the charge would be conducted away at once. When a body is electrified by friction, the nature of the charge produced on it depends on the substance with which it is rubbed; thus glass, which acquires a positive charge when rubbed with silk, acquires a negative charge when rubbed with catskin.

Absolutely pure water would be a non-conductor, but water usually contains small quantities of dissolved matter which make it a conductor. Thus if the silk supporting the pith ball is damp, the latter does not retain its charge at all. Indeed, it is impossible to do experiments in electrostatics in a damp atmosphere; the charges conduct away so rapidly.

Gold-Leaf Electroscope.—Fig. 163 represents the gold-leaf electroscope, which is an instrument used for indicating the presence of an electric charge. It consists of two gold leaves hung from the lower end of a metal rod. This rod passes through an insulating plug in a glass shade, which protects the leaves from mechanical injury and air currents, and terminates above in a metal disc. The gold leaves, rod, and disc may be regarded as one conductor.

If the disc is touched for an instant by a glass rod which has been electrified in the usual way, a positive charge is communicated to the leaves, and they repel one another and diverge. If they diverge too far, they touch strips of tinfoil pasted inside the shade, which conduct away their charge to the earth, and cause them to collapse together again. The electroscope can be discharged at any instant by merely touching the plate with a finger; the charge then escapes through the body of the experimenter to the earth. If a charged electroscope is left to itself, the charge gradually leaks away through the atmosphere.

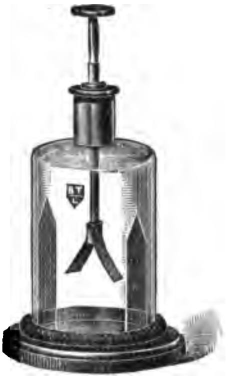

FIG. 163.

If the electroscope is discharged, and then a glass rod electrified in the usual manner is brought near the disc, but not allowed to touch it, the leaves diverge. They are then said to be electrified by induction. For the negative electricity in the disc, rod, and leaves is attracted to the disc by the positive electricity in the glass rod, and the leaves diverge with positive electricity. If now, while the glass rod is kept in position, the plate is touched for an instant with the finger, the positive charge on the leaves escapes through the body of the experimenter to the earth, and the leaves come together, while the negative charge on the disc is held in position by the attraction of the positive charge on the glass rod. Suppose now that the glass rod is removed; the negative electricity is no longer confined to the plate by the attraction of the positive charge on the glass rod, but spreads over the plate, rod, and leaves, so that the leaves diverge with a charge of negative electricity. The glass rod has thus been made the means of giving the electroscope a charge of the opposite nature to that which it possessed itself. When charged in this way the electroscope is said to be charged by induction.

Electrophorus.—Fig. 164 represents a section of the electrophorus, which is a simple apparatus used for producing and collecting larger quantities of electricity than can be obtained by rubbing sealing wax or glass.

It consists of a cake of resin or some similar substance about 12 inches in diameter and a half-inch thick, contained in a metal mould AB, and a metal disc or cover CD, of a diameter somewhat less than that of the cake, and provided with an insulating glass handle E. To use the electrophorus the cake is first well dried, and then rubbed with catskin. This gives its surface a charge of negative electricity. The cover is then placed on the top of the cake. Owing to the surface of the latter not being quite plane contact is made only at a few points; consequently, since the resin is not a conductor, its electricity does not pass to the cover, but acts inductively on the latter, causing a positive charge to separate out on the under surface of the

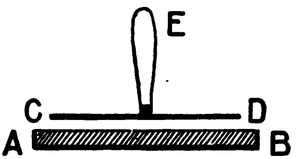

FIG. 164.

plate and a negative charge to appear on its upper surface. If the upper surface is then touched with the finger, this negative charge escapes through the body of the experimenter, and if the cover is raised by its insulating handle the positive charge is retained, and diffuses itself over the surface. If a finger is then brought near the edge of the cover, a sharp spark passes, or if the edge is brought near an unlit burner out of which gas is streaming, a spark passes to the burner and lights the gas.

The cover may be charged a considerable number of times without recharging the resin.

EXAMPLES 36

1. A piece of shellac is rubbed with flannel and used to touch a pith ball, which is suspended by a silk thread. A stick of sealing wax, which has also been electrified by rubbing it with flannel, is then made to approach the pith ball, and the latter is repelled. What is the sign of the charge upon the shellac?
2. If an insulated brass rod is rubbed with silk, it becomes electrified; if a brass rod held in the hand is rubbed with silk, it shows no signs of electrification. Why is this?
3. An electrified pith ball is attracted by an unknown body. Does this prove that the unknown body is initially charged with electricity?
4. A positive charge is given to a gold-leaf electroscope, and the leaves diverge. A charged body is made to approach the disc; the leaves approach each other and hang down straight, but when the body is brought closer still, without touching the disc, they diverge again. State the sign of the charge on the body, and explain the behaviour of the leaves.

37. CONDENSERS AND ELECTRICAL MACHINES

Distribution of Electrical Charge upon Conductors.—A proof plane consists of a small metal disc, about the same size as a sixpence, but thinner, fixed by sealing wax to the end of a thin glass rod; the glass rod forms an insulating handle. Figs. 165, 166, and 167 all represent metal conductors mounted on insulating glass pillars; in Fig. 165 the conductor is spherical, in Fig. 166 it is pear shaped and drawn out to a point on one side, and in Fig. 167 it is spherical and hollow, and has a little hole at the top.

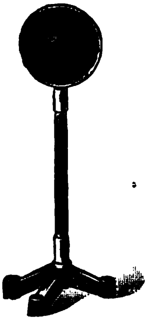

FIG. 165.

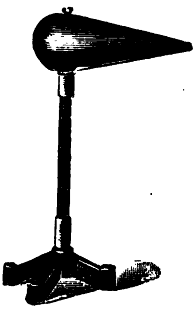

FIG. 166.

Now suppose that the spherical conductor is charged by an electrophorus, and that its surface is touched by the proof plane. Some of the electricity then flows into the proof plane; if the proof plane is then made to touch the disc of a gold-leaf electroscope, the leaves of the latter will diverge. If the electroscope and proof plane are discharged by touching them with the finger, and the proof plane is employed to bring a charge to the electroscope from any other point on the surface of the conductor, the gold leaves will diverge again to the same extent as before. Now the extent of the divergence is a measure of the quantity of electricity communicated to the electroscope. Hence the quantity of electricity taken from the two points on the surface of the conductor is the same, *i.e.* the density of the charge is the same over the whole surface, the electricity distributes itself uniformly over the whole surface.

If, however, the conductor represented in Fig. 166 is charged, and charges are taken from different points on its surface to the electroscope, the divergence is not the same in the different cases, but is greatest when the charge is taken from the point. This is a particular case of a general rule: if the conductor has any sharp points or edges, the electricity collects there, and is very apt to break away from the conductor into the air in virtue of its own self-repulsion.

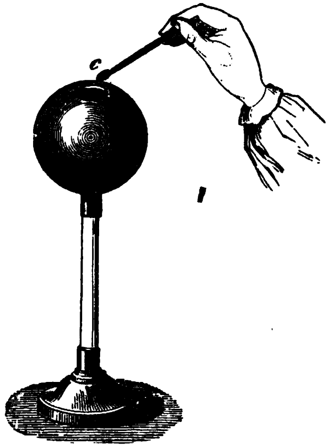

FIG. 167.

If the hollow conductor shown in Fig. 167 is charged, and the proof plane is inserted to touch the inside of the conductor, then withdrawn very carefully so as to prevent its touching the edge of the hole, and made to touch the disc of an electroscope, the leaves do not diverge at all. This shows that there is no charge on the inside of the sphere; the electricity resides solely on the outside of a conductor.

Potential.—Fig. 168 represents a pair of discharging tongs; the distance between the two knobs can be altered by changing the inclination of the arms. The handle is an insulating one of glass.

Now let us suppose that two insulated conductors of the pattern shown in Fig. 165 are each charged, and then put into communication by means of the discharging tongs, that is, one knob is made to touch the one conductor and the other knob the other conductor. In general electricity flows through the tongs from the one conductor to the other, and the question arises as to the direction in which this flow takes place.

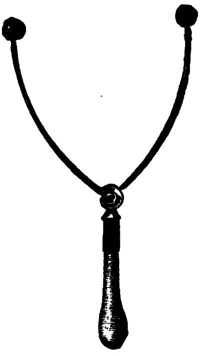

FIG. 168.

This experiment is analogous to two experiments in other branches of physics. Fig. 169 represents a section of two cylindrical vessels containing water connected by a tube, which is closed by a stopcock S.

When the stopcock is opened, water flows through the tube from the one vessel to the other. The water flows from the vessel in which the surface has the higher level, even though that vessel may contain less water than the other one.

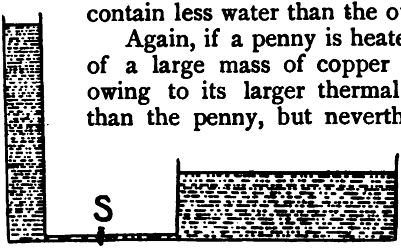

Fig. 169.

Again, if a penny is heated red hot and placed on the top of a large mass of copper at room temperature, the latter, owing to its larger thermal capacity, may contain more heat than the penny, but nevertheless the flow of heat is from the penny to the large mass, because the penny is at the higher temperature.

To return now to the electrical problem. When the two conductors are put into communication we attribute the direction of flow to a condition of the electricity called its potential, which bears very much the same relation to quantity of electricity, that pressure bears to quantity of water, or temperature bears to quantity of heat. The electricity flows from the conductor which is at the higher potential to the conductor which is at the lower potential, even though the latter may contain a larger quantity of electricity. The flow continues until the difference of potential between the two conductors is annulled. It follows that all parts of a conductor, in which the electricity is at rest, must be at the same potential.

Just as measurements of temperature are made with reference to some standard such as the melting point of ice, and as measurements of level are made with reference to sea level, so measurements of potential are made with reference to the potential of the earth. The earth is at the same potential throughout, at least those parts of it which are large in extent and conduct well, for example, the sea, or the gas and water pipes; its potential is taken as zero. The potential of a positive charge, of glass rubbed by silk, is above the potential of the earth; the potential of a negative charge, of sealing wax rubbed with flannel, is below the potential of the earth.

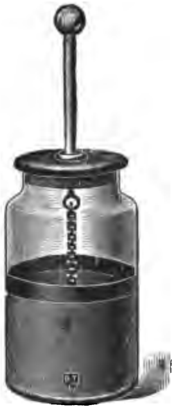

Fig. 170.

Capacity and Condensers.—The quantity of heat given to a body is obtained by multiplying its increase of temperature and its thermal capacity. If we have two bodies in contact, and an additional quantity of heat is given to them, it is distributed between them in proportion to their thermal capacities.

Just in the same way all conductors have electrical capacities. The quantity of electricity given to a conductor is equal to the product of its increase of potential and its capacity. The larger the conductor, the greater is its capacity, provided its shape is unchanged. But there is another method of increasing the capacity of a conductor, better than increasing its size, which forms the principle of the condenser. The best-known condenser, the Leyden jar, is shown in Fig. 170.

Fig. 171 illustrates the principle of the condenser. Let us first suppose that the plate CD is away. A plate AB is given a large charge of positive electricity. Owing to the self-repulsion of this charge it tends to escape from the plate given the least opportunity; the greater the potential, the greater is the tendency of the charge to escape, just as in the case of a heated body the higher the temperature, the greater is the tendency of the heat to escape from the body. Suppose now that the plate CD is placed parallel and close to AB, and put into conducting communication with the earth. The positive charge on AB induces a negative charge on CD; the attraction of this negative charge keeps the positive charge together and lowers its potential, consequently enabling AB to hold a greater quantity of electricity and thus increasing its capacity.

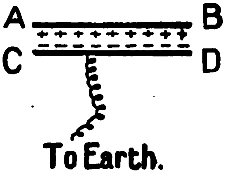

FIG. 171.

The Leyden jar consists of a glass jar, the inner surface of which is coated with tinfoil halfway up. This tinfoil corresponds to the plate AB. It is put into conducting communication by means of a chain with a brass rod surmounted by a knob, which is mounted in the insulating cover of the jar. The outer surface of the glass is also covered with tinfoil halfway up, this coating corresponding to the plate CD.

Frictional Machine.—Fig. 172 represents Winter's frictional electrical machine. It consists of a circular glass plate, which is made to revolve rapidly while pressed against a cushion covered with an amalgam of tin and zinc. As a result of the friction against this cushion positive electricity is generated on the surface of the glass. As the electrified part of the glass revolves, it passes between two wooden rings. The inner sides of these rings are each furnished with a row of sharp metal points, which are in conducting communication with the metal sphere supported on the insulating glass pillar to the left. The metal points come very close to the glass surface; consequently the positive electricity sparks from the surface of the glass, and collects on the sphere. After the handle has been rotated some time the charge on the sphere attains a maximum; any gain beyond this is balanced by leakage to the atmosphere and down the glass pillar.

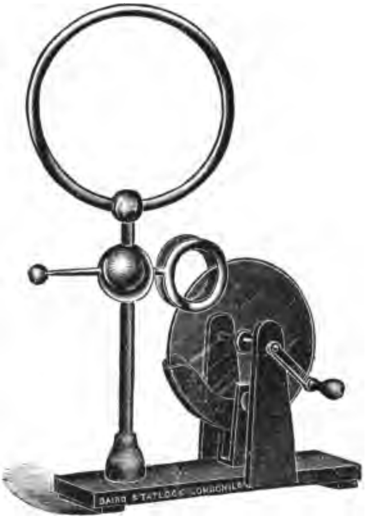

FIG. 172.

Experiments with a Leyden Jar.—If a Leyden jar is held with the hand touching its outer coating and the knob close to the spherical collector of the friction machine, or better still, close to the little conducting sphere to the left of the latter, sparks of positive electricity pass from the machine to the inner coating of the jar, and at the same time a negative charge is induced from the earth through the body of the experimenter to the outer coating of the jar. If the jar is then placed on the table, the outer coating touched with one knob of the discharging tongs, and the other knob of the tongs brought up to the knob of the condenser, when the distance between the two knobs is small, a bright spark passes between them with a sharp report. The charges on the two coatings of the jar then annul one another, and the jar is said to be discharged.

If the jar is charged and placed on a table, and the knob touched by the experimenter, a shock, which may be a severe one, is obtained, for the inner charge passes to the outer one through the body of the experimenter.

If, however, after charging the jar it is placed on an insulating stool, *i.e.* a little stool with glass legs, or placed on a large block of paraffin wax, which is a very good insulating material, the experimenter may touch the inner coating with impunity. The charge does not move; it is held in position by the attraction of the charge on the outer coating. But if the experimenter touches both coatings at the same time, he receives a shock the same as before.

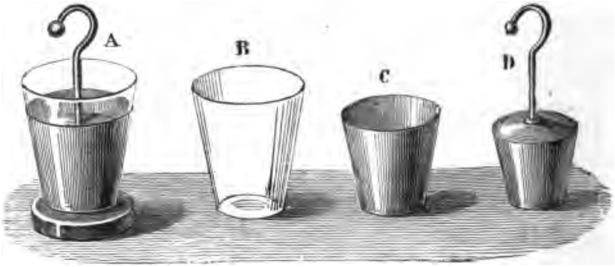

FIG. 173.

From Ganot's Physics.

When a Leyden jar is discharged, the energy of the jar, that is, part of the work which was done in turning the frictional machine, is dissipated in the heat, light, and sound of the discharge. The question arises as to where this energy was when the jar was charged. The answer to this question is given by an interesting experiment performed with the particular kind of jar shown in Fig. 173, both the inner and outer coatings of which, D and C, are made of thin metal sheeting, and can be detached from B, the glass jar itself.

After the jar is charged in the usual way, it is placed on an insulating block. The inner coating is next removed by the hand, or better, by a glass rod; then the glass vessel is removed, and finally the outer coating itself. The two coatings are next put into contact, and little or no spark passes. The jar is then built up again, and its coatings put into communication by means of the discharging tongs, when a spark and report are obtained, almost as strong as if the coating had not been removed. The energy of the charge is thus apparently stored up in the glass itself.

Wimshurst Electrical Machine.—Fig. 174 represents the Wimshurst electrical machine, which gives a much more powerful supply of electricity than the frictional machine shown in Fig. 172. It consists of two varnished glass discs on which strips of tinfoil are pasted. The discs are rotated at a high speed in opposite directions by a handle shown at the back of the figure. Two horseshoe conductors provided with rows of points collect the positive electricity from the tinfoil strips at the one side, and the negative electricity from the tinfoil strips at the other side; the electricity flows through chains to the Leyden jars, where it is stored up. In front of the front plate there is a diametral conductor provided with brushes at its ends; these brushes graze against the strips of tinfoil. The presence of a similar conductor, which makes contact with the tinfoil on the other plate, is indicated by dotted lines. In the front of the instrument are the sparking knobs with their vulcanite handles for regulating the length of the spark gap. When the machine is working, after the jars are filled, a constant stream of sparks passes between the knobs.

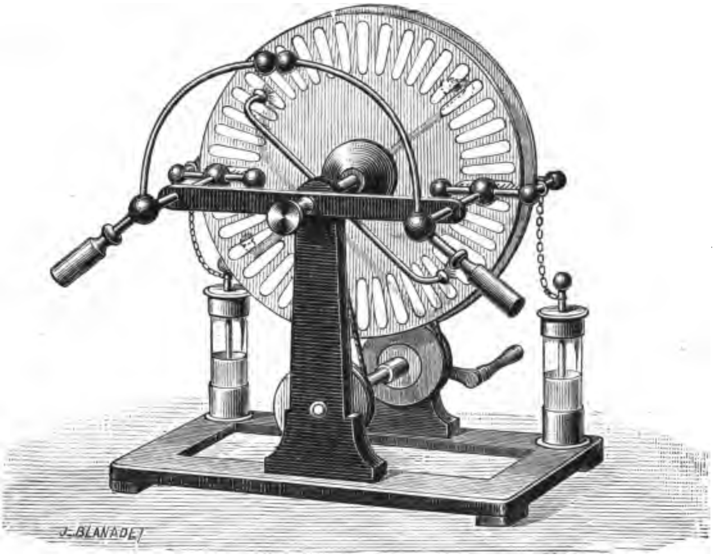

FIG. 174.

From Foster & Porter's Electricity.

Fig. 175 explains the working of the machine. The front plate is represented by the inner ring, and the back plate by the outer ring. MN and PQ are the two diametral conductors, E and F the combs for collecting the electricity, H and J the Leyden jars, and K the sparking knobs. The directions of rotation of the two discs are shown by the arrows. Let us suppose that initially the strip A has a negative charge, which may have been induced on it by the neighbourhood of a charged body; then this negative charge induces a positive charge on A_1, and a negative charge on C_1. As the discs rotate the negative charge on A is delivered up at F. When A_1 arrives at B_1 and C_1 at D_1, they induce a negative charge on B and a positive charge on D; then they themselves give up their charges to the combs at E and F. The positive charge on D is given up to the comb at E. Thus when the machine is working the outer ring receives a negative charge at B and a positive one at D, while the inner ring receives a positive charge at A_1 and a negative one at C_1, also the machine acts more powerfully as the motion goes on.

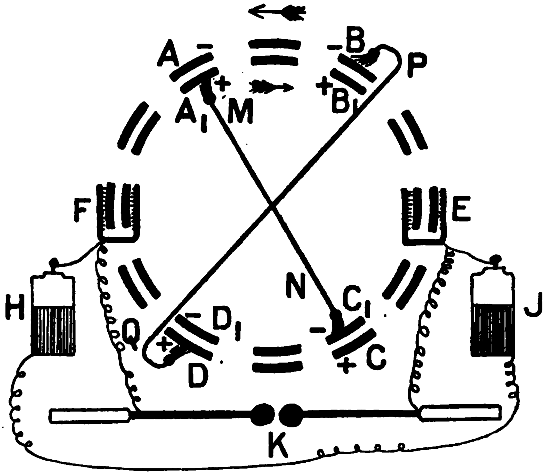

FIG. 175.

Lightning.—Clouds can become charged bodily with electricity. When a disruptive electric discharge takes place between two clouds or a cloud and the ground, we have lightning. Lightning is thus just a huge electric spark. The reverberation of thunder is the echoing and re-echoing of the original report from the clouds.

The lightning conductor, which was invented by Franklin, consists of a metal rod projecting above the highest point of the building which is to be protected, and in good conducting communication with a point in the earth, which is deep enough to be always moist. When a charged cloud is above the building, it induces an opposite charge into the lightning conductor, which escapes silently from the sharp point of the latter and neutralizes the cloud, or, if a discharge does take place, it strikes the rod in preference to the building, and the electricity is conducted to the earth without doing any damage.

EXAMPLES 37

1. Will a solid sphere hold a larger charge of electricity than a hollow one of the same diameter?

2. An experimenter wraps an insulating sheet of rubber round the outside of a Leyden jar, and holds it up to receive a charge from a Wimshurst machine, but the charge obtained is very small indeed. Why is this?

38. THE ELECTRIC CURRENT AND OHM'S LAW

Leclanché Cell.—Fig. 176 represents a Leclanché cell. It consists of a glass jar, inside which there is a porous pot containing a plate of carbon, packed round with a mixture of powdered carbon and manganese dioxide. To the top of the carbon plate is fastened a piece of lead in which a brass binding screw is fixed. The carbon plate is called the positive pole, or positive electrode, or positive terminal, of the cell. The negative pole consists of the zinc rod to the left. The cell is filled one-third full with a solution of ammonium chloride (sal ammoniac).

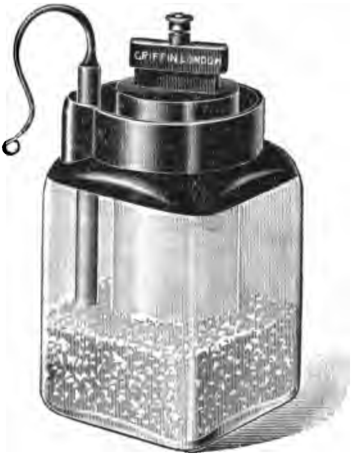

FIG. 176.

Electric Current.—If a magnet *ns* is pivoted so that it is free to rotate about a vertical axis, and a wire is fixed up with a straight piece AB in the magnetic meridian, horizontal, and close to *ns*, and if the ends of the wire are connected to the poles of the cell, the magnet is at once deflected. This experiment was first performed by Oersted in 1820. If instead of one cell two cells are taken in series, that is, with the positive pole of the one cell joined by a short piece of wire to the negative pole of the other and with the ends of the long wire joined to the free poles, then the deflection of the magnetic needle is increased.

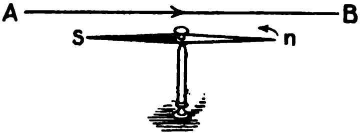

FIG. 177.

If the poles of a Leclanché cell are joined by a short piece of thin iron wire, the latter is felt to become warm as soon as the poles are joined.

Fig. 178 represents a voltameter, consisting of a U-shaped tube with stopcocks at the ends of its arms, with short pieces of platinum wire K and A sealed through the glass at the corners, and with a thistle funnel T at the middle for filling it. It is filled originally to the top of both arms with dilute sulphuric acid. If several Leclanché cells are arranged in series, and the free positive pole connected to the platinum wire at A, and the free negative pole connected to the platinum wire at K, as soon as the connections are made, bubbles begin to be given off inside the tube at K and A. These bubbles ascend the arms and collect, the volume of the gas formed in the one branch being exactly twice the volume of gas in the other. The gases are hydrogen and oxygen, the volume of the hydrogen being twice the volume of the oxygen, and the gases are formed by the decomposition of water. When the cells are disconnected the formation of bubbles stops.

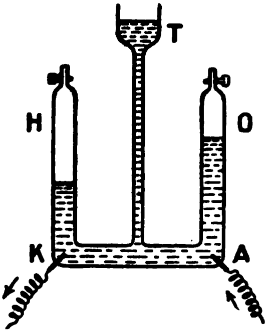

FIG. 178.

To sum up, then. If the two poles of a Leclanché cell are joined by a wire,

(i) a magnetic field is produced in the neighbourhood of the wire,

(ii) heat is evolved in the wire,

(iii) if the wire is cut in two, and its ends dipped into acidulated water, or indeed into any solution of an inorganic salt, chemical change takes place.

When these effects are produced, an electric current is said to pass in the wire. The direction of the current is said to be from the positive to the negative electrode of the cell along the wire.

The Swimming Man Rule.—The direction of the deflection of the needle in Oersted's experiment (Fig. 177) is given by Ampère's rule, which runs as follows:—

If a man is swimming in the wire in the direction of the current above the needle, breast stroke, then the north pole is deflected towards his left hand, the south pole being deflected in the opposite direction.

If the direction of the current is reversed, the direction of the deflection is reversed. If the wire is stretched below the needle instead of above it, the direction of the deflection is reversed. Hence if the wire is formed into a vertical rectangular coil, as in Fig. 179, and the needle placed at the centre, the effects of the lower sides of the rectangles reinforce the effects of the upper sides. A coil of wire arranged like this used to be termed a galvanic multiplier.

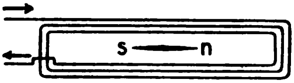

FIG. 179.

Magnetic Field of Electric Current.—Since a wire carrying a current deflects a compass needle just as a magnet does, it has a magnetic field just as a magnet has, and this magnetic field can be plotted in the same way as the magnetic field of a bar magnet, namely, by placing a compass needle on a sheet of paper, making marks opposite its ends, shifting it so that the first end comes opposite the second mark, and so on. The simplest case is when the paper is pinned on a table, and the wire is a long, vertical, straight one passing through a hole in the table and carrying a very large current. In this case the lines of force are simply a series of concentric circles with their common centre in the point in which the wire intersects the plane of the paper.

Unit of Current.—The magnitude of an electric current is defined by the strength of the magnetic field it produces. Unit current is such, that, when it flows in an arc 1 cm. long of a circle of radius 1 cm., it acts on a unit magnetic pole placed at the centre of the circle with a force of 1 dyne. The unit of current as defined in this way is called the theoretical unit, and is found to be rather large for practical use, so in practical work a unit one tenth as large, called the ampere, is used instead.

The practical unit of quantity of electricity is the coulomb, which is defined as the quantity of electricity passing any point in a wire in one second, when a current of one ampere is flowing in the wire. The theoretical unit of quantity is equal to 10 coulombs.

Unit of Electromotive Force.—Unit electromotive force is said to exist between two points in a wire, when work equal to one erg is done on unit quantity of electricity in moving it from the one point to the other point in the wire.

In this definition the unit quantity is the theoretical unit, and the unit of electromotive force thus defined is the theoretical unit. The latter is, however, found to be too small for practical use, and instead of it a practical unit called the volt and equal to 10^8 theoretical units is employed.

Connection between Frictional and Current Electricity.—It was stated on p. 176 that when two charged conductors were put into conducting communication electricity flowed from the one conductor to the other, from the conductor at the higher potential to the conductor at the lower potential. In the same way, when the Wimshurst machine shown in Fig. 174 is working, if the knobs on both sides of the spark gap are joined by a wire, electricity passes from the one knob to the other along the wire instead of passing across in sparks. The question arises, as to what is the difference between the passage of electricity in these two cases and the electric current produced by joining the terminals of a Leclanché cell by means of a wire.

Exactly the same thing takes place in each case, only the numerical values of the quantities involved are different. Difference of potential is exactly the same as electromotive force, and can be measured in volts. If we represent difference of potential by difference of level, the electric current produced by the Leclanché cell can be compared to a broad river flowing slowly and steadily with a fall of level of only a few feet per mile, while the current produced by joining the terminals of the Wimshurst machine is like a mountain torrent in which there is not much water, but the gradient of which may be 45°. The current produced by putting the conductors into conducting communication has the additional difference, that it lasts only for a very short time.

The maximum difference of potential of which the Leclanché cell is capable is about 1·5 volts. The smallest difference of potential that will affect an ordinary gold-leaf electroscope is about 100 volts. A rod of glass rubbed by silk may easily be excited up to some thousands of volts. The potential difference necessary to make a spark pass in air between two metal balls 1 cm. in diameter is 4800 volts when the surfaces are 1 mm. apart, and 26,700 volts when the surfaces are 1 cm. apart.

As has been stated on p. 171, the negative electric fluid, which consists of very small charged particles called electrons, much smaller than atoms, can be investigated apart from matter, and what really happens in an electric current is that these electrons travel along the wire from molecule to molecule, from the point at the lower to the point at the higher potential. It has, however, become customary to consider the current as passing the other way, from the carbon to the zinc electrode of the Leclanché cell along the wire, and to assume in current electricity that something called positive electricity is passing in this direction, and it is not desirable to attempt to change the convention.

Ohm's Law.—If the poles of a cell are connected by a great length of iron wire, and then the iron wire replaced by an equal length of copper wire of the same diameter, it is found that the current is not the same in each case, but that it is much smaller in the case of the iron wire. We express this by saying that the resistance of the iron wire is greater.

In 1826 Ohm annnounced the law, that if a current C passes through a wire when the e.m.f. or electromotive force between its ends is E, then E/C is equal to a constant R, no matter how E and C vary, as long as the temperature of the wire remains the same. This constant R is defined to be the resistance of the wire. If C is measured in amperes and E measured in volts, R is measured in ohms, the unit being named after the discoverer of the law. Thus, if the e.m.f. of a cell is 1·45 volts, and it produces a current of ·112 amp., the total resistance of the circuit is 1·45/·112 = 12·9 ohms.

It is found that the resistance of a wire varies directly as its length and inversely as the area of its cross-section. This result may be put in the form of an equation,

$$R = \frac{kl}{a}$$

where l is the length of the wire, a the area of its cross-section, and k a factor depending on the nature of the material of which the wire is composed. k is defined as the specific resistance of the material of the wire, is measured in ohm-cms., and in the case of metals it increases with the temperature.

The reciprocal of the resistance of a wire is called its conductance, and the reciprocal of the specific resistance of a material is called its specific conductivity.

Resistance of Two Wires in Series.—When two wires are connected, the one after the other, they are said to be arranged in series, and their total resistance is then equal to the sum of the resistances of the separate wires. This may be proved as follows:—

Let R_1 be the resistance of the first and R_2 the resistance of the second wire, C the current, E_1 the e.m.f. between the ends of the first wire, and E_2 the e.m.f. between the ends of the second wire. Then the total resistance of the two wires is

$$\frac{E_1 + E_2}{C} = \frac{E_1}{C} + \frac{E_2}{C} = R_1 + R_2$$

Resistance of Two Wires in Parallel.—Let us suppose that two wires ADB and AFB are connected in parallel, *i.e.* as shown in Fig. 180, and let the resistance of ADB be R_1 and the resistance of AFB be R_2; it is required to find the resistance of a single wire which could be substituted for the two wires without altering the difference of potential of A and B, or, as it is called, the equivalent resistance of the two wires.

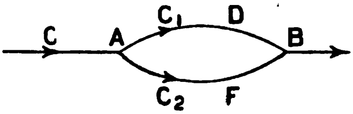

FIG. 180.

Let E be the difference of potential between A and B, and let us suppose a current C approaches A in the manner indicated in the diagram, and divides at A into a current C_1 along ADB and a current C_2 along AFB. Then, since the electricity does not accumulate at A,

$$C = C_1 + C_2 = \frac{E}{R_1} + \frac{E}{R_2} = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Now let R be the equivalent resistance. Then

$$\frac{E}{R} = C = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

i.e.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

which gives

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

EXAMPLES 38

1. If a penny is lying on the table tails up, and a current is flowing round its edge from Britannia's trident to her helmet, will a north pole placed at the centre of the penny be raised or lowered by the action of the current?

2. A 16 candle-power carbon glow lamp takes a current of ·22 amp. when run off a 250-volt lighting circuit. What is its resistance when in use? (In the case of carbon the resistance decreases as the temperature increases.)

3. Two flash lamp bulbs are marked 2·5 V ·3A and 3·5 V ·3A respectively. If the V stands for volts, and the A for amperes, what are the resistances of these two bulbs when lit up? They give 1·5 and 2 candles.

4. The resistance of a bobbin of wire is measured, and found to be 52·7 ohms; a portion 121 cms. long is cut off, and its resistance is found to be ·49 ohm. What was originally the total length of wire on the bobbin?

5. One end of a wire ABC is connected to earth, and the other is raised to a potential of 1·48 volts. If the resistance of AB is 10 ohms and the resistance of BC 5 ohms, find the current in the wire and the potential of B.

The current in the wire = 1·48/15 = ·099 amp.
The potential of B is $\frac{5}{15}$ × 1·48 = ·49 volt

6. The specific resistance of platinum is 11·0 × 10^{-6} ohm-cms. at 18° C. Calculate the resistance of a platinum wire 1 metre long and $\frac{1}{4}$ mm. diameter at this temperature.

$$R = \frac{kl}{a} = \frac{11{\cdot}0 \times 10^{-6} \times 100 \times 40^2}{\pi}$$
$$= {\cdot}56 \text{ ohm.}$$

7. The specific resistance of platinum can be represented by the formula

$$R = R_0(1 + at)$$

where R_0 is the resistance at 0° C., t the temperature in degrees Centigrade, and $a = ·0035$. If the resistance of a platinum wire is 100 ohms. at 0° C., what is it at 15° C?

8. The specific resistance of platinum is 6 times as great as the specific resistance of copper. How much greater is the resistance of a platinum wire $\frac{1}{2}$ mm. diameter and 328 cms. long than the resistance of a copper wire 10 metres long and ·42 mm. diameter?

9. One wire is 20 feet long and ·021 inch diameter, and another 13 feet 3 inches long and ·030 inch diameter. They are made of the same material. Compare their resistances.

10. A uniform glass tube 106·3 cms. long and one square millimetre in cross-sectional area is filled with pure mercury at a temperature of 0° C. The specific resistance of mercury at this temperature is $94·07 \times 10^{-6}$ ohm-cms. Calculate the resistance of the mercury column.

11. Two points are connected by two wires, the resistance of one of which is 20 ohms, and the resistance of the other of which is 10 ohms. Calculate their equivalent resistance.

12. Two points are connected by a wire the resistance of which is 10·78 ohms. What resistance must be arranged in parallel with this wire in order to make the resistance of the combination exactly equal to 10 ohms?

13. Three wires of resistance R_1, R_2, and R_3 are connected in parallel. Show that R, the equivalent resistance of the combination, is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

14. Show that the conductance of two wires placed in parallel is equal to the sum of their conductances.

39. THE MEASUREMENT OF CURRENT AND RESISTANCE

Theory of the Tangent Galvanometer.—The tangent galvanometer is both theoretically and historically the most important method of measuring electric current, although instruments based on another principle are in more frequent commercial use nowadays.

Let us suppose that a current C (in theoretical units) is flowing round a wire wound in the form of a circular coil of radius r of n turns, and that the plane of the coil is vertical and in the magnetic meridian. Let us suppose also, that a small magnet of length $2l$ and pole strength m is placed at the centre of the coil, and pivoted on an agate point, so that it is free to rotate about a vertical axis. Let Fig. 181 represent a section of the coil by a horizontal plane through its centre; NS is the magnetic needle, TF the direction of the magnetic meridian, *i.e.* of the horizontal component of the earth's magnetic field H, and T and F are the points in which the coil intersects the plane of the diagram.

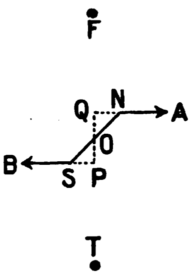

FIG. 181.

When a current passes round the coil, in agreement with Oersted's experiment, forces are exerted on each of the poles. Each of these forces is equal to $2\pi n Cm/r$, a part $2\pi Cm/r$ being due to each of the turns of the coil, and they are represented in direction and magnitude by the lines NA and SB.

NA and SB constitute a couple tending to turn the magnet out of the plane of the meridian. Let the ∠QON through which the magnet has already been deflected be denoted by θ. Then $QP = 2QO = 2ON \cos\theta = 2l \cos\theta$, and the magnitude of the deflecting couple is

$$\frac{2\pi n Cm QP}{r} = \frac{2\pi n Cm 2l \cos\theta}{r} = \frac{2\pi n CM \cos\theta}{r}$$

where M denotes the magnetic moment of the magnet.

But it has been shown on p. 166 that there is a restoring couple due to the earth's field equal to

$$HM \sin\theta$$

which tends to bring the magnet back into the meridian. Now, as θ increases, the restoring couple increases, and the deflecting couple diminishes, until, when θ is 90°, it has become zero. Consequently there is always a position in which they balance one another; this position is given by

$$\frac{2\pi n CM \cos\theta}{r} = HM \sin\theta$$

which becomes

$$C = \frac{r}{2\pi n} H \tan\theta$$

Thus the current is proportional to the tangent of the deflection; hence the name of the instrument.

Tangent Galvanometer.—Fig. 182 represents a tangent galvanometer. The compass needle has a long, thin pointer, the ends of which move over a scale graduated in degrees. The pointer is at right angles to the needle. The vertical circle carries the coils, of which in the case of the instrument depicted there are three, one of a single turn of thick wire, one of 50 turns of thinner wire, and one of 500 turns of thinner wire still. Each of these coils has its own two terminals, the terminals being visible on the front of the instrument. Which coil should be used depends on the strength of the current to be measured; it is obvious from the formula that, if C is small, in order to get a large deflection, n should be as large as possible.

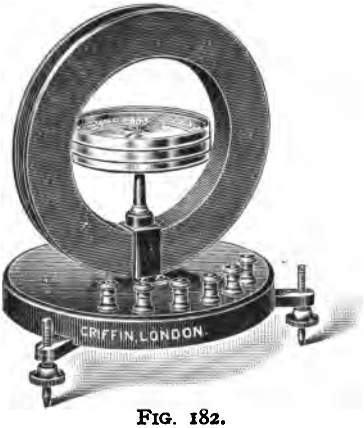

FIG. 182.

Before using the galvanometer it must first be levelled. Then the dial must be turned until the 90° marks lie in the plane of the coil. The galvanometer itself must next be rotated until the ends of the pointer are at the 0° marks on the dial; the coil is then in the magnetic meridian. This is an important adjustment, and must be verified by sending a current through the galvanometer and reversing its direction; the magnitude of the deflection should then be the same both ways. If it is not, the adjustment should be repeated until it is.

If the current is desired in amperes, the formula for the current must be multiplied by 10.

Resistance Box.—It is sometimes necessary to vary the resistance in an electrical circuit. Different types of apparatus are employed for this purpose according to the size of the currents used. When the current is small, a resistance box of the type shown in Fig. 183 is used.

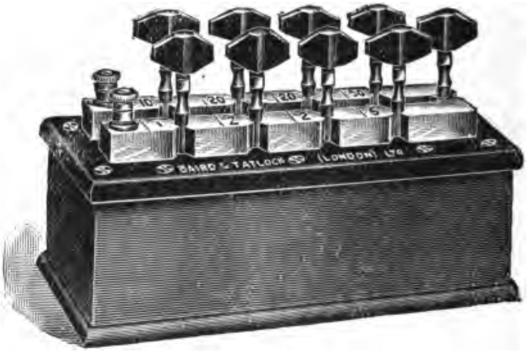

Fig. 183.

The wires are connected to the binding screws on the left. The box has a vulcanite top. On this top are fixed ten brass blocks, connected by nine brass plugs with vulcanite handles. When all the plugs are in position, the current simply flows through the blocks and plugs from the one terminal to the other, and, since the cross-sectional area of the blocks is large, the resistance is practically zero.

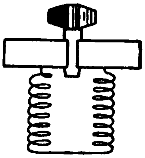

Fig. 184.

But if a plug is out (cf. Fig. 184), the current can pass from the one block to the other only through a coil of wire inside the box. The lengths of these coils are adjusted, so that their resistances are exact numbers of ohms; thus in the case of the box depicted in Fig. 183 the resistances are 1, 2, 2, 5, 10, 20, 20, 50 ohms. Thus the box gives any whole number of ohms up to 110. The end plug gives an infinite resistance, *i.e.* the end blocks are not joined by a coil inside the box.

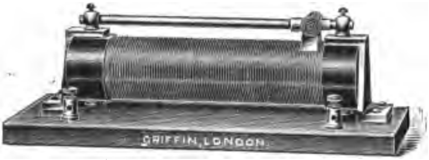

Fig. 185.

For larger currents (1–10 amps.) an uncovered wire with a high specific resistance wound firmly on a slate cylinder is used, as shown in Fig. 185. The neighbouring turns of the wire do not touch one another. The left-hand terminal is connected to the left-hand end of the wire. The right-hand terminal is connected to the top bar; on this bar there is a sliding piece which makes contact with the wire on the cylinder. The length of wire on the cylinder through which the current flows, and consequently the resistance, can be altered by moving the sliding contact, for the resistance of the bar itself is so small that it can be neglected.

For still higher currents the wires are wound on an open frame. A variable resistance for high currents is called a rheostat.

Keys.—In carrying out electrical experiments it is often necessary to make or break an electric current, or to reverse the direction of a current through a galvanometer. Special pieces of apparatus called keys are used for this purpose, since it would be inconvenient to be continually connecting and disconnecting the wires. Figs. 186 and 187 show two forms of key, a simple contact key and a plug key switch.

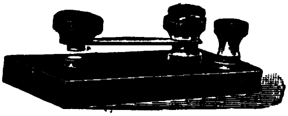

FIG. 186.

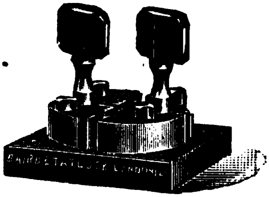

FIG. 187.

The former of these is used simply for making and breaking a current. The wires are connected to the two terminals on the right. The terminal at the corner is connected below the wood with the stud. When the key is pressed, current is made; when the finger is withdrawn, the metal strip springs up, and current is broken.

The plug key switch can be used for various purposes. Fig. 188 shows it arranged for reversing a current through a galvanometer; G represents the galvanometer, R is a resistance box, and B is the cell or battery. When the plugs are in the positions P, the current flows through the galvanometer in the direction indicated; when they are in the positions Q, it can be shown, by drawing another diagram, that the current flows through the galvanometer the other way.

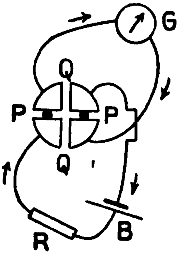

FIG. 188.

Wheatstone's Network.—The diamond-shaped system of conductors AHDF shown in Fig. 189, with two opposite corners H and F connected through a galvanometer G and a contact key k, and the other two corners A and D connected through a cell B and a contact key K, is called Wheatstone's network.

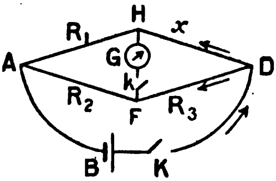

FIG. 189.

Let us suppose that the key K is pressed down. Then a current flows from the battery in the direction indicated by the arrow, and divides at D into two parts, one part of strength C_1 which flows along

DH, and the other part of strength C_2 which flows along DF. Let us suppose that the points H and F are exactly at the same potential; then, if the key k is pressed down, no current will flow from F to H. The two currents C_1 and C_2 unite again at A, and flow back to the battery.

Let R_1, R_2, R_3, and x be respectively the resistances of the branches AH, AF, FD, and HD, let E_1 be the potential difference between D and H or D and F, and let E_2 be the potential difference between F and A or H and A. Then, by Ohm's law,

$$E_1 = C_1x = C_2R_3$$

and

$$E_2 = C_1R_1 = C_2R_2$$

This gives

$$\frac{E_1}{E_2} = \frac{x}{R_1} = \frac{R_3}{R_2}, \text{ whence } x = \frac{R_1R_3}{R_2}$$

Hence the resistance x can be calculated, if either of the adjacent resistances R_1 or R_3 and the ratio of the other two resistances is known.

The Wire Bridge.—The wire bridge is a piece of apparatus based on the principle of Wheatstone's network and used for determining the value of an unknown resistance. A simple form of wire bridge is represented diagrammatically in Fig. 190. AD is a straight uncovered wire of uniform cross-sectional area and high specific resistance. The thick black lines are copper strips of negligible resistance; R_1 is a known resistance of constant amount and x an unknown resistance, both attached to the copper strips by binding screws. G is a galvanometer connected to a binding screw at H and to a contact key at F. A and D are connected to a battery B and key K.

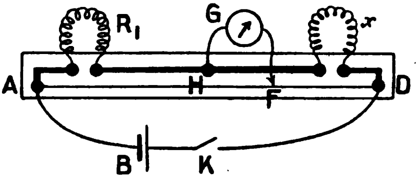

FIG. 190.

Corresponding points in Figs. 189 and 190 have the same letter, so that A, H, D, and F are the corners of the diamond, and the ratio of FD to AF gives the ratio of R_3 to R_2, since the resistance of the wire is proportional to its length. Directly below the wire there is a metre stick, not shown in the diagram, on which the lengths of FD and AF can be read off. In order to determine the resistance x we have consequently only to press the key K, then move the contact F along the wire until a point is found for which there is no deflection of the galvanometer. Then F and H are at the same potential, and

$$x = R_1\frac{FD}{AF}$$

EXAMPLES 39

1. Find in amperes the strength of a current which produces a deflection of 45° in a tangent galvanometer with a single turn of wire of 8 cms. radius, given that H = ·17 c.g.s. unit.

$$C = \frac{10r}{2\pi n} H \tan\theta \text{ amps.} = \frac{10 \times 8 \times \cdot 17}{2\pi} = 2\cdot 16 \text{ amps.}$$

2. A current of ·72 amp. flows through a tangent galvanometer consisting of 5 turns of wire each of 30 cms. diameter. What is the strength at the centre of the coil of the field due to the current, and through what angle is the needle deflected, given that H is ·17?

3. Compare the strengths of the fields produced at the centres of a coil of 10 turns and 6 cms. radius and a coil of 20 turns and 9 cms. radius, when the same current is passing through both.

4. A tangent galvanometer has a resistance of 1 ohm, and gives a deflection of 45° when traversed by a current of $\frac{1}{50}$ amp. What resistance must be placed in series with the galvanometer so that it may give the same deflection, when a potential difference of 1·43 volts is applied?

5. A tangent galvanometer has a resistance of 1 ohm, but the current through it is too great to be measured with accuracy. The galvanometer is consequently shunted with a coil of wire of resistance $\frac{1}{9}$ ohm, *i.e.* a coil of wire of this resistance is arranged in parallel with it. What fraction of the current now goes through the galvanometer?

Let g denote the current through the galvanometer and s the current through the shunt. Then the potential difference between the terminals of the galvanometer is given by

$$g = \tfrac{1}{9}s$$

Hence $s = 9g$, and $\frac{1}{10}$ of the total current now goes through the galvanometer.

6. What shunt is required to reduce to one hundredth the sensitiveness of a galvanometer of 1 ohm resistance?

7. A battery, a resistance box, an unknown resistance, and a tangent galvanometer are connected in series. When the 10-ohm resistance is out in the box, the deflection is exactly 45°. The unknown resistance is then taken out the circuit, and it is found that in order to restore the same deflection the box has to give a resistance of 87 ohms. How great is the unknown resistance?

8. An unknown resistance is measured against a 5-ohm resistance coil by means of a wire bridge. It is found to be greater than the latter, contact having to be made either at the 28·9 or the 70·9 cm. mark on the scale—the wire is 100 cms. long—according to the side of the bridge on which the unknown resistance is placed. What is its value? To what do you attribute the slight difference in the two results.

9. An unknown resistance is measured against a 5-ohm resistance by means of a one-metre wire bridge. In order to obtain balance contact must be made at 39·7 cms. A 10-ohm resistance is substituted for the 5-ohm resistance, and balance is then obtained at 24·7 cms. What is the value of the unknown resistance?

10. The resistance of a galvanometer is 1 ohm, and its terminals are connected by a wire 1 metre long, the resistance of which is also 1 ohm. One of the poles of a battery is connected to a terminal A of the galvanometer, and a wire attached to the other pole is connected to a point B on the metre wire. What must be the length of AB, in order that one-tenth of the current supplied by the battery may pass through the galvanometer?

40. INDUCED CURRENTS. PRINCIPLE OF THE ELECTRIC MOTOR

Faraday's Experiments.—If a cylindrical coil of wire is connected with a sensitive galvanometer, and a bar magnet is suddenly thrust into the coil, as shown in Fig. 191, the galvanometer indicates a momentary current through the coil. The current lasts only as long as the magnet is moving, and stops when the magnet comes to rest. If the magnet is suddenly withdrawn, we have again a momentary current, but this time in the opposite direction. Also thrusting in the south pole of the magnet produces a current in the same direction as withdrawing the north pole.

If we have two cylindrical coils, one called the secondary with a sensitive galvanometer in circuit with it, and another called the primary inside the first with a battery and key in circuit with it, when the key is pressed down we have an instantaneous current through the galvanometer. When the current is broken we have again an instantaneous current through the galvanometer, this time in the opposite direction.

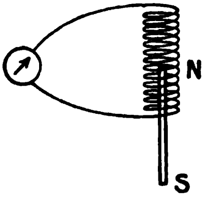

FIG. 191.

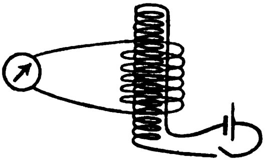

FIG. 192.

If the key is pressed down all the time, and the primary is suddenly withdrawn from the secondary, there is an instantaneous current in the secondary in the same direction as when the primary current is broken. This instantaneous current ceases as soon as the relative motion ceases. If the primary is thrust into the secondary with the key pressed down, an instantaneous current is again produced in the secondary, this time in the opposite direction.

The instantaneous currents produced in the above experiments are all said to be induced currents. They were discovered by Faraday in 1831. It is not necessary that the circuit containing the galvanometer should be at rest; the induced currents are produced equally well when the coil is slipped over the magnet or when the secondary is slipped over the primary, in fact, whenever there is relative motion of a circuit and a magnetic field.

Lenz's Law.—The direction of the induced current is always such as to oppose the relative motion. This rule, which is known as Lenz's law, includes all cases, since starting the current in the primary is equivalent to moving the latter into the secondary. Although difficult to apply in particular cases, it is very easy to remember; here it will be illustrated only by consideration of one particular case.

Let us suppose that a penny is lying on the table "tails up," and that a current is flowing round the edge from Britannia's trident to her helmet. Then, by the swimming man rule, such a current would raise a north pole placed at the centre of the penny. Consequently, by Lenz's law, if a north pole were thrust down towards the penny, the induced current would take place in the above-mentioned direction.

Magnitude of the Induced e.m.f. Earth Inductor.—The electromotive force induced in a circuit is numerically equal to the rate at which the number of lines of force passing through the circuit is altering. The meaning of this rule will be made evident by the consideration of one particular case, namely, the earth inductor.

There are different forms of earth inductor, but we shall consider only a very simple type, namely, one consisting of a rectangular wooden frame round which is wound a coil of n turns of wire, the total resistance of the wire being R. Let the area of the coil be A. The frame is set up with the plane of the coil vertical and at right angles to the magnetic meridian. It is then suddenly rotated through 180° about a vertical axis.

The horizontal component of the earth's magnetic field is given by H. The direction of H can be represented by lines in space of three dimensions, just as the direction of the field of a magnet is represented by lines on a plane in Fig. 158. The strength of the field can also be represented by lines of force, by supposing them drawn at such intervals, that if any plane surface is held up at right angles to their direction, the number received per sq. cm. of the surface is numerically equal to H.

When the frame, then, is set up at right angles to the direction of H, the number of lines of force passing through it is AH. But as the circuit goes round the frame n times, the total number of lines of force passing through the circuit is nAH. As the frame rotates, the number of lines of force passing through it decreases, becomes zero when the frame has turned through 90°, and increases to its former value, but in the opposite direction through the coil, when the frame has turned through 180°. Hence during the rotation the number of lines of force passing through the circuit has changed from nAH to $-n$AH. If the rotation takes place in t seconds, the average e.m.f. induced in the inductor is $2n\text{AH}/t$ and the average current $2n\text{AH}/(t\text{R})$, both in theoretical units.

Dynamo and Motor.—If the rotation of the earth inductor is continued through another 180°, the number of lines of magnetic force passing through the circuit changes from $-n$AH back to nAH, and a current is again induced, but this time in the opposite direction. Let us suppose now that the frame is rotated continuously with uniform angular velocity. There is then a current in the circuit, the direction of which changes every half-revolution, an alternating current, as it is called. The intensity of this current varies from a maximum value in the one direction through zero to a maximum of the same numerical value in the other direction, then back through zero to its first value again.

The intensity of the induced current will be increased if, instead of placing the rotating coil in the earth's field, it is placed between the poles of a magnet; this gives a much greater number of lines of force through the coil.

All the electric current which is used nowadays for lighting, driving machinery, tramcars, etc., is generated by machines called dynamos; these consist essentially of coils, which are rotated by a steam engine in an intense magnetic field. The induced alternating current thus produced can be converted into a continuous direct current, *i.e.* one without change of direction, by an arrangement called a commutator.

If a coil is rotated in a magnetic field, a current is induced. The converse of this is also true; *if a current of a suitable strength is passed through the same coil in a magnetic field, it rotates.* This is the principle of the electric motor. A motor is essentially a dynamo run backwards. At the power station the energy of the coal is converted into the kinetic energy of the steam engine; the latter is converted into electrical energy by the dynamo, and conveyed along the wire, perhaps to a tramcar, and is there converted into the kinetic energy of the tramcar by the motor, which is situated below the body of the car. The electric current enters the car by the trolley, and escapes through the wheels to the rails.

Ammeter.—The readings of the tangent galvanometer are given in degrees, and a calculation is necessary before they can be evaluated in amperes. Any instrument for measuring current which has its scale graduated directly in amperes is called an ammeter.

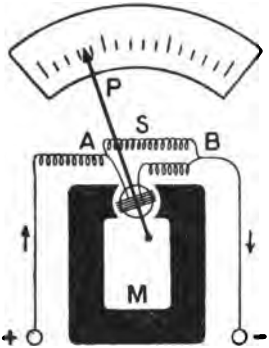

FIG. 193.

The best commercial ammeters are based on the same principle as the electric motor; they are termed moving-coil instruments. The tangent galvanometer has the disadvantage, that if the earth's field is altered in its neighbourhood by turning off or on an electric current, or by the presence of iron pipes, etc., the reading of the instrument changes; moving-coil instruments are free from this disadvantage.

A moving-coil ammeter is represented diagrammatically in Fig. 193. Its field is given by a horseshoe magnet M. Between the poles of this magnet is fixed a cylindrical core of soft iron, represented by the circle, to concentrate the lines of force as much as possible. Round this core rotates a coil of insulated copper wire. The current enters the instrument at the positive terminal T; when it reaches A it divides, part passing through the resistance ASB, which is called a shunt, part passing through the coil; it reunites again at B, and leaves the instrument by the negative terminal. The coil is pivoted on jewel bearings and held at its zero position by a spiral spring not shown in the figure. When the current passes, it rotates the coil against the spring, and the pointer moves over the scale. If it were not for the shunt S, the pointer would oscillate about a bit before coming to rest.

Barlow's Wheel.—Fig. 194 represents a piece of apparatus known as Barlow's wheel. It consists of a star-shaped copper disc, one point of which dips into a mercury trough. There are two terminals, one of which is connected to the brass pillar, and the other of which is connected to the mercury trough; when the terminals are connected to a battery, a current passes up the pillar to the axle of the disc, and then down a radius of the disc to the mercury. If a horseshoe magnet is held with its poles on each side of the lowest blade of the star, the lines of force passing between the poles meet the disc at right angles, and the star begins to rotate. This causes the point to move out of the mercury, and breaks the current. The rotating force consequently ceases, but the momentum acquired is sufficient to bring the next point into the mercury, and the star receives another impulse forward. And so it rotates in a series of jerks. If the direction of the current is altered, the direction of rotation is altered; also, if the poles of the magnet are interchanged, the direction of rotation is altered.

The principle involved in Barlow's wheel is the same as that of the electric motor or of the moving-coil ammeter. We have a current flowing at right angles to a magnetic field. It experiences a force in a direction at right angles both to the current and to the field. It is this force acting on each element of the coil which causes the latter to rotate, both in the case of the motor and of the moving-coil galvanometer.

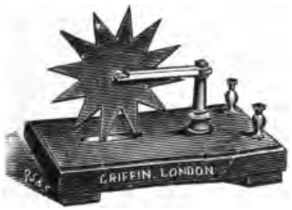

FIG. 194.

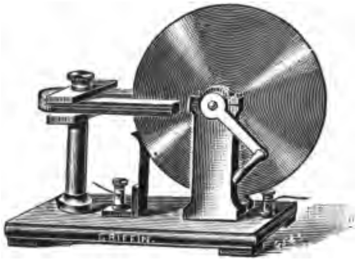

FIG. 195.

Fig. 195 represents an apparatus due to Faraday for performing the converse experiment to Barlow's wheel. A copper disc is rotated by a handle; it passes between the poles of a magnet, cutting the lines of force at right angles, and as a consequence of this an e.m.f. is set up between the centre and circumference of the disc. A thin strip of copper makes contact with the rim of the wheel, and is connected to the one terminal, and the axle of the disc is connected to the other. So if the terminals are connected by a wire, a current passes through it when the disc is rotated.

EXAMPLES 40

1. A circular coil is suddenly thrust over the N. pole of a magnet which is fixed in a vertical position with the N. pole up. Is the direction of the induced current clockwise or counter-clockwise when viewed from above?

2. A straight horizontal wire 1 metre long is falling vertically with a velocity of 500 cms./sec. and its length is at right angles to the magnetic meridian. What is the magnitude in volts of the e.m.f. induced between its ends, given that $H = {\cdot}17$?

Number of lines of force cut per second

$$= 100 \times 500 \times {\cdot}17 = 8500$$

$$\text{e.m.f.} = 8500/10^8 = 8{\cdot}5 \times 10^{-5} \text{ volts}$$

41. CELLS. ELECTROLYSIS

Simple Cell.—Let us suppose that we have a copper plate and a zinc plate dipping into a dilute solution of sulphuric acid. If the difference of potential of the two plates is measured, it is found to be 1·08 volts, the copper plate being at the higher potential. Small bubbles of gas form on the surface of the zinc plate, but these can be eliminated by using chemically pure zinc; no bubbles form on the copper plate.

If, however, the copper plate and zinc plate are joined by a wire, a current flows from the copper plate through the wire to the zinc plate, there is a copious evolution of bubbles at the copper plate, and the zinc plate wastes away. The arrangement acts as a cell, and can be used for giving current. The H_2SO_4 molecules in solution dissociate into parts called ions, an SO_4 ion and two hydrogen ions; the SO_4 attacks the zinc, forming zinc sulphate, and the hydrogen ions are given off as gas on the copper plate. It is the energy given out by the formation of the zinc sulphate which keeps the current going.

If the action is allowed to go on for some time, an accumulation of hydrogen bubbles takes place on the copper plate. This acts in two ways; it increases the resistance of the cell by diminishing the effective area of the plate, and it lowers the e.m.f. of the cell. As a result of these changes the current in the circuit diminishes, and the cell is said to polarize. But if the copper plate is removed and dried, and dipped in the solution again, the polarization is found to have disappeared.

The disadvantage of the simple voltaic cell described here is that it polarizes too rapidly. All the primary cells in use, Daniell cell, Leclanché cell, etc., have devices for preventing polarization. Thus in the Leclanché cell, when the chlorine ion attacks the zinc forming zinc chloride, the NH_4 ion liberated at the carbon plate breaks up into NH_3, which dissolves in the solution, and hydrogen, which reacts with the manganese dioxide to form water and manganous oxide. The Leclanché cell possesses the advantage, that the solution does not attack the zinc electrode when no current is passing. Consequently it is much used for electric bells and other intermittent work.

When the simple cell consisting of zinc and copper plates dipped into a dilute solution of sulphuric acid is giving a current, before polarization sets in the potential falls from the copper to the zinc outside the cell, makes a sudden rise of ·62 volt from the zinc to the acid, falls in the acid, and makes a sudden rise of ·46 volt from the acid to the copper. After polarization begins, the potential rise from the acid to the copper diminishes. When no current is flowing, the acid is at the same potential throughout.

Dry Cell.—The disadvantage of a cell containing a liquid is that the latter may dry up or be spilled. For this reason cells containing no liquid, called dry cells, are in considerable use now; they have different fancy names, but in composition are all practically the same, not differing much from the Leclanché cell. Their positive pole is a carbon rod surrounded by a paste, the average composition of which is

10 parts by weight of manganese dioxide, 10 parts of carbon or graphite, 2 parts sal ammoniac, and 1 part zinc chloride. This paste is called the depolarizing paste; it is held firmly round the carbon electrode by a sack, and it occupies almost all the volume of the cell. The negative electrode is the zinc box containing the cell. Between the zinc and depolarizing paste is the electrolytic paste, usually white, made of flour, plaster of Paris, sal ammoniac, and zinc chloride; the layer of electrolytic paste is generally very thin. The carbon and depolarizing paste must not touch the bottom of the zinc box. There is a small hole in the top of the cell for the escape of ammonia.

The e.m.f. of such a cell when fresh is 1·45 volts. The batteries used in the pocket flash lamps consist simply of three such cells in series, and consequently ought to have an e.m.f. of 4·35 volts when fresh. Dry cells deteriorate in the course of time, even though not in use, owing to chemical changes taking place inside them.

Storage Cell.—If two lead plates are immersed in a solution of one part of sulphuric acid to ten parts of water, and a current passed through the solution, hydrogen bubbles rise from the plate at which the current leaves the solution, and the other plate becomes brown. This is due to the formation of lead peroxide (PbO_2) by the action of the oxygen, which is liberated by the current at this plate in exactly the same way as oxygen was liberated in the case of the voltameter represented in Fig. 178. If the current passing through the liquid is measured by an ammeter, it is found to decrease as the lead peroxide is formed.

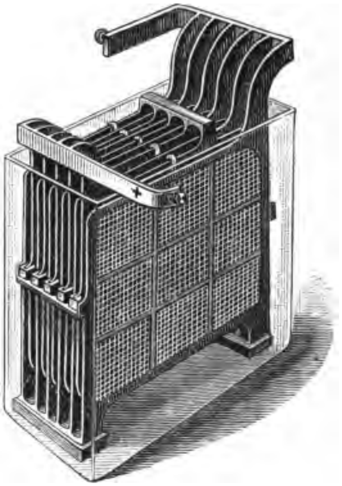

FIG. 196.
From Ganot's Physics.

If the battery is now removed, and the lead plates joined through the ammeter, it will be found that a current passes through it the other way. The original current virtually changed the two lead plates into a lead plate and a plate of lead peroxide, and thus made the arrangement into a voltaic cell, for whenever we have two dissimilar metals dipping into an acid we have a voltaic cell. It was the e.m.f. of the cell thus formed acting against the charging e.m.f. that made the ammeter reading fall in the first case. The arrangement acts as a voltaic cell only until the peroxide is used up.

The principal difference between the cell just described and the commercial storage cell or accumulator in common use is, that in the storage cell lead grids are used instead of plates, that the interstices in the grids are filled with lead sulphate before charging, and that, after the cells are charged, the interstices in the positive grid are filled with lead peroxide and in the negative grid with a porous spongy lead. The e.m.f. of a freshly charged storage cell is 2·1 volts, and it gives back

75 or 80 per cent. of the energy spent in charging it. It has the disadvantage that the grids disintegrate rapidly and have to be renewed. The test for knowing when to stop charging an accumulator is that, when the cell is fully charged, when all the lead sulphate on the negative grid is converted into lead, hydrogen gas begins to be given off; products of decomposition are then being lost and the cell is no longer working reversibly. The lead grids of an accumulator are shown in Fig. 196.

The accumulator is called a secondary cell in contradistinction to cells of the type of the Leclanché, which are primary cells, because it gives the current, not at the cost of its own chemical energy, but at the expense of the electrical energy which has been put into it.

Electrolysis.—Let us suppose that two copper plates dip into a solution of copper sulphate, and that a current is passed between them. The plate connected to the positive pole of the battery is called the anode, and the plate connected to the negative pole the kathode. It is found that, when the current passes, copper is deposited on the kathode, copper goes into solution from the anode, that the strength of the solution remains constant, and that there is no polarization.

According to the modern theory a certain proportion of the molecules in the copper sulphate solution is dissociated into parts, a Cu ion, which carries two charges of positive electricity, and an SO_4 ion, which carries two charges of negative electricity. When the current passes, the Cu ion or kation moves towards the kathode, gives up its charges there, and then deposits on the kathode, while the SO_4 ion moves towards the anode, gives up its charges to the anode, and attacks the latter, forming more $CuSO_4$. Any solution which carries a current by dissociating in this way is called an electrolyte, and the decomposition of the electrolyte is termed electrolysis. In the voltameter represented in Fig. 178 the wire A is the anode, K is the kathode, the kation is H, and the anion SO_4. If two silver electrodes dip into a solution of silver nitrate ($AgNO_3$), the ions are Ag and NO_3; the Ag ion goes with the current and is deposited on the kathode, while the NO_3 ion attacks the anode. Thus the strength of the solution is kept constant. It is useful to remember that in all cases of electrolysis the metal goes with the current, and that hydrogen counts as a metal. An arrangement of copper plates dipping into a copper sulphate solution or any similar arrangement is termed an electrolytic cell.

Ohm's law holds for electrolytic conduction, provided that the difference of potential is measured between two points inside the liquid. Electrolytic conductors also produce a magnetic field in the same way as metallic conductors do.

The laws of electrolysis were discovered by Faraday and hence are known as Faraday's laws. They can be combined into the following statement:—

The mass of any ion liberated is equal to the quantity of electricity which passes measured in coulombs, multiplied by the electro-chemical equivalent of the ion. The electro-chemical equivalents of the ions are to one another as their chemical combining weights, and the electro-chemical equivalent of silver is ·001118 gram.

Thus suppose that we have copper plates dipping into a vessel containing copper sulphate, and silver plates dipping into a vessel containing silver nitrate, that the same current passes through both liquids, and that it is required to find how much copper is deposited, when one gram of silver is deposited.

The atomic weight of copper is 63·57, and the atomic weight of silver is 107·88, but the copper ion is divalent, while the silver ion is monovalent. Hence their chemical combining weights are in the ratio 31·78 to 107·88, and the quantity of copper deposited is 31·78/107·88 = ·295 gram.

The laws of electrolysis hold for primary cells and secondary cells as well as for simple electrolytic cells. Thus if a battery of Leclanché cells arranged in series is used to pass a current through a zinc sulphate solution, into which two zinc rods dip, the quantity of zinc eaten away from the negative electrode of each of the Leclanché cells is exactly equal to the quantity deposited on the kathode of the electrolytic cell.

The essential difference between the primary voltaic cell, the secondary cell, and the electrolytic cell is, that in the first of these the chemical change goes of itself, in the second the current produces the chemical change during charging, and the chemical change goes of itself when the cell discharges, while the third is neutral as regards the chemical change.

Applications of Electrolysis.—Electrolysis is used in the arts for electroplating, electrotyping, and the refining of metals.

In electroplating—copper plating, nickel plating, silver plating—the article to be plated is placed in a bath, and forms the negative electrode of an electrolytic cell. The other electrode is of the metal which is to be deposited from the solution; thus in the case of silver plating it is of silver, and the solution consists of 2 parts of silver cyanide and 2 parts of potassium cyanide dissolved in 250 parts of water. When the current passes, silver is deposited on the article to be plated, silver goes into solution from the anode, and the strength of the electrolyte remains constant.

In electrotyping the page is set up in ordinary type, a mould taken in wax, powdered with graphite in order to render it conducting, and then copper plated by electrolysis. The wax is then removed, replaced by a metal backing, and the sheets printed off from the copper-faced plate thus formed, which stands wear much better than ordinary type, and is always used for large editions.

In purifying copper, for example, by electrolysis, unrefined copper is used as the anode, and pure copper sulphate used as the electrolyte. As the anode goes into solution, the impurities fall to the bottom of the bath; consequently pure copper is deposited on the kathode. Electrolytic copper is the purest on the market.

EXAMPLES 41

1. How long will it take a current of half an ampere to deposit 1 gram of silver from a solution of silver nitrate?

2. An acidulated water voltameter and an electrolytic cell, which contains copper sulphate and has copper electrodes, are connected in series, and it is

found that 1·00 gram of hydrogen is given off, when 31·75 grams of copper are deposited on the copper kathode. Calculate the atomic weight of copper.

Since the chemical combining weight of hydrogen is 1, that of copper is 31·75, and the atomic weight of copper consequently 63·5.

3. A steady current is sent for 20 mins. through (i) a solution of hydrochloric acid (HCl), (ii) a solution of cuprous chloride (Cu_2Cl_2). The volume of hydrogen liberated in (i) shows that ·000621 gram of hydrogen was separated from the solution. Find the weight of copper deposited in (ii). (An atom of copper is 63·5 times the weight of an atom of hydrogen.)

4. A constant current was sent for half an hour through the coil of a tangent galvanometer, producing a deflection of 35°, and also through a copper voltameter containing copper sulphate in which 1·112 grams of copper were deposited. What is the constant of the galvanometer, *i.e.* the factor with which the tangent of the deflection must be multiplied in order to give the current in amperes?

5. A Leclanché cell supplies a current of one ampere for five minutes. Calculate the diminution in weight of its zinc electrode, given that zinc is divalent, and that its atomic weight is 65, that the electro-chemical equivalent of silver is ·001118 gram, that silver is monovalent, and that the atomic weight of silver is 108.

42. VOLTMETER. JOULE'S LAW

Resistance of a Battery.—If a current is flowing round a circuit consisting of a Leclanché cell, a resistance box, and an ammeter, the resistance of the circuit can be divided into two parts. First of all there is the external resistance, which we shall denote by R, the sum of the resistances of the connecting wires, ammeter, and resistance box; secondly there is the internal resistance of the cell, which we shall denote by r, the sum of the resistances of the zinc rod, electrolyte, and carbon plate. Let E be the electromotive force of the cell, and C be the current it produces in the circuit. Then by Ohm's law

$$E = C(R + r)$$

Let the external resistance in the circuit be changed to R_1, and let the current change at the same time to C_1. Then

$$E = C_1(R_1 + r)$$

These two equations give

$$r = \frac{CR - C_1R_1}{C_1 - C}$$

Hence r can be calculated. It should be noted that here R and R_1 are the whole external resistances, not merely the resistances out in the box.

Voltmeter.—Let us suppose that a cell of electromotive force E and internal resistance r is connected to an ammeter of high resistance R, and that the current reading is C. Then

$$E = C(R + r)$$

The product CR is less than E, but is almost equal to E, if r is small in comparison with R. An instrument used for measuring the electromotive force of a cell, *i.e.* for measuring E, is called a voltmeter, and the type in most common use is simply an ammeter with a very high resistance, and its dial marked not to read C, but to read CR. Such instruments always give E slightly too low, but the error is very small indeed. Thus the internal resistance of a fresh Leclanché cell is less than ·5 ohm, and of a fresh flash lamp battery about 2 ohms, while the resistance of a good voltmeter reading to 6 volts is about 700 ohms. If CR is taken for E, the error is much less than 1 per cent.

Joule's Law.—If a current C flows through a wire of resistance R for a time t, the heat evolved in the wire is equal to C^2Rt. This was proved by Joule experimentally. It can also be derived theoretically from the principle of energy.

For let the electromotive force between the ends of the wire be E. The quantity of electricity which passes through the wire in time t is Ct. Then by the definition of electromotive force the work done in carrying this quantity from the one end of the wire to the other is $E \times Ct$. But by Ohm's law $E = CR$. And the work done represents a disappearance of electric energy, which reappears as heat. Hence the heat evolved is $ECt = C^2Rt$. The expression ECt is analogous to mgh, the expression for potential energy, Ct being analogous to m, and E to gh.

If C is measured in amperes, R in ohms, and t in seconds, the heat evolved is given in joules. One joule, it will be remembered, equals 10^7 ergs, and 1 calorie equals 4·2 joules. Joule's law is obeyed by electrolytes as well as by metallic conductors.

Experimental Verification of Joule's Law.—Fig. 197 represents a calorimeter for verifying Joule's law. First of all the resistance of the coil of high-resistance wire inside the inner vessel is measured, then the inner vessel is filled more than half full with water, a battery, plug key, ammeter, and variable resistance of the type shown in Fig. 185, connected to the terminals, the temperature of the water noted, and the current started. After the current has been run for a measured interval of time, it is broken, the temperature again noted, and the amount of heat evolved calculated both from the rise of temperature and from the formula C^2Rt. The two results should agree. The purpose of the variable resistance is to keep the current constant, in case the e.m.f. of the battery falls during the experiment.

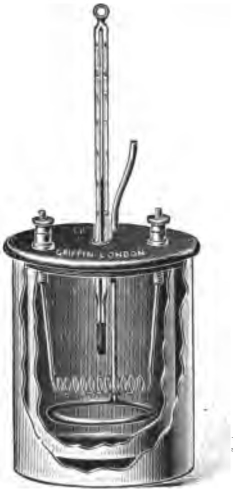

FIG. 197.

Incandescent Electric Lamp.—The incandescent electric lamp consists of a thin filament of carbon or tungsten sealed into a glass bulb from which the air has been evacuated. This filament has a high resistance; consequently, when a current is passed through it, much heat is evolved, and the filament becomes heated to a stage between red heat and white heat, and gives out light. The filament is enclosed in a vacuum to prevent it oxidizing; the vacuum has also the effect of preventing loss of heat by conduction to the sides of the bulb.

It is customary to rate the performance of an incandescent lamp or glow lamp in watts per candle. A 16 candle-power carbon lamp, when run off a 250 volt lighting circuit usually takes about ·22 amp. Its activity, *i.e.* the amount of heat produced in it per second, is $CE = {\cdot}22 \times 250 = 55$ watts, and the number of watts required per candle is consequently $55/16 = 3{\cdot}4$. The tungsten or wire-filament lamp—the Osram, Mazda, etc., are tungsten lamps—requires only 1·25 watts per candle, and so gives roughly three times as much light for the same expenditure of energy. Since its introduction in 1906 it has displaced the carbon filament lamp to a very considerable extent. Its disadvantage is that the lamp itself costs more. Originally the tungsten filaments were made by "squirting" a paste consisting of the metal and a binding material through a hole in a diamond, and afterwards burning out the binding material. They were then very fragile. They are now wire-drawn, and are consequently much tougher.

The bulbs used in the pocket flash lamps are intended for $3\frac{1}{2}$ volts. The battery supplied when fresh should give 4·35 volts. Consequently, at first the current is too great, the lamp is very bright, and is said to be over-run. As the battery becomes exhausted, the lamp becomes dimmer, until finally the filament is just luminous. Occasionally $2\frac{1}{2}$ volt bulbs are sold in pocket flash lamps; a perfectly fresh battery may burn such lamps out, *i.e.* melt their filaments through.

Arc Lamps.—If two rods of carbon, which are at a potential difference of 70 or 80 volts, are brought into contact for an instant, and then drawn a short distance apart, an electric discharge characterized by intense light and heat passes between them. This constitutes the electric arc. After the discharge has passed some time, the end of the positive carbon or anode becomes hollowed out, and the end of the negative carbon or kathode becomes pointed. Both the carbons are consumed by the arc, the positive one twice as fast as the negative one. The light comes chiefly from the positive crater, where a temperature of about 4000° abs. is attained.

EXAMPLES 42

1. The e.m.f. of a Daniell's cell is 1·07 volts, and its internal resistance is 4 ohms. Its poles are joined by a wire, the resistance of which is 20 ohms. Calculate the current produced, and the difference of potential between the ends of the wire.

2. The internal resistance of a cell is 4 ohms. It is connected by short thick wires to a tangent galvanometer of 1 ohm resistance, and the deflection is 60°. What will the deflection be if the thick wires are replaced by thin ones of resistance 2 ohms?

3. Three cells, X, Y and Z, the e.m.fs. and resistances of which are as follows :—

	X	Y	Z
e.m.f. . . .	1·07	1·50	2·00 volts
resistance . .	4	·4	·3 ohms

are connected in series through a resistance of 83 ohms. Find the current; find also the current when the cell Z is reversed.

4. A fresh flash-light battery is run for 20 minutes through an external resistance of 8·85 ohms. The current is at first ·42 amp., but at the end of the test it has fallen to ·27 amp. The e.m.f., as measured on a voltmeter, was 4·35 volts before, and 3·0 volts after the test. Find the initial and final values of the resistance of the cell.

5. An ammeter is connected in series with a battery and resistance box. When the resistance unplugged in the box is 100, 200, 300, and 400 ohms, the ammeter reading is 99·0, 65·0, 48·3, and 38·6 divisions respectively. Find the other resistance in the circuit as accurately as possible.

Let G be the other resistance in the circuit, *i.e.* the resistance of galvanometer, battery, and wires, let C be the current, when R is the resistance unplugged, let C_1 be the current, when R_1 is the resistance unplugged, and let E be the e.m.f. of the battery. Then

$$E = C(G + R) = C_1(G + R_1)$$

Consequently
$$G = \frac{C_1R_1 - CR}{C - C_1}$$

If we substitute for C and C_1 the number of divisions read on the ammeter, and combine the first and third, and second and fourth observations, we obtain for G 90·53 and 92·43 ohms, giving a mean result of 91·48 ohms.

Or the result may be obtained by a neat graphical method, by plotting $1/C$ against R. The curve obtained is a straight line, and G is equal to the abscissa of the point of intersection of this straight line with the R axis.

6. A current from a battery is flowing through an external circuit, which has a resistance 6 times that of the battery. In what ratio does the current increase when the value of the external resistance is halved?

7. A cell, which has an e.m.f. on open circuit of 1·46 volts, and the internal resistance of which is 2 ohms, has its terminals joined by two wires of 4 and 10 ohms placed in parallel. Find the current in each wire.

Let C and C_1 be the currents in the 4 and 10 ohm wires respectively.

Then $C4 = C_1 10$

and $1·46 = (C + C_1)2 + C4$

These give $C = ·215, \quad C_1 = ·086$ amp.

8. A battery has an e.m.f. of 15 volts and an internal resistance of 5 ohms. Its terminals are connected by a wire, and it is found that, when the circuit is closed, the difference of potential between the ends of the wire is 12 volts. Calculate, in calories per minute, the rate at which heat is being produced in the wire.

9. A Leclanché cell deflects a tangent galvanometer 30° when 200 ohms resistance are introduced into the circuit, and 15° when 530 ohms are introduced. A bichromate cell is then substituted for the Leclanché, and it is found that when 295 ohms resistance are introduced into the circuit, the deflection is again 30°. Find the ratio of the electromotive forces of the two cells. The internal resistances of the cells can be disregarded.

Let G be the resistance of the galvanometer and connecting wires. Then, since the current is proportional to the tangent of the deflection,

$$(200 + G) \tan 30° = (530 + G) \tan 15°$$

which gives G = 85·6 ohms. The e.m.fs. of the two cells are as the resistances which produce the same deflection, 30°, *i.e.*

$$\frac{\text{e.m.f. of bichromate}}{\text{e.m.f. of Leclanché}} = \frac{295 + 85·6}{200 + 85·6} = 1·332$$

10. A wire of 3 ohms resistance is immersed in 60 grams of water in a calorimeter of water equivalent 6 grams, and a current of 4 amps. passed for 5 mins. Find the rise in temperature of the water.

11. A wire of 2·5 ohms resistance is immersed in 55 grams of water in a calorimeter of water equivalent 6 grams, and a current of ·5 amp. passed for 10 mins. Find the rise in temperature produced.

12. A 16 candle-power carbon lamp run off a 250-volt circuit takes ·22 amp. The calorific value of coal is $3{\cdot}6 \times 10^6$ cals. per lb. What quantity of coal measured in lbs./hour gives out as much heat as the lamp?

13. If a Leclanché cell working at 1·46 volts gives a current of 1 amp., what is its rate of expenditure in watts? How many such cells would be equivalent to 1 H.P.?

14. Show that the amount of heat in calories developed per second in an iron wire, the length of which is 1 metre and the diameter of which is d mms., when a current of C amps. is flowing in it, can be written in the form $a(C/d)^2$, where a is a constant, and calculate the numerical value of this constant, given that the specific resistance of iron is 12×10^{-6} ohm-cm.

15. The unit of electrical energy used in connection with electricity supply for power and lighting is the kilowatt-hour, *i.e.* the amount of energy supplied in an hour by an agent working at an activity of 1000 watts. If the price of electricity is 4*d.* per kilowatt-hour, what is the cost of keeping six 16 candle-power lamps burning 4 hours, given that the electricity is supplied at 250 volts, and that each lamp takes ·22 amp.?

16. A desk electric fan makes 1400 revolutions per minute and takes 45 watts. Given that the kilowatt-hour costs 4*d.*, what is the expense of running it for one hour?

43. APPLICATIONS OF ELECTRICITY

Electromagnet.—Fig. 198 represents one form of electromagnet. It consists of a horseshoe core of soft iron, round which are wound two coils of wire A and B. When a strong electric current passes through these coils, the horseshoe core becomes a powerful magnet. In the figure a pan carrying weights is shown suspended from the keeper or armature of the magnet. When the current is broken, the iron demagnetizes almost completely, and the weights fall. If the current is made again, but this time in the opposite direction, the core magnetizes the opposite way, *i.e.* with the former north pole now a south pole, and vice versa.

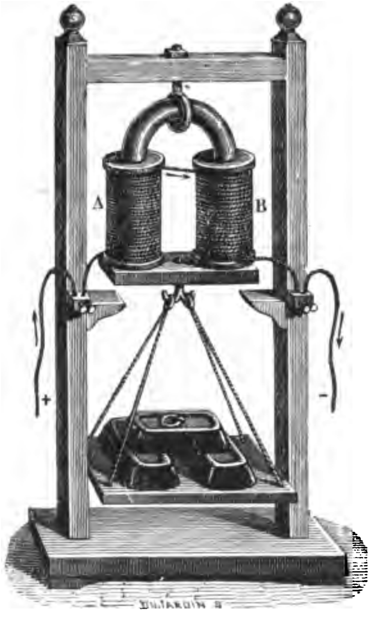

FIG. 198.
From Ganot's Physics.

All the large magnets used in the construction of dynamos, motors, etc., are electromagnets.

It was stated on p. 162 that artificial permanent magnets were made by means of an electric current. The method of procedure is to insert the unmagnetized hard steel bar inside a cylindrical coil of wire, and pass as strong a current through this coil as the wire can stand. When the bar is removed it is found to have become a permanent magnet. To harden the bar it is heated to a bright red heat, then quenched in water.

Electric Bell.—The electric bell forms one of the simplest applications of the electromagnet. It is represented in Fig. 199, and is set up in circuit with a battery and a bell push. The latter is merely a contact key; as long as it is pressed with the finger, contact is made. The current from the battery arrives at the terminal *m*, flows round both coils of the magnet, then along the armature of the magnet to the spring contact C, and thence to the terminal *n*. The armature is itself mounted on a spring, and carries the hammer P.

When current is made by the bell push, the core magnetizes, attracts the armature, and the hammer strikes the gong. But, when the armature is attracted, the circuit breaks at C, the core demagnetizes, and the armature flies back. Contact is then made again, the armature is attracted, and the hammer strikes the gong, and so on; the bell keeps ringing as long as the bell push is pressed.

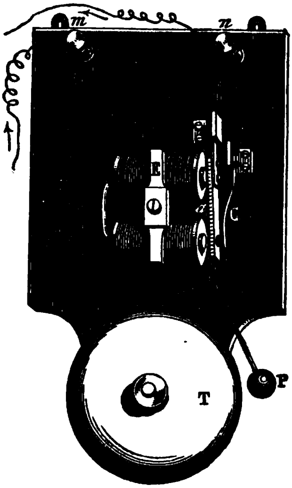

FIG. 199.
From Ganot's Physics.

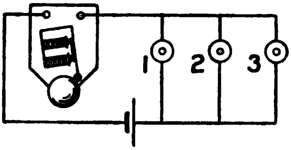

FIG. 200.

Fig. 200 shows the connections for a bell which is to be rung from different rooms, 1, 2, and 3. Such a bell is usually fitted with an indicator telling from which of the rooms the bell was rung.

Electric Telegraph.—In the following table the Morse code ordinarily employed in telegraphic signalling is given.

A	▪ ▬	J	▪ ▬ ▬ ▬	S	▪ ▪ ▪
B	▬ ▪ ▪ ▪	K	▬ ▪ ▬	T	▬
C	▬ ▪ ▬ ▪	L	▪ ▬ ▪ ▪	U	▪ ▪ ▬
D	▬ ▪ ▪	M	▬ ▬	V	▪ ▪ ▪ ▬
E	▪	N	▬ ▪	W	▪ ▬ ▬
F	▪ ▪ ▬ ▪	O	▬ ▬ ▬	X	▬ ▪ ▪ ▬
G	▬ ▬ ▪	P	▪ ▬ ▬ ▪	Y	▬ ▪ ▬ ▬
H	▪ ▪ ▪ ▪	Q	▬ ▬ ▪ ▬	Z	▬ ▬ ▪ ▪
I	▪ ▪	R	▪ ▬ ▪		

Call ▬ ▪ ▬ ▪ ▬ One dash = three dots. Space between signals forming same letter = one dot, space between letters = three dots, space between words = five dots.

If a circuit contains a battery, key, and galvanometer, the galvanometer being distant some miles from the battery and key, signals can be sent from the key to the galvanometer by deflecting the needle either to the right or to the left. This was the principle of some of the older forms of telegraph; the needle was vertical, a deflection of the upper end to the left being equivalent to a dot, and a deflection of the upper end to the right being equivalent to a dash. Nowadays an electromagnet with its armature supported by a spring is used instead of the galvanometer. Whenever the key at the sending station is pressed, the armature at the receiving station is pulled down against the magnet; when the circuit is broken, it springs back. The armature can be made to carry a pen, below which a tape is drawn at a uniform rate, so that when the current is made, the pen presses down on the paper and writes dots and dashes, but in practice the operators read the messages by ear alone. Whenever the armature strikes the magnet, and whenever it springs up, it makes a click, and the letters can be read by the intervals between the clicks.

In the early days of telegraphy a double circuit was used, but it was found in 1837, that the return wire might be dispensed with, and the earth itself used as return conductor. This is the universal practice nowadays; Fig. 201 shows the arrangement, the ends of the wires being attached to copper plates which are buried at some depth in the earth, so as to make good contact with the latter.

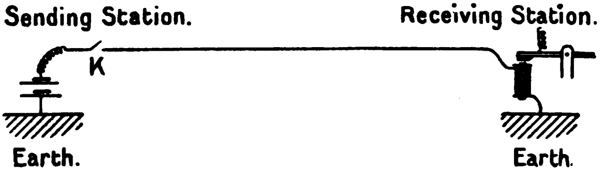

FIG. 201.

Telephone.—The telephone, which was invented by Graham Bell in 1875, consists at each end in its simplest form of a bar magnet round which there is a coil of wire, these coils forming part of a circuit of which the earth is the return wire. In front of each coil is a thin iron diaphragm, which vibrates in somewhat the same way as the diaphragm of a gramophone. When the diaphragm at the one end is spoken into, it vibrates and alters the number of lines of force which pass through the coil. Induced currents are thus set up in the circuit; these alter the strength of the field in which the iron diaphragm at the other end of the line is situated, it executes the same vibrations as the first diaphragm, and consequently reproduces the sounds which caused the latter to vibrate.

In modern practice an instrument of this type is still used as receiver, a horseshoe magnet being substituted for the bar magnet, but a return wire is provided, and the transmitter works upon a different principle altogether. At the back of the diaphragm is a little chamber filled with granulated carbon. As the diaphragm vibrates, the electrical resistance of this carbon alters, and the strength of a primary current which flows through the carbon also alters. This primary current then flows round a soft iron core, round which the telephone line is wound as secondary, and its variations induce currents in the telephone line, which actuate the diaphragm at the other end. The primary current is provided by a battery. This type of instrument carries much farther than the original instrument.

Induction Coil.—Fig. 202 represents the induction coil, which is a piece of apparatus based on the experiment represented in Fig. 192.

It consists of a bundle of straight iron wires termed the core, round which is wound a coil of insulated wire called the primary. The secondary is wound round the primary, and consists of a very large number of turns of fine wire insulated with silk and shellac. In the primary circuit there is an interrupter. This in its simplest form consists of a spring to the upper end of which a piece of soft iron is attached on one side; on the other side is a platinum point which rests against a platinum surface. The current passes up the spring and through the platinum point to the surface, and then round the primary. This magnetizes the core, the core attracts the piece of soft iron, contact between the platinum point and surface is broken, and the current is interrupted. The core then demagnetizes, the spring flies back, contact is made, and the same cycle takes place over again; the action is similar to that of the electric bell. Whenever the primary current is made or broken, an e.m.f. is induced in the secondary, and if the two terminals of the secondary are placed close together, sparks pass between them in the same way as sparks pass between the knobs of a Wimshurst machine. Since the break takes place much more sharply than the make, the induced electromotive force is much greater one way, and the stream of sparks passes between the points mainly in the one direction.

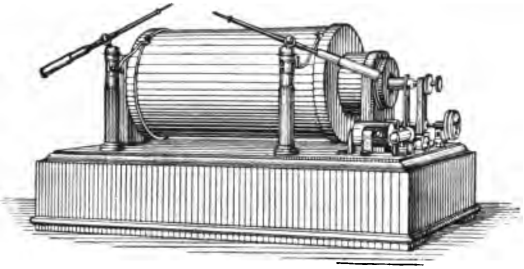

FIG. 202.

From Foster & Porter's Electricity.

The presence of the iron core in the primary increases the induced e.m.f. enormously; if a soft iron core is inserted in the primary in the experiment represented in Fig. 192, it increases the induced current in a very marked manner. The largest induction coils can be made to give sparks between the terminals of the secondary, when the latter are 18 or 24 inches apart. When an induction coil is passing sparks, it changes oxygen of the atmosphere into ozone; ozone is a powerful bleaching agent, and can easily be recognized by its very characteristic smell.

Very small induction coils are sometimes used for the purpose of giving electric shocks. The person to receive the shock simply holds in each of his hands a handle connected to one of the terminals of the secondary.

Kathode Rays. X Rays.—If two platinum wires are sealed through the walls of a glass tube and attached to aluminium electrodes inside the tube, if the wires are connected to the secondary of an induction coil, and the air gradually exhausted from the tube, it is found that at atmospheric pressure the tube conducts badly or not at all, but that as the pressure decreases, the conductivity improves, and the tube lights up. The light is brightest at a pressure of 1 or 2 mms. At higher rarefactions the conductivity diminishes, and the character of the discharge alters, until finally at $\frac{1}{1000}$ mm. of mercury we have rays proceeding in a straight line out from the kathode and producing a fluorescent glow where they impinge on the side of the tube. These rays are called kathode rays. They can be deflected by a magnet. It was shown, towards the end of last century, that they consist of small negatively charged particles called electrons, which have been referred to already on p. 183, and which are identical with the two negative charges ascribed to the SO_4 ion on p. 198. All molecules contain electrons, and when a molecule loses one electron it acquires a positive charge. The discovery of the electron has been one of the greatest advances in physics, and it was the study of the kathode rays which led principally to this discovery.

It was discovered by Prof. W. K. Röntgen in 1895, that when kathode rays strike a metal with a high atomic weight, such as platinum, it emits rays called X rays, which travel in straight lines, penetrate media such as paper, cloth, leather, aluminium, etc., which are opaque to ordinary light, but are stopped by the denser metals, especially lead. Flesh is transparent to them, but the bones are opaque, and by their aid we can see the bones through the flesh. They have thus an important application in surgery.

The simplest method of using them is to fix up a screen of barium platino-cyanide near the X ray tube, so that the rays fall on it. The rays make the screen luminous, but objects held between the tube and the screen cast a shadow on the latter. If a hand and wrist are placed between the tube and screen, the bones are distinctly visible in the shadow through the flesh, and the sleeve links are visible through the sleeve. If a purse is held up, the coins in it are visible through the leather. The brass weights in a wooden box are visible through the wood, and the coils in a resistance box are visible through the box.

EXAMPLES 43

1. If the electromagnet depicted in Fig. 198 is looked at from below, in what direction (clockwise or counter-clockwise) does the current flow round (i) the north pole, (ii) the south pole?

2. Electric bells are to be installed in separate rooms, each with its own

bell push, but they are to be run off the same battery. Show how you would make the connections.

3. Why are copper wires used in trolley systems and iron wires used in telegraph lines?

4. The instruments on a telegraph circuit have a resistance of 110 ohms, and the line offers a resistance of 12 ohms per mile. The length of the line is 100 miles. Assuming that the current is given by Daniell cells of e.m.f. 1·07 volts and internal resistance 4 ohms, and that a current of ·01 amp. is required to work the instruments, find how many cells are required.

APPENDIX

Approximate Calculation.—The numerical examples in this book are best solved by the use of a table of four-figure logarithms. Sometimes, however, when we are dealing with two quantities which do not differ much from unity, four-figure logarithms are not accurate enough; in these cases recourse may be had to the approximate formulæ,

$$\frac{1}{1+\alpha} = 1 - \alpha \quad . \; . \; . \; . \; . \; . \; . \; . \qquad \text{I}$$

$$(1+\alpha)(1+\beta) = 1 + \alpha + \beta \quad . \; . \; . \; . \; . \; . \; . \qquad \text{II}$$

$$\sqrt{(1+\alpha)} = 1 + \tfrac{1}{2}\alpha \quad . \; . \; . \; . \; . \; . \; . \qquad \text{III}$$

which hold when α and β are small in comparison with 1.

By ordinary algebraic division

$$\frac{1}{1+\alpha} = 1 - \alpha + \alpha^2 - \alpha^3 \; . \; . \; .$$

hence I holds, if α^2 and higher powers of α can be neglected in comparison with 1. By direct multiplication we have

$$(1+\alpha)(1+\beta) = 1 + \alpha + \beta + \alpha\beta$$

hence II holds, if $\alpha\beta$ can be neglected in comparison with 1. If the right-hand side of III is squared we have

$$(1 + \tfrac{1}{2}\alpha)^2 = 1 + \alpha + \tfrac{1}{4}\alpha^2$$

hence III holds, if $\frac{1}{4}\alpha^2$ can be neglected in comparison with 1.

Examples on Approximate Formulæ.

(1) $\sqrt{50} = \sqrt{(49+1)} = \sqrt{49}\sqrt{(1 + \frac{1}{49})} = 7(1 + \frac{1}{98})$ by III
$= 7{\cdot}07.$

(2) $12{\cdot}17 \times \dfrac{1{\cdot}00316}{1{\cdot}00293} = 12{\cdot}17(1 + {\cdot}00316)(1 - {\cdot}00293)$ by I
$= 12{\cdot}17(1 + {\cdot}00316 - {\cdot}00293)$ by II
$= 12{\cdot}17(1 + {\cdot}00023)$
$= 12{\cdot}17 + 12{\cdot}17 \times {\cdot}00023$ (Note method of multiplying here)
$= 12{\cdot}17 + {\cdot}00280$
$= 12{\cdot}1728$

Four-figure logarithms would have shown no difference between 1·00316 and 1·00293.

(3) $$\frac{530000 + 31}{530000 - 31} = \frac{1 + \dfrac{31}{530000}}{1 - \dfrac{31}{530000}}$$

$$= \left(1 + \frac{31}{530000}\right)\left(1 + \frac{31}{530000}\right) \text{ by I}$$

$$= 1 + \frac{31}{530000} + \frac{31}{530000} \text{ by II}$$

$$= 1 + \frac{62}{530000} = 1{\cdot}000117$$

Meaning of Numbers.—In physics we attach a meaning to every decimal place to which a result is stated. Thus

4 means nearer 4 than 5, nearer 4 than 3,
4·0 „ „ 4·0 „ 4·1, „ 4·0 „ 3·9,
4·00 „ „ 4·00 „ 4·01, „ 4·00 „ 3·99.

How many decimal places should be given depends on the accuracy of the data from which the result is calculated. If, for example, in a specific gravity determination a solid is weighed in air and in water to $\frac{1}{100}$ gram, the weight in air being about 10 grams, the result cannot be accurate to more than four figures, and it is misleading, if not actually dishonest, to give it to six figures.

Regard should also be paid to the values of constants introduced. For example, if we write $\pi = \frac{22}{7} = 3{\cdot}143$, which is not accurate to four figures, the result cannot be right to four figures; if the data are right to four figures, a more accurate value of π must be used. In this book g is usually taken = 32 ft./sec.^{2}, since this number is convenient for calculation, and, when British units are used, a high degree of accuracy is not wanted; results based on this value cannot be accurate to more than three figures.

If a result comes out 3·1728, and it is desired to state it only to four figures, the proper contraction is 3·173, not 3·172, as it is nearer the former value than the latter. Repeating decimals are not used in physics; if a result comes out $1{\cdot}\dot{6}$, it is written 1·667 if correct to four figures, and 1·67 if correct to three. A large number like 3,252,304 should be written $3{\cdot}25 \times 10^6$, if accurate to three figures, and $3{\cdot}252 \times 10^6$, if accurate to four figures. When it is written this way, the mind grasps its magnitude more readily.

ANSWERS

CHAPTER I.—Dynamics

SECTION 1

2 88 ft./sec., 96·58 kms./hr., 26·82 metres/sec. 3 6·533 kgs./km., 23·17 lbs./ml.
4 17·52 kgs./metre^2, 32·29 lbs./yd.^{2} 6 No.

SECTION 2

2 10 gms. 3 141·4 gms. in first thread, 100 gms in second. 4 24 lbs.
6 (i) 10 units in direction 36° 52′ S. of W. (ii) 5·291 units in direction making angle 220° 54′ with first force. (iii) 2·828 units in direction making angle 225° with first force. 7 No; they cause rotation.

SECTION 3

2 210 lbs. slung $\frac{3}{7}$ of AB from A. 3 At 36·04 cms. 5 77·48 gms.
6 At 43·66 cms. 7 6 lbs., 10 lbs. 8 11·16 ft.

SECTION 4

2 ·2051 inch from centre of large sphere.
3 Take a point D on AB, so that AD = 2DB. 3DG = 4GC.
4 In axis of symmetry $\frac{1}{9}a$ from centre.
5 $a\sqrt{2}/6$ from centre along axis of symmetry.
6 1·06 cms. from centre of large cylinder.

SECTION 5

4 43·8 ft./sec., 1·37 secs. 5 $\frac{5}{8}$ ft./sec.^{2}, 54 ft. 6 1·56 ft. 7 7·81 ft
8 53·8 ft. 9 44 ft./sec., $\frac{11}{60}$ ft./sec.^{2} 10 1, 3, 5, 7 and 9 ft.
11 (i) $7{\cdot}857 \times 10^4$ mls./hr.^{2}, (ii) 975·4 cms./sec.^{2}, (iii) $1{\cdot}264 \times 10^5$ kms./hr.^{2}
12 (i) $1{\cdot}296 \times 10^4$ kms./hr.^{2}, (ii) 3·28 ft./sec.^{2}, (iii) $8{\cdot}056 \times 10^3$ mls./hr.^{2}
13 $b(\sqrt{2} - 1)$ secs. 14 144 ft.
15 (i) 6·5 mls./hr., (ii) 4·61 mls./hr., (iii) 5·40 mls./hr. 16 $2\frac{2}{15}$ mls.

SECTION 6

3 25 gms. 4 80 ft./sec. 5 9·6 ft./sec. 6 (i) 12 dynes, (ii) ·0122 gm.
7 128 lbs. 8 (i) 4·58 tons, (ii) 5·04 tons. 9 55 : 96. 10 ·937 ton.
11 $\frac{1}{2}$ ton, 4 lbs. 12 (i) 150·55 gms., (ii) 150·80 gms.
14 6·38 ft./sec. 15 $\frac{1}{1000}$ cm./sec., $\frac{1}{5000}$ cm./sec.

SECTION 7

2 $3{\cdot}22 \times 10^3$ gms. 3 14·34 ft./sec.^{2}.
4 (i) ·175 ft./sec.^{2}, (ii) ·702 ft./sec.^{2}, (iii) 1·403 ft./sec.^{2} 5 ·1112 ft./sec.^{2}
6 ·0624 ft./sec.^{2} 8 8·041 cms./sec., $2{\cdot}586 \times 10^4$ cms./sec.^{2}
9 ·897, 1·269, 1·553, 1·794, 2·005 secs. 10 1·566, 1·917, 2·214 secs.
12 Loses 18 per day. 13 40·3 inches. 14 0·4 per cent.

SECTION 8

1 24·6 ft., 24·6 ft./sec., 9·23 lbs. 2 ·625. 4 401 cms./sec., 501 cms.
5 4·72 lbs., 5·09 lbs. 7 ·58. 8 135·9 cms./sec., 248·6 cms./sec.
9 16·4 ft./sec., 396 ft.

SECTION 9

2 2909 ft.-lbs. 3 1·357 × 10^7 ergs. 4 792 ft.-tons.
5 (i) 17600 ergs., (ii) 3200 gm. cms., (iii) 9 ft.-lbs.
6 5·843 × 10^4 ft.-lbs./cub. ft. 7 340 ft.-lbs. 8 1·227 × 10^8 ergs.
9 4114 ergs.
10 (i) 4·735 × 10^5 ergs, 97·29 cms./sec., (ii) 3·542 × 10^5 ergs, 84·14 cms./sec.
12 1056 cub. ft. 13 616 H.P.

CHAPTER II.—**Hydrostatics**

SECTION 11

2 102·5. 3 12486. 4 11·09. 5 60°.

SECTION 12

1 12·10 metres. 2 9·97 lbs./sq. in. 4 1800 ft. 5 106.
6 7·99 kms. 7 53390.

SECTION 13

1 72·81 cms. 2 150·6 cms.; 57·24 cms. 3 11 : 1 neglecting thickness of piston.
5 8·15 cms. 6 30·7 inches. 7 74·1 cms. 8 20·9 cms.
10 4·97 : 1. 11 ·543 gm.
12 2·5 times the original density, the latter being supposed atmospheric.

SECTION 14

2 10·24 ft. 3 25 tons. 4 3·6 tons; ·087 inch.

SECTION 15

1 22·610 gms. 2 (i) 67 tons, (ii) 43 tons. 3 208 kgs. 5 515·8 kgs.
6 32·694 gms. 8 11·40. 9 8·934. 10 1·740,
11 1·069. 13 0·909. 14 ·6784. 15 8·97.
16 ·653. 17 8·38. 18 1·056. 19 1·088.
20 11·35. 21 7·59. 23 3·422 kgs.
24 5·5 cms. below surface of the oil. 25 Height of barometer is unaltered.
26 9196 cubic yds. 28 1·0252; 1·0254.
29 ·9874 gm./c.c.

CHAPTER III.—**Heat**

SECTION 16

2 (i) 24·8°, 71·6°, 190·4°, 233·6° F., (ii) −3·2°, 17·6°, 70·4°, 89·6° R.
3 −30°, −14·4°, 8·3°, 44·4° C. 4 (i) −40° C., (ii) −25·6° R.
5 (i) 98·10° C., (ii) 98·79° C., (iii) 100·91° C.
7 (i) About 95° C., (ii) about 77° C. using the rule 1 inch for 900 ft.; the rule is, however, not applicable for such great heights, and the correct value for Mont Blanc is 84° C. 8 14·6° C.; 54·1° C. 9 1·000 : 1·089.

SECTION 17

2 174° C.; 150·488 cms. 3 59·4 ft. 4 Loses 4·1 secs. per day.
5 77·94 cms. 6 123·564 sq. cms. 7 ·552.
9 ·248 c.c. 10 13·449 gms./c.c. 11 ·708 gm. ✓
13 98·7° C. 14 ·203 mm. 15 ·000486. ✓
16 134·3° C. 17 1·26 gms.

SECTION 18

2 80·5 cms. 3 ρ at bottom; ρ at top = 1·0024 : 1·0000. 4 70·7° C.
5 187 gms. 6 Volume 1024 cubic feet. Yes. 7 81 atmospheres.

SECTION 19

1 (a) 31·0° C., (b) 31·7° C. 2 78·4. 3 82·7. 4 ·0325.
5 518. 6 ·0668. 8 2·0 gms. 9 246·9° C.
10 ·463. 12 About ·075 lb. 13 22·8° C. 14 29·2° C.
15 28·8° C. 16 997·6 cals. 18 543·9 and 538·5.
19 252.

SECTION 20

5 62·8 per cent.; ·00876 gm./litre. 6 11·8° C.; ·01045 gm./litre.
7 11·0 per cent. 9 57·6 per cent. 10 4661. 11 66·3 per cent.
13 1·04 mm.

SECTION 21

3 1·06° C. 4 ·057° C. 5 428 metres. 6 228° C.
7 357 metres/sec. 9 ·177 lb. 10 ·561° C. 11 ·525° C.
12 ·0169° C.

CHAPTER IV.—Sound

SECTION 24

1 340·3 metres/sec., 1116 ft./sec. 2 4400 ft. 3 33·9 secs.
5 3·92 mls./hr. 6 1774 ft. 7 60·6 ft.
8 3300 ft. 10 334·8 metres/sec.

SECTION 25

1 102·4. 2 8·88. 3 27·4, 3512. 4 1241 cms., 9·68 cms.
6 Natural 391·5, 489·4, 522·0; equally tempered 391·1, 492·8, 522.
9 648 before, 552 after. 10 543·5 before, 463·0 after. 11 68·2 mls./hr.

SECTION 26

2 36 lbs. 3 5·406 × 10^{6} dynes. 4 335·3. 5 About 53 cms.
7 316. 8 33·4° C.

CHAPTER V.—Light

SECTION 27

1 15·6. 2 41·0.
3 57·8 and 370·3 cms. from 30 C.P. lamp measured towards other lamp.
4 2·714 (mean). 5 1·003 × 10^{17}. 7 13·56 kms. 8 5·865 × 10^{12}
9 8 mins. 20 sec. 11 3rd, 4th, 5th, and 6th, 2·993 × 10^{10} cms./sec.
12 7·5 metres. 13 215 times earth's radius; 5770 mls.
14 30·51′; 1 hr. 41 mins.; 3 hrs. 43 mins. 16 4400 mls.

SECTION 28

3 1·22 cms. 4 2·26 and 1·97 × 10^{10} cms./sec. 6 3·04 cms.
7 3·42 cms. above paper. 8 It emerges. 9 48° 36′.
10 6000 ft. 12 4·91 cms. 13 6 mls./hr.
14 Water 1·309, glass 1·482.

SECTION 29

1 (i) 20 cms., 1·33 cms. high, real, inverted; (ii) 24 cms. behind mirror, 6 cms. high, virtual, erect.

2 (i) 30 cms., 2 cms. high, real, inverted; 10 cms. behind mirror, 2 cms. high, virtual, erect.

3 (i) 8·57 cms. behind, ·57 cm. high, virtual, erect; (ii) 4·8 cms. behind, 1·2 cms. high, virtual, erect.

4 (i) 3·33 cms. behind, ·33 cm. high, virtual, erect; (ii) 2·22 cms. behind, ·55 cm. high, virtual, erect.

5 12 cms. 6 18 inches from mirror; inverted. 9 11·08 cms.

SECTION 30

1 (i) 33·33 cms., ·67 cm. high, real, inverted ; (ii) 20·0 cms. on same side, 2 cms. high, virtual, erect.

2 (i) 19·5 cms., ·39 cm. high, real, inverted ; (ii) 46·7 cms., 2·33 cms. high, real, inverted.

3 (i) 14·3 cms. on same side, ·29 cm. high, virtual, erect ; (ii) 6·66 cms. on same side, ·67 cm. high, virtual, erect.

4 (i) 10·9 cms. on same side, ·22 cm. high, virtual, erect ; (ii) 8·2 cms. on same side, ·41 cm. high, virtual, erect.

5 11·68 cms. 6 25·00 cms. 7 21·0 cms. 11 + 1·5 cms.
12 Halfway between candle and wall. 14 $\frac{4}{3}f$. 16 88·7 cms.

SECTION 31

2 12·4. 3 Image not so bright ; size of field unaltered.
4 ·49, 1·89, 4·00, 7·56, 16·00 in hundredths of a second.

SECTION 32

2 1·586. 3 38° 39′. 4 19° 20′. 5 $2·53 \times 10^{-15}$ sec., $1·33 \times 10^{-15}$ sec.

SECTION 33

1 + 1·33 diopters. 2 23·1 cms. 3 −6·67 diopters.
4 Black, if the colours are pure.

CHAPTER VI.—**Electricity and Magnetism**

SECTION 34

1 (i) 2·5 dynes ; (ii) 1·2 dynes. 2 13·9 dynes.
4 ·26 dyne in a direction making 49° 38′ with meridian.
8 (i) 5·40 c.g.s. units ; (ii) 9·35 c.g.s. units. 9 229 units.
10 105 units. 11 14·2 cms. from magnet.

SECTION 35

1 No. 2 6° 0′ E. and 24° 6′ W.
3 ·450 and ·479 c.g.s. units. 4 It balances in any position.

SECTION 38

1 Raised. 2 1136 ohms. 3 8·33 and 11·67 ohms.
4 130·1 metres. 7 105·25 ohms. 8 1·39 times.
9 3·08 : 1. 10 1·000 ohm. 11 6·67 ohms.
12 138·2 ohms.

SECTION 39

2 ·149 c.g.s., 41·2°. 3 3 : 4. 4 70·5 ohms. 6 $\frac{1}{99}$ ohm.
7 77 ohms. 8 12·24 ohms. 9 3·286 ohms. 10 20 cms.

SECTION 40

1 Clockwise.

SECTION 41

1 29 mins., 49 secs. 3 ·03944 gm. 4 2·68. 5 ·1009 gm.

SECTION 42

1 ·0446 amp., ·892 volt. 2 51° 3′. 3 ·0521 amp., ·0065 amp.
4 1·5 and 2·3 ohms. 6 4 : 7. 8 103.
10 52° C. 11 1·46° C. 12 ·0131.
13 1·46 watts, 511 cells. 14 ·0364. 15 5·28*d*. 16 ·18*d*.

SECTION 43

1 (i) Counter-clockwise, ii) clockwise. 4 13.

CONSTANTS

π = 3·14159.
1 foot = 30·48 centimetres.
1 lb. = 453·6 grams.
Earth's equatorial radius = 3962 miles.
Sidereal day = 86164 seconds.
J = 42 × 10^6 ergs per calorie.

g at equator = 978·0 cms./sec.^{2}
„ Greenwich = 981·18 „
„ Glasgow = 981·56 „
„ north pole = 983·2 „
1 Horse-power = 33,000 ft.-lbs./min.
1 Kilowatt = 1·34 horse-power.

1 litre of dry air weighs 1·293 grams at 0° C., 76 cms.
Velocity of sound in air is 331·3 metres/sec. or 1087 ft./sec. at 0° C.
Velocity of light is 3·00 × 10^{10} cms./sec. = 186,000 miles/sec.
Index of refraction of crown glass 1·52, of water 1·33, of air under standard conditions 1·0003.
Electro-chemical equivalent of silver (number of grams deposited by 1 coulomb) = ·001118.

Substance.	Specific gravity.	Coeff. of expansion.	Specific heat.	Specific resistance.
				ohm—cms.
Aluminium	2·65	25·5 × 10^{-6}	·21	3 × 10^{-6}
Brass	8·4 to 8·7	18·7 „	·090	6 to 9
Copper	8·93	16·7 „	·093	1·7
Iron	7·86	11 „	·119	9 to 15
Lead	11·37	27·6 „	·030	
Platinum	21·50	8·9 „	·032	11·0
Steel	7·7 to 7·9	11 „		
Tin	7·29	21·4 „	·055	
Glass	About 2·5	8 „	·12 to ·2	10^{11} to 10^{14}
Mercury	13·6	182 „ (cubical)	·033	95 × 10^{-6}
Methyl alcohol	·80		·60	
Turpentine	·87		·42	
Ice	·916		·50	

1 cubic foot of water weighs 62·43 lbs. Specific gravity of paraffin wax ·91, of cork ·25, of sea-water 1·01 to 1·05.

Saturation pressure of aqueous vapour on pp. 73 and 96. Wave-lengths of light from frontispiece. Standard pressure on p. 50.

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